

## 2-6

Formalizing Relations  
and Functions

**F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns each element of the domain exactly one element of the range . . . Also F.IF.2, F.IF.5

**Objectives** To determine whether a relation is a function  
To find domain and range and use function notation



**Solve It!** Write your solution to the Solve It in the space below.

A **relation** is a pairing of numbers in one set, called the **domain**, with numbers in another set, called the **range**. A relation is often represented as a set of ordered pairs  $(x, y)$ . In this case, the domain is the set of  $x$ -values and the range is the set of  $y$ -values.

**Essential Understanding** A function is a special type of relation in which each value in the domain is paired with exactly one value in the range.



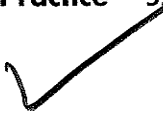
### Problem 1 Identifying Functions Using Mapping Diagrams


**Got It?** Identify the domain and range of each relation. Represent the relation with a mapping diagram. Is the relation a function?

✓ a.  $\{(4.2, 1.5), (5, 2.2), (7, 4.8), (4.2, 0)\}$

**When is a relation *not* a function?**


✓ b.  $\{(-1, 1), (-2, 2), (4, -4), (7, -7)\}$

 **A Practice** 5. **Shopping** You are buying orange juice for \$4.50 per container and have a gift card worth \$7. The function  $f(x) = 4.50x - 7$  represents your total cost  $f(x)$  if you buy  $x$  containers of orange juice and use the gift card. How much do you pay to buy 4 containers of orange juice?

 **STEM** 6. **Physics** Light travels about 186,000 mi/s. The function  $d(t) = 186,000t$  gives the distance  $d(t)$ , in miles, that light travels in  $t$  seconds. How far does light travel in 30 s?



**Problem 4** Finding the Range of a Function

 **Got It?** The domain of  $g(x) = 4x - 12$  is  $\{1, 3, 5, 7\}$ . What is the range?

**A Practice** Find the range of each function for the given domain.

7.  $h(x) = x^2; \{-1.2, 0, 0.2, 1.2, 4\}$

8.  $f(x) = 8x - 3; \left\{-\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}\right\}$



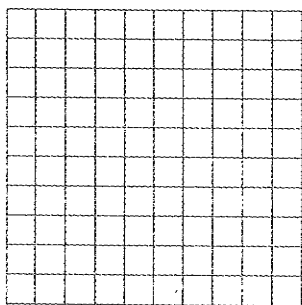
**Problem 5** Identifying a Reasonable Domain and Range

**Got It?** a. What domain and range are reasonable if you have 7 qt of paint instead of 3 qt in Problem 5?

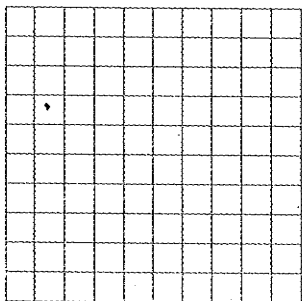
© b. **Reasoning** Why does it *not* make sense to have domain values less than 0 or greater than 3 in Problem 5?

**A Practice** Find a reasonable domain and range for each function. Then graph the function.

9. **Fuel** A car can travel 32 mi for each gallon of gasoline. The function  $d(x) = 32x$  represents the distance  $d(x)$ , in miles, that the car can travel with  $x$  gallons of gasoline. The car's fuel tank holds 17 gal.



10. **Nutrition** There are 98 International Units (IUs) of vitamin D in 1 cup of milk. The function  $V(c) = 98c$  represents the amount  $V(c)$  of vitamin D, in IUs, you get from  $c$  cups of milk. You have a 16-cup jug of milk.

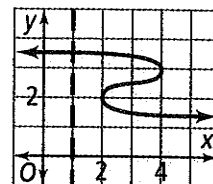


## Lesson Check

### Do you know HOW?

11. Identify the domain and range of the relation  $\{(-2, 3), (-1, 4), (0, 5), (1, 6)\}$ . Represent the relation with a mapping diagram. Is the relation a function?

- ✓ © 17. **Error Analysis** A student drew the dashed line on the graph shown and concluded that the graph represented a function. Is the student correct? Explain.



## More Practice and Problem-Solving Exercises



### B Apply

Determine whether the relation represented by each table is a function. If the relation is a function, state the domain and range.

18.

$x$	0	3	3	5
$y$	2	1	-1	3

19.

$x$	-4	-1	0	3
$y$	-4	-4	-4	-4

- ✓ © 20. **Open-Ended** Make a table that represents a relation that is not a function. Explain why the relation is not a function.

✓ © 21. **Reasoning** If  $f(x) = 6x - 4$  and  $f(a) = 26$ , what is the value of  $a$ ? Explain.

✓ © 22. **Think About a Plan** In a factory, a certain machine needs 10 min to warm up. It takes 15 min for the machine to run a cycle. The machine can operate for as long as 6 h per day including warm-up time. Draw a graph showing the total time the machine operates during 1 day as a function of the number of cycles it runs.

- What domain and range are reasonable?
- Is the function a linear function?

23. **Carwash** A theater group is having a carwash fundraiser. The group can only spend \$34 on soap, which is enough to wash 40 cars. Each car is charged \$5.

- If  $c$  is the total number of cars washed and  $p$  is the profit, which is the independent variable and which is the dependent variable?
- Is the relationship between  $c$  and  $p$  a function? Explain.
- Write an equation that shows this relationship.
- Find a reasonable domain and range for the situation.

✓ © 24. **Open-Ended** What value of  $x$  makes the relation  $\{(1, 5), (x, 8), (-7, 9)\}$  a function?

Determine whether each relation is a function. Assume that each different variable has a different value.

25.  $\{(a, b), (b, a), (c, c), (e, d)\}$

26.  $\{(b, b), (c, d), (d, c), (c, a)\}$

27.  $\{(c, e), (c, d), (c, b)\}$

28.  $\{(a, b), (b, c), (c, d), (d, e)\}$

- © 29. **Reasoning** Can the graph of a function be a horizontal line? A vertical line? Explain.

### Challenge

30. To form the inverse of a relation written as a set of ordered pairs, you switch the coordinates of each ordered pair. For example, the inverse of the relation  $\{(1, 8), (3, 5), (7, 9)\}$  is  $\{(8, 1), (5, 3), (9, 7)\}$ . Give an example of a relation that is a function, but whose inverse is *not* a function.

Use the functions  $f(x) = 2x$  and  $g(x) = x^2 + 1$  to find the value of each expression.

31.  $f(3) + g(4)$

32.  $g(3) + f(4)$

33.  $f(5) - 2 \cdot g(1)$

34.  $f(g(3))$