HL2 Math

REVIEW LESSON – COUNTING PRINCIPLE, PROBABILITY & DIFFERENTIAL EQUATIONS

REVIEW SET 1 – CHAP 14.3

1. Five different coloured flags can be run up a mast.
(a) How many different signals can be produced if all five flags are used?
(b) How many different signals can be produced if any number of flags is used?

2. In how many different ways can 7 books be arranged in a row?

3. In how many different ways can three boys and four girls be seated in a row?
In how many ways can this be done if
(a) no two girls are sitting next to each other,
(b) the ends are occupied by girls?

4. In how many different ways can 7 books be arranged in a row if
(a) 3 specifed books must be together,
(b) two specified books must occupy the ends.

5. A school council consists of 12 members, 6 of whom are parents, 2 are students, the Principal and the remainder are teachers. The school captain and vice–captain must be on the council. If there are 10 parents and 8 teachers nominated for positions on the school council, how many different committees can there be?

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11. A committee of 4 is to be selected from 7 men and 6 women. In how many ways can this be done if
(a) there are no restrictions?
(b) there must be an equal number of men and women on the committee?
(c) there must be at least one member of each sex on the committee?

12. Prove that (a) $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$. (b) ${}^{n+1}P_r = {}^nP_r + r \times {}^nP_{r-1}$.

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15. (a) Show that $2^n = \sum_{r=0}^{n} \binom{n}{r}$.
(b) In how many ways can 8 boys be divided into two unequal sets?

16. Whilst at the library, Patrick decides to select 5 books from a group of 10. In how many different ways can Patrick make the selection?

17. A fish tank contains 5 gold coloured tropical fish and 8 black coloured tropical fish.
(a) In how many ways can five fish be selected?
(b) If a total of 5 fish have been selected from the tank, how many of these contain two gold fish?

18. In how many ways can 4 people be accommodated if there are 4 rooms available?

19. A car can hold 3 people in the front seat and 4 in the back seat. In how many ways can 7 people be seated in the car if John and Samantha must sit in the back seat and there is only one driver?

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24. In how many ways can 5 maths books, 4 physics books and 3 biology books be arranged on a shelf if subjects are kept together?

25. How many even numbers of 4 digits can be formed using 5, 6, 7, 8 if
(a) no figure is repeated?
(b) repetition is allowed?

26. 5 men and 5 women are to be seated around a circular table. In how many ways can this be done if the men & women alternate?

27. A class of 20 students contains 5 student representatives. A committee of 8 is to be formed. How many different committees can be formed if there are
(a) only 3 student representatives?
(b) at least 3 student representatives?

REVIEW SET 2 – CHAP 15.3

7. A money box contains 10 discs, 5 of which are yellow, 3 of which are black and 2 green. Two discs are selected in succession, with the first disc not replaced before the second is selected.
(a) Draw a tree diagram representing this process.
(b) Hence find the probability that the discs will be of a different colour.
(c) Given that the second disc was black, what is the probability that both were black?

8. Two dice are rolled. Find the probability that the faces are different given that the dice show a sum of 10.

9. Given that $p(A) = 0.6$, $p(B) = 0.7$ and that A and B are independent events.
Find the probability of the event
(a) $A \cup B$ (b) $A \cap B$ (c) $A|B'$ (d) $A' \cap B$

10. The probability that an animal will still be alive in 12 years is 0.55 and the probability that its mate will still be alive in 12 years is 0.60.
Find the probability that
(a) both will still be alive in 12 years.
(b) only the mate will still be alive in 12 years.
(c) at least one of them will still be alive in 12 years.
(d) the mate is still alive in 12 years given that only one is still alive in 12 years.

REVIEW SET 2 – 15.3

12. The probability that Rory finishes a race is 0.55 and the probability that Millicent finishes the same race is 0.6. Because of team spirit, there is an 80% chance that Millicent will finish the race if Rory finishes the race.
Find the probability that
(a) both will finish the race.
(b) Rory finishes the race given that Millicent finishes.

13. If A and B are independent events, show that their complementary events are also independent events.

14. A student runs the 100 m, 200 m and 400 m races at the school athletics day. He has an 80% chance of winning any one given race. Find the probability that he will
(a) win all 3 races.
(b) win the first and last race only.
(c) win the second race given that he wins at least two races.

15. Dale and Kritt are trying to solve a physics problem. The chances of solving the problem are Dale—65% and Kritt— 75%. Find the probability that
(a) only Kritt solves the problem.
(b) Kritt solves the problem.
(c) both solve the problem.
(d) Dale solves the problem given that the problem was solved.

CONSOLIDATION SET 1 – CHAP 15.5

1. Five red cubes and 4 blue cubes are placed at random in a row. Find the probability that
(a) the red cubes are together.
(b) both end cubes are red.
(c) the cubes alternate in colour.

2. Five books of different heights are arranged in a row. Find the probability that
(a) the tallest book is at the right end.
(b) the tallest and shortest books occupy the end positions.
(c) the tallest and shortest books are together.
(d) the tallest and shortest books are never next to each other.

3. Three cards are dealt from a pack of 52 playing cards. Find the probability that
(a) 2 of the cards are kings.
(b) all three cards are aces.
(c) all three cards are aces given that at least one card is an ace.

CONSOLIDATION SET 1 – CHAP

8. Three red cubes, four blue cubes and six yellow cubes are arranged in a row. Find the probability that
(a) the cubes at each end are the same colour.
(b) the cubes at each end are of a different colour.

9. A sample of three light bulbs is selected from a box containing 15 light bulbs. It is known that five of the light bulbs in the box are defective.
(a) Find the probability that the sample contains a defective.
(b) Find the probability that the sample contains at least two defectives.

10. Eight people of different heights are to be seated in a row. What is the probability that
(a) the tallest and shortest persons are sitting next to each other?
(b) the tallest and shortest occupy the end positions?
(c) there are at least three people sitting between the tallest and shortest?

11. Eight people of different heights are to be seated in a row. The shortest and tallest in this group are not seated at either end. What is the probability that
(a) the tallest and shortest persons are sitting next to each other?
(b) there is one person sitting between the tallest and shortest?

12. A committee of four is to be selected from a group of five boys and three girls. Find the probability that the committee consists of exactly two girls given that it contains at least one girl.

REVIEW SET 3 – CHAP 24.2

2. Solve the following differential equations

(a) $\sqrt{t}\frac{dy}{dt} - t^2 = 0, y = 1, t = 1$
(b) $\frac{dx}{dt} = 4\tan 2t, x(0) = 1$
(c) $\frac{dy}{dx} = \frac{2x}{(x-1)(x+1)}, y(2) = 4$
(d) $(1-x)^2\frac{dy}{dx} + 1 = 0, y(2) = \frac{4}{3}$
(e) $\frac{dv}{dt} = \frac{k}{(t+t_0)^3}, v(0) = u$
(f) $\frac{dy}{dx} = x\cos(x^2), x = 0, y = 0$
(g) $\frac{1}{\cos t}\left(\frac{dx}{dt}\right) = 4\sin^3 t, x(0) = 1$
(h) $\frac{dN}{dt} = t\sqrt{25+t^2}, N = 5, t = 0$
(i) $\frac{dT}{d\theta} = \sin 2\theta\sqrt{8+\cos 2\theta}, T(0) = 0$
(j) $\frac{dP}{dt} = te^{-3t}, P(0) = 1$

REVIEW SET 3 – CHAP 24.2

5. Find the general solution of the differential equations

(a) $\frac{dy}{dx} = \frac{y}{x+1}$
(b) $\frac{dy}{dx} = x(y+1)$
(c) $\frac{dy}{dx} = \frac{x}{y+1}$
(d) $\frac{dy}{dx} = \frac{\sin x}{e^y}$
(e) $\frac{dy}{dx} = \frac{x\sin x}{e^y}$
(f) $\frac{dy}{dx} = \frac{x\sin x}{ye^y}$
(g) $x\sqrt{1-y^2}\frac{dx}{dy} = 1$
(h) $x\sqrt{1-y^2}\frac{dy}{dx} = 1$
(i) $\frac{\sqrt{1-y^2}}{x}\frac{dy}{dx} = 1$

REVIEW SET 3 – CHAP 24.2

6. Solve the following differential equations

(a) $y(4+x^2)\frac{dy}{dx} = 1, x = 0, y = 1$

(c) $\frac{dy}{dx} = xe^{y-x}, x = 0, y = 0$

REVIEW SET 3 – CHAP 24.2

11. Solve the differential equation $(1+y)\frac{dy}{dx} + \sin^2 x = 1, x = 0, y = 0$.

12. By using the substitution $u = x + y$, show that the differential equation $\frac{dy}{dx} = x + y$ can be reduced to the d.e. $\frac{du}{dx} = u + 1$.

Hence, show that the general solution is given by $x + y + 1 = ke^x$.

13. Using the substitution $u = x + y + 1$, show that the differential equation $\frac{dy}{dx} = \frac{1}{x+y+1}$ can be reduced to the d.e. $\frac{du}{dx} - 1 = \frac{1}{u}$.

Hence, show that the general solution is given by $x + y + 2 = ke^y$.

REVIEW SET 3 – CHAP 24.2

14. By using the substitution $y = ux$, show that the differential equation $x\frac{dy}{dx} = y^2 + x^2 + y$ can be reduced to the d.e. $\frac{du}{dx} = u^2 + 1$.

Hence, show that the general solution is given by $y = x\tan(x + c)$.

15. By using the substitution $y = xu$, show that the differential equation $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}, x > 0$ can be reduced to the d.e. $x\frac{du}{dx} = \sqrt{u^2 + 1}$.

Hence, show that if the curve passes through the point $(1, 0)$, the particular solution is given by $y = \frac{1}{2}(x^2 - 1)$.