LESSON 71 – Testing for Covergence

HL Math –Santowski

OBJECTIVES

- (a) How do we estimating Truncation Errors & how do we make use of that idea?
- (b) Deciding on which method to use when testing series for convergence……

EXPLORATION ….. TRUNCATION ERRORS

- Let's work with the known series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$
- Determine the EXACT value of this sum
  (use wolframalpha please!) ➔
- Then write out the value approximated the sum, correct to 9 decimal places
- ➔

EXPLORATION ….. TRUNCATION ERRORS

- Let's work with the known series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$
- Determine the EXACT value of this sum
  (use wolframalpha please!) ➔ $\frac{\pi^2}{12}$
- Then write out the value approximated the sum, correct to 9 decimal places
- ➔ 0.822467033……

EXPLORATION ….. TRUNCATION ERRORS

- Let's work with the known series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$
- Use wolframalpha to determine the following partial sums (round answers again to 9 decimal places – so we have estimated the sum of the series)

$\sum_{n=1}^{5} \frac{(-1)^{n+1}}{n^2}$	$\sum_{n=1}^{10} \frac{(-1)^{n+1}}{n^2}$	$\sum_{n=1}^{15} \frac{(-1)^{n+1}}{n^2}$	$\sum_{n=1}^{20} \frac{(-1)^{n+1}}{n^2}$	$\sum_{n=1}^{25} \frac{(-1)^{n+1}}{n^2}$

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EXPLORATION ….. TRUNCATION ERRORS

- Now let's find the ***truncation errors***, $R_n = |S_\infty - S_n|$ when we do these approximations

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- Now, let's find u_{n+1}

$u_{n+1} = \frac{1}{36}$ .027777777	$u_{n+1} = \frac{1}{121}$ .008264462	$u_{n+1} = \frac{1}{216}$ .00390625	$u_{n+1} = \frac{1}{441}$ .002267573	$u_{n+1} = \frac{1}{576}$ .001479289

EXPLORATION ..... TRUNCATION ERRORS

- Now, what ***general observation*** do we make about the truncation error and u_{n+1}

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EXPLORATION ..... TRUNCATION ERRORS

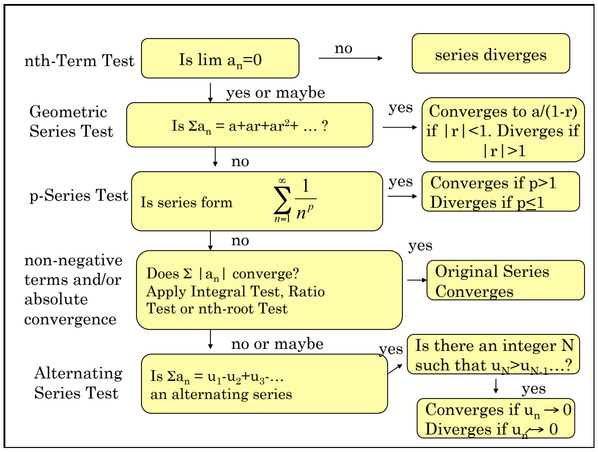
- Now, what ***general observation*** do we make about the truncation error and u_{n+1} ?
- In general, $|u_{n+1}| > |S_\infty - S_n|$
- WHY?
- & how do we make use of this "mathemagics"?

EXAMPLES

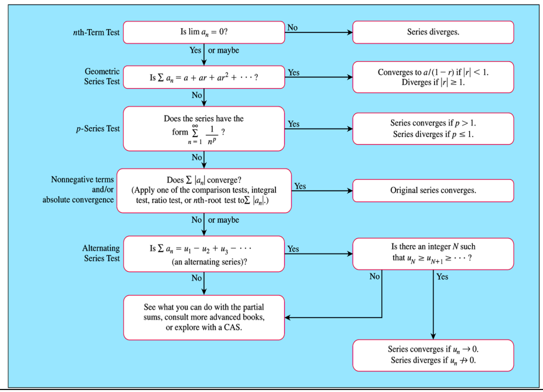
- EX 1. What is the least number of terms in the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$ such that the error in the estimate is smaller than 10^{-6}?
- EX 2. How many terms of the series below must be taken in order for the error to be smaller than 10^{-8}?

$$S_n = \frac{1}{2} - \frac{1}{2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} - \frac{1}{2\cdot 3\cdot 4\cdot 5} + ...$$

TESTS TO USE ....



PROCEDURE FOR DETERMINING CONVERGENCE



EXAMPLES

- See handout