

$$
\begin{aligned}
& \text { Review } \\
& \text { We have seen a definition/formula for a } \\
& \text { definite integral as } \\
& A(x)=\lim _{n \rightarrow \infty} \sum_{i=1}^{n} f\left(x_{i}\right) \Delta x=\int_{a}^{b} f(x) d x=\left.F(x)\right|_{a} ^{b}=F(b)-F(a) \\
& \text { where } \mathrm{F}(\mathrm{x})=\mathrm{f}(\mathrm{x}) \text { (or the antiderivative } \\
& \text { of } \mathrm{f}(\mathrm{x}) \\
& \text { And have seen an interpretation of the } \\
& \text { definite integral as a "net/total change" }
\end{aligned}
$$

FTC, PART 1

- Some textbooks/resources refer to this statement as the Integral Evaluation Theorem $\Rightarrow$ as it tells us HOW to evaluate the definite integral

$$
\int_{a}^{b} f(x) d x=\left.F(x)\right|_{a} ^{b}=F(b)-F(a)
$$

by finding antiderivatives


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$$
\begin{aligned}
& \text { A Graphic Representation of the Area } \\
& \text { Function } \\
& \text { Let's work with the area under the curve } \mathrm{f}(\mathrm{t})=\mathrm{t}^{2} \text {, } \\
& \text { starting from } \mathrm{x}=0 \text { so our integral will be } A(x)=\int_{0}^{? ?} t^{2} d t \\
& A(x)=\int_{0}^{1} t^{2} d t=1 / 3 \\
& \text { Now, lets set the upper limit to } \mathrm{x}=1 \text { and we get } \\
& 6
\end{aligned}
$$

## A Graphic Representation of the Area Function

- Now, let's set the upper limit to $x=2$ and we get

$$
A(x)=\int_{0}^{2} t^{2} d t=8 / 3
$$



- Now, let's set the upper limit to $x=3$ and we get

$$
A(x)=\int_{0}^{3} t^{2} d t=27 / 3
$$


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## A Graphic Representation of the Area

 Function- Now, let's set the upper limit to $x=4$ and we get

$$
A(x)=\int_{0}^{4} t^{2} d t=64 / 3
$$



- Now, let's set the upper limit to $x=5$ and we get


A Graphic Representation from Before

- Now that we have seen a few calculations and some "data points", let's see what we really have ???

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Area | 0 | $1 / 3$ | $8 / 3$ | $27 / 3$ | $64 / 3$ | $125 / 3$ |

- We have the function

$$
A(x)=\frac{1}{3} x^{3}
$$



FTC, Part 2

- So what does $g(x)=\int_{a}^{x} f(t) d t$ REALLY mean??
- Try this as a simple contrast $\rightarrow$ we have looked as integration as a process that generates fixed numbers (that correspond to net areas under curves)
- Now, with a different notion $\rightarrow$ the value of the upper limit is variable, we are now considering $\quad g(x)=\int f(t) d t$ as a FUNCTION, not a single number
- What does the function represent $\rightarrow$ the antiderivative of $y=f(t)$
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FTC, Part 2

- So here's the shift in our understanding $\boldsymbol{\rightarrow}$ the expression

$$
g(x)=\int_{a}^{x} f(t) d t
$$

- IS AN EXPRESSION for the antiderivative of $y=f(t)$ which graphically could be understood as a cumulative AREA function


## A Clarifying Example (I Hope)

- Let's go back to $y=f(t)=t^{2}$ between $a=0$ and $b=5$
- So we have worked out the integral $\int_{0}^{5} t^{2} d t=\left.\frac{1}{3} t^{3}\right|_{0} ^{5}=\frac{125}{3}$
- So now as the value of $b$ changes, we get

$$
\int_{0}^{x} t^{2} d t=\left.\frac{1}{3} t^{3}\right|_{0} ^{x}=\frac{1}{3} x^{3}+0=\frac{1}{3} x^{3}
$$

- Where the antiderivative $\left(1 / 3 x^{3}\right)$ can be used to evaluate for the area under the curve
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A Clarifying Example (I Hope)

- So we need to recognize the equivalence of the following 2 statements:

$$
A(x)=\frac{1}{3} x^{3} \text { and } A(x)=\int_{0}^{x} t^{2} d t
$$

- In one statement, the antiderivative is given as a simple, explicit function, in the other, the antiderivative function is defined as an integral of $f(t)$
- To become consistent in our notations, I will switch the statements to

$$
g(x)=\frac{1}{3} x^{3} \text { and } g(x)=\int_{0}^{x} t^{2} d t
$$

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## FTC, Part 2

- If $f$ is continuous on $[\mathrm{a}, \mathrm{b}]$ then the fon $F(x)=\int^{x} f(t) d t$ has a derivative at every point in $[\mathrm{a}, \mathrm{b}]$ and

$$
\frac{d}{d x} F(x)=\frac{d}{d x} \int_{a}^{x} f(t) d t=f(x)
$$

- So what does this really mean $\Rightarrow$ Every continuous
function $f(x)$ HAS an antiderivative (which simply happens
to be expressed as an integral as: $\left.\quad \int^{x} f(t) d t\right)$ rather than explicitly in terms of elementary functions
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## Using the FTC, Part 2

- Find the derivative of the following functions

$$
\begin{aligned}
& g(x)=\int_{0}^{x} \sin t d t \\
& g(x)=\int_{2}^{x} \sin t d t \\
& g(x)=\int^{\pi} \sin t d t
\end{aligned}
$$

- For all real numbers of x , define $F(x)=\int_{0} \sin (\pi t) d t$

Evaluate $F^{\prime}\left(\frac{3}{4}\right)$ and $F^{\prime}\left(-\frac{1}{2}\right)$ and interpret

## Using the FTC, Part 2

- Find the derivative of the following functions

$$
\begin{aligned}
& g(x)=\int_{1}^{x}\left(\sqrt{t^{3}+1}\right) d t \\
& g(x)=\int_{0}^{3 x^{2}} \sin t d t \\
& g(x)=\int_{1}^{\sin x}\left(\sqrt{t^{3}+1}\right) d t
\end{aligned}
$$

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## Using the FTC, Part 2

| - Find dy/dx if: | Find a function, $\mathrm{y}=\mathrm{f}(\mathrm{x})$ <br> with a derivative of |
| :--- | :--- |
| $y=\int_{1}^{x^{2}} \cos t d t$ | $\frac{d y}{d x}=\tan x$ |
| $y=\int_{x}^{5} 3 t \sin t d t$ | that satisfies the condition <br> $f(3)=5$ |
| $y=\int_{2 x}^{x^{2}} \frac{1}{2+e^{t}} d t$ |  |

Using the FTC

- Given the function $f$ be as shown and let $g(x)=\int_{0} f(t) d t$
- (a) Evaluate $g(0), g(2), g(4), g(7)$ and $g(9)$ and $g(I I)$
p (b) On what intervals is $g$ increasing!
- (c) Where does $g$ have a maximum value?
- (d) Sketch a rough graph of $\mathrm{g}(\mathrm{x})$


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| Using the FTC, Part 2 |
| :--- |
| Graph the function defined by $\quad F(x)=\int_{0}^{x} e^{-t^{2}} d t$ |
| Address the following in your solution: |
| (i) determine and discuss $\mathrm{F}^{\prime}(\mathrm{x})$ |
| (ii) determine and discuss F "(x) |
| (iii) find symmetry of $\mathrm{F}(\mathrm{x})$ |
| (iv) estimate some points using trapezoid sums |

Using the FTC, Part 2

- We can use the methods of differential calculus to analyze these functions!
- Find the interval on which the curve $y(x)=\int_{0}^{x} \frac{1}{1+t+t^{2}} d t$
, is concave up
- Find the interval on which the curve $y(x)=\int_{0}^{x} \frac{1}{1+t+t^{2}} d t$ is increasing

Using the FTC, Part 2
FTC, Part 2

- What is the advantage of defining an antiderivative as an integral? => we can then simply use our numerical integration methods (RRAM, LRAM etc) to estimate values
- (a) Find all critical points of F
- (b) Determine the interval on which $F$ increase and $F$ decreases
- (c) Determine the intervals of concavity and inflection points of $F$
- (d) Sketch a graph of $F$

