

Lesson 34 – Polynomial Function Analysis using Derivatives

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POP QUIZ

- Given the following polynomial functions, use your TI-84 to determine:
 - (a) the co-ordinates of the extrema
 - (b) the intervals of increase & decrease

$$(a) f(x) = x^3 + 6x^2 + 9x + 2$$

$$(b) g(x) = x^4 - 4x^3 - 8x^2 - 1$$

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Lesson Objectives

- 1. Apply the power rule to real world problems
- 2. Apply the power rule to determine characteristics of polynomial function (increase/decrease & extrema)
- 3. Explain what the First Derivative Test is and why it “works”
- 4. Use Calculus methods to determine the absolute and relative extrema of a continuous & differentiable function

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(A) Examples - Applications

- A ball is dropped from the top of the Empire State building to the ground below. The height in feet, $h(t)$, of the ball above the ground is given as a function of time, t , in seconds since release by $h(t) = 1250 - 16t^2$
- (a) Determine the velocity of the ball 5 seconds after release
- (b) How fast is the ball going when it hits the ground?
- (c) what is the acceleration of the ball?

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(A) Examples - Economics

- Suppose that the total cost in hundreds of dollars of producing x thousands of barrels of oil is given by the function $C(x) = 4x^2 + 100x + 500$. Determine the following.
 - (a) the cost of producing 5000 barrels of oil
 - (b) the cost of producing 5001 barrels of oil
 - (c) the cost of producing the 5001st barrel of oil
 - (d) $C'(5000)$ = the marginal cost at a production level of 5000 barrels of oil. Interpret.
 - (e) The production level that minimizes the average cost (where $AC(x) = C(x)/x$)

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(B) Tangent Lines

- Ex 1: Find the equation of the tangent line at $x = 2$ to the function $y = x^3 + 2x^2 - 4$. Verify on TI-84
- Ex 2: Find the equation of the line which is normal/orthogonal to the curve $y = x^2 - 2x + 4$ at $x = 3$.
- Ex 3: Given the parabola $y = 2x^2 + 6x + 5$, find the point at which the line $y = -4x + b$ is tangent to the parabola. Hence, find the value of b .
- Ex 4: Given an external point $A(-4, 0)$ and a parabola $f(x) = x^2 - 2x + 4$, find the equations of the 2 tangents to $f(x)$ that pass through A

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(C) Increasing/Decreasing Functions

- A function is said to be increasing if
- A function is said to be decreasing if
- Q? How does CALCULUS/DERIVATIVES help us determine where a function is increasing/decreasing?

(C) Increasing/Decreasing Functions

- Test for Increasing or Decreasing Functions
- 1. If $f'(x) > 0$ for all x in an interval, I , then $f(x)$ is increasing on that interval
- 2. If $f'(x) < 0$ for all x in an interval, I , then $f(x)$ is decreasing on that interval
- Q? How can we determine that INTERVAL?

(C) Analyzing Functions - Extrema

- Define the term "extrema" as it relates to functions (like polynomial functions)
- Q → How does CALCULUS/DERIVATIVES help us determine where a function has extrema?
- Ex 1: At which x -values does the function $f(x) = x^4 - 4x^3$ have extrema?
- Ex 2: Determine the co-ordinates of the extrema of the function $g(x) = x^3 - 16x^2$

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Classifying Extremas – First Derivative Test

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(A) First Derivative Test

- Let f be a differentiable function with $f'(c) = 0$ then:
- 1. If $f'(x)$ changes from positive to negative, then f has a relative maximum at c .
- 2. If $f'(x)$ changes from negative to positive, then f has a relative minimum at c .
- 3. If $f'(x)$ does not change sign at c , then f has neither an maximum or minimum at c . (Stationary point)

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(A) First Derivative Test

- Your task: Write an explanation/clarification/rationalization of the FDT. Include diagrams in your explanation

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(A) First Derivative Test

- Ex 1. Use the FDT to classify all extrema of the function $f(x) = x^4 - 4x^3 + 4x^2$
- Ex 2. Find and classify all extrema using the FDT given the function $f(x) = 3x^5 - 25x^3 + 60x$.

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More on Extreme Values

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(A) Terms

- Given a function, $f(x)$, that is defined on a given interval and let c be a number in the domain
- $f(c)$ is the ABSOLUTE or GLOBAL maximum of $f(x)$ on the interval if $f(c) \geq f(x)$ for every x in the interval
- Now, sketch an example of what has just been described.

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(A) Terms

- Given a function, $f(x)$, that is defined on a given interval and let c be a number in the domain
- $f(c)$ is the ABSOLUTE or GLOBAL minimum of $f(x)$ on the interval if $f(c) \leq f(x)$ for every x in the interval
- Now, sketch an example of what has just been described.

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(B) Extreme Value Theorem

- A function, $f(x)$, that is CONTINUOUS on a CLOSED interval $[a,b]$ will have BOTH an absolute maximum value and an absolute minimum value on the closed interval
- Now, sketch an example of what has just been described.

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(C) Terms – Relative or Local

- We say that $f(x)$ has a relative (or local) maximum at $x = c$ if $f(x) \leq f(c)$ for every x in some open interval around $x = c$.
- Now, sketch an example of what has just been described.
- We say that $f(x)$ has a relative (or local) minimum at $x = c$ if $f(x) \geq f(c)$ for every x in some open interval around $x = c$.
- Now, sketch an example of what has just been described.

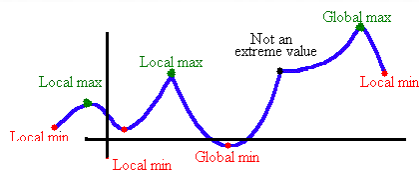
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(D) Terms - Diagram

- To understand the terms, a visualization will help:



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(E) How to “do it”

- Problem solving approach for finding absolute extrema of $f(x)$ on $[a,b]$.
- 1. Verify that the function is continuous on $[a,b]$.
- 2. Find all critical points of $f(x)$ that are in the interval $[a,b]$. This makes sense if you think about it. Since we are only interested in what the function is doing in this interval we don't care about critical points that fall outside the interval.
- 3. Evaluate the function at the critical points found in step 1 and the end points.
- 4. Identify the absolute extrema.

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(F) Examples

- Determine the absolute extrema for the following function and interval:

(i) $g(x) = 3x^4 - 4x^3 - 12x^2 + 2$

(ii) $g(x) = 2x^3 + 3x^2 - 12x + 4$ on $[-4, 2]$

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