

## Lesson Objectives

- I. Introduce piecewise functions algebraically, numerically and graphically
- 2. Define continuity and know the 3 conditions of continuity
- 3. Understand the conditions under which a function is NOT continuous on both open and closed intervals
- 4. Use algebraic, graphic, \& numeric methods to determine continuity or points of discontinuity in a function
- 5.Apply continuity to application/real world problems
> 2
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(A) Piecewise Functions
, Sketch/graph the following

$$
f(x)=\left\{\begin{array}{ll}
x+3 & x<1 \\
1-x^{2} & x \geq 1
\end{array} \quad f(x)= \begin{cases}\frac{1}{x^{2}} & x \neq 0 \\
1 & x=0\end{cases}\right.
$$

$$
f(x)= \begin{cases}\frac{x^{2}-x-2}{x-2} & x \neq 2 \\ 1 & x=2\end{cases}
$$

- 4

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## (B) Continuity

- We can understand continuity in several ways:
- (1) a continuous process is one that takes place gradually, smoothly, without interruptions or abrupt changes
- (2) a function is continuous if you can take your pencil and can trace over the graph with one uninterrupted motion
- (3) We will develop a more mathematically based definition later


## (C) Types of Discontinuities

- (1) Jump Discontinuities:
, ex $f(x)= \begin{cases}x+3 & x<1 \\ 1-x^{2} & x \geq 1\end{cases}$
- Determine the function values (from the left and from the right) at $x=1$.
- We notice our function values "jump" from 4 to 0



## (C) Types of Discontinuities <br> - (II) Infinite Discontinuities <br> - ex. $f(x)= \begin{cases}\frac{1}{x^{2}} & x \neq 0 \\ 1 & x=0\end{cases}$ <br> - determine the function values ( $\mathrm{L} \& \mathrm{R}$ ) at $x=0$. <br> - The left hand value and right hand value do not exist (both are $+\infty$ ) although the function value is I <br> > 7 <br> 

(C) Types of Discontinuities
(III) Removable
Ex ${ }_{f(x)}= \begin{cases}\frac{x^{2}-x-2}{x-2} & x \neq 2 \\ 1 & x=2\end{cases}$
Determine the function
values (L \& R) at $x=2$.
The left value and right
value are equal to 3
although the function
value is I
8

## (E) Conditions for Continuity

- a function, $f(x)$, is continuous at a given number, $x=c$, if:

Find all numbers, $x=a$, for which each function is discontinuous. For each discontinuity, state which of the three conditions are not satisfied.
(i) $f(x)=\frac{x}{(x+1)^{2}}$
(ii) $f(x)=\frac{x^{2}-9}{x-3}$
( (ii)

$$
f(x)= \begin{cases}2 x^{4}-3 x^{3}-x^{2}+x-1 & x \leq 2 \\ \frac{x^{2}+2 x-3}{x-1} & x>2\end{cases}
$$

(iv) $f(x)= \begin{cases}\frac{x^{2}+3 x-10}{x-2} & x \neq 2 \\ 7 & x=2\end{cases}$
-1/25/15
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, (i) $f(c)$ exists

- (ii) $\lim _{x \rightarrow c} f(x)$ exists
- (iii) $\lim _{x \rightarrow c} f(x)=f(c)$
- In other words, if I can evaluate a function at a given value of $x=c$ and if I can determine the value of the limit of the function at $x=c$ and if we notice that the function value is the same as the limit value, then the function is continuous at that point.
- A function is continuous over its domain if it is continuous at each point in its domain.

1/25/15
Calculus - Santowski 10

## (F) Importance of Continuities

- Continuity is important for three reasons:
- (I) Intermediate Value Theorem (IVT)
- (2) Extreme Value Theorem (EVT)
- (3) differentiability of a function at a point - for now, the basic idea of being able to draw a tangent line to a function at a given point for $x$


## (G) Internet Links for Continuity

- Calculus I (Math 2413) - Limits - Continuity from Paul Dawkins
- A great discussion plus graphs from Stefan Waner at Hofstra U $\rightarrow$ Continuity and Differentiability $\rightarrow$ then do the Continuity and Differentiability Exercises on this site
- Here are a couple of links to Visual Calculus from UTK
- General discussion plus examples and explanations: Continuous Functions
- Quiz to take on continuous functions: Continuity quiz
- And a second, different type of quiz:

Visual Calculus - Drill - Continuity of Piecewise Defined Functions

- 12

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| (I) "A" Level Work <br> • Research the Intermediate Value Theorem <br> " Tell me what it is, why it is important, what continuity has <br> to do with it and be able to use it <br> - MAX 2 page hand written report (plus graphs plus <br> algebra) +2 Q quiz |
| :--- |

## (I) "A" Level Work

- A function is defined as follows:

$$
f(x)=\left\{\begin{array}{cc}
e^{x}+a & x<-2 \\
x+2 & -2 \leq x \leq 3 \\
b+e^{x-3} & x>3
\end{array}\right.
$$

(i) Evaluate fcn value(s) if $a=1 \quad\left(\lim _{x \rightarrow-2}\right)$

- (ii) Evaluate fcn value(s) if $b=1 \quad\left(\lim _{x \rightarrow 3}\right)$
- (iii) find values for $a$ and $b$ such $f(x)$ is continuous at BOTH $x=-2$ and $x=3$
> 14 Calculus - Santowski $1 / 25 / 15$


## (I) "A" Level Work

- Define a piecewise function as $\mathrm{f}(\mathrm{x})$ where

$$
f(x)=\left\{\begin{array}{c}
x-c, \quad x<-3 \\
2-b x, \quad-3<x<1 \\
x^{3}+b x, \quad x>1
\end{array}\right.
$$

- (a) Find a relationship between $b$ and $c$ such that $f(x)$ is continuous at -3. Then give a specific numerical example of values for $b$ and $c$
- (b) Find value(s) for $b$ such that $f(x)$ is continuous at I - (c) Find values for $b$ and $c$ such that $f(x)$ is continuous on $\times \mathcal{R}$

