Lesson 29 – Sine Law & The Ambiguous Case

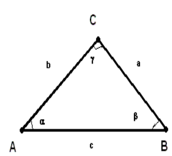
IB Math HL - Santowski

Lesson Objectives

- Understand from a geometric perspective WHY the ambiguous case exists
- Understand how to identify algebraically that their will be 2 solutions to a given sine law question
- Solve the 2 triangles in the ambiguous case
- See that the sine ratio of a acute angle is equivalent to the sine ratio of its supplement

(A) Sine Law – Scenario #1

- Let's work through 2 scenarios of solving for ∠B :
- Let ∠A = 30°, a = 3 and b = 2 (so the longer of the two given sides is opposite the given angle)



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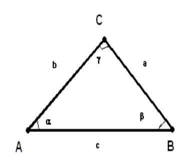

- Then sin β = b sin α / a
- And sin β = 2 sin 30 / 3
- So ∠B = 19.5°

(B) Sine Law – Scenario #2

- In our second look, let's change the measures of a and b, so that a = 2 and b = 3 (so now the shorter of the two given sides is opposite the given angle)
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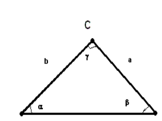

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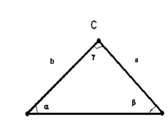
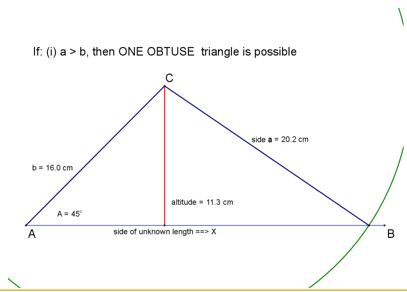


- Then sin β = b sin α / a
- And sin β = 3 sin 30 / 2
- So ∠B = 48.6°
- BUT!!!!! …….. there is a minor problem here …..

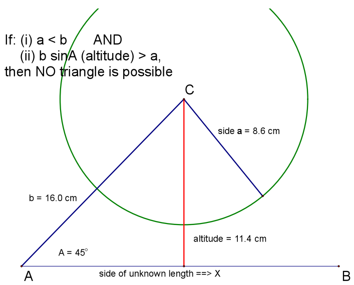
(F) Considerations with Sine Law

- If you are given information about non-right triangle and you know 2 angles and 1 side, then ONLY one triangle is possible and we never worry in these cases
- If you know 2 sides and 1 angle, then we have to consider this "ambiguous" case issue
  - If the side opposite the given angle IS THE LARGER of the 2 sides ➔ NO WORRIES
  - **If the side opposite the given angle IS THE SHORTER of the 2 sides ➔ ONLY NOW WILL WE CONSIDER THIS "ambiguous" case**
- WHY????

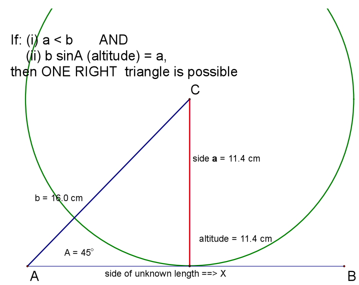
Case #1 – if a > b



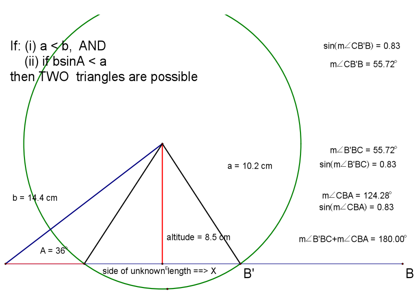
Case #2 - if a < b



Case #3 – if a < b



Case #4 – the Ambiguous Case



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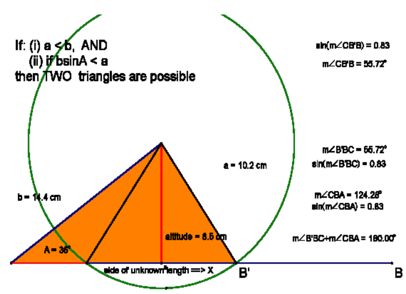

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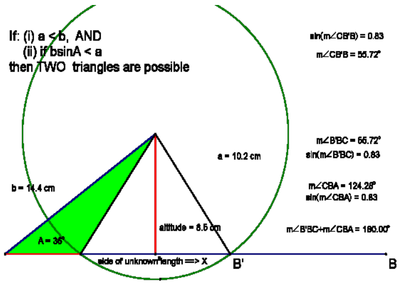
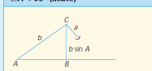
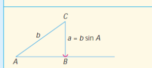
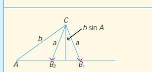
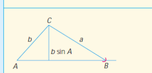


Summary

- Case 1 ➔ if we are given 2 angles and one side ➔ proceed using sine law **(ASA)**
- Case 2 ➔ if we are given 1 angle and 2 sides and the side opposite the given angle is LONGER ➔ proceed using sine law
- if we are given 1 angle and 2 sides and the side opposite the given angle is SHORTER ➔ proceed with the following "check list"
- Case 3 ➔if the product of "bsinA > a", NO triangle possible
- Case 4 ➔ if the product of "bsinA = a", ONE triangle
- Case 5 ➔ if the product of "bsinA < a" TWO triangles
- RECALL that "bsinA" represents the altitude of the triangle

Summary

∠A < 90° (acute)	Conditions	Number and Type of Triangles Possible
	$a < b \sin A$	no triangle
	$a = b \sin A$	one right triangle
	$b \sin A < a < b$	two triangles—one acute, one obtuse
	$a \geq b$	one triangle

Examples of Sine Law

- if ∠ A = 44° and ∠ B = 65° and b=7.7 find the missing information.

Examples of Sine Law

- if ∠ A =44.3° and a=11.5 and b=7.7 find the missing information.

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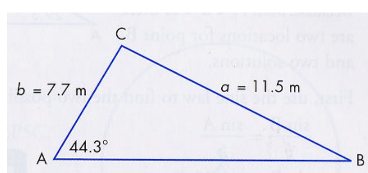
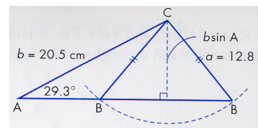


Examples of Sine Law

- if ∠ A=29.3º and a=12.8 and b = 20.5

Examples of Sine Law

- if ∠ A=29.3 and a=12.8 and b = 20.5
- All the other cases fail, because bsinA<a<b 10<a (12.8)<20.5, which is true.
- Then we have two triangles, solve for both angles



Examples of Sine Law

- ex. 1. In ΔABC, ∠ A = 42°, a = 10.2 cm and b = 8.5 cm, find the other angles
- ex. 2. Solve ΔABC if ∠ A = 37.7°, a = 30 cm, b = 42 cm

Examples of Sine Law

- ex. 1. In Δ ABC, ∠ A = 42°, a = 10.2 cm and b = 8.5 cm, find the other angles
- First test ➔side opposite the given angle is longer, so no need to consider the ambiguous case ➔ i.e. a > b ➔ therefore only one solution
- ex. 2. Solve Δ ABC if ∠ A = 37.7, a = 30 cm, b = 42 cm
- First test ➔ side opposite the given angle is shorter, so we need to consider the possibility of the "ambiguous case" ➔ a < b ➔ so there are either 0,1,2 possibilities.
- So second test is a calculation ➔ Here a (30) > b sin A (25.66), so there are two cases