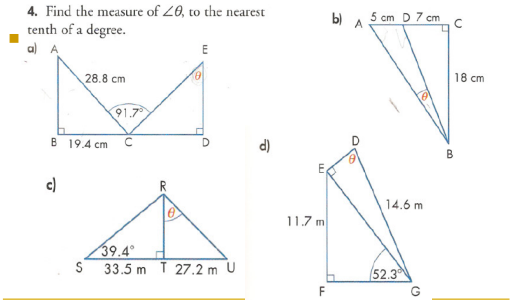
Lesson 28 - Review of Right Triangle Trig & the Sine Law & Cosine Law

IB Math HL1 – Santowski

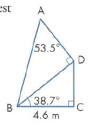
Examples – Right Triangle Trigonometry

- **4.** Find the measure of $\angle\theta$, to the nearest tenth of a degree.

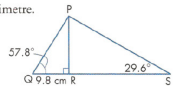


Examples – Right Triangle Trigonometry

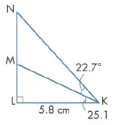
- **5.** Find AB, to the nearest tenth of a metre.



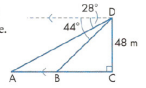
- **6.** Find RS, to the nearest tenth of a centimetre.



- **9.** Find MN, to the nearest tenth of a centimetre.



- **10.** Find AB, to the nearest metre.

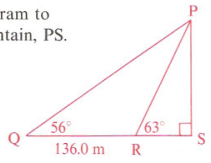


Examples – Right Triangle Trigonometry

- (6) Assuming that the Earth has a radius of 6380 km, determine the length of the 35th parallel.
- (7) To determine the width of a river, a surveyor marks a point on the bank of the river, A. Her partner is standing directly across the river from her at point C. The surveyor then walks 100 m downstream to point B, where she now has a line of sight to her partner at an angle of 58° relative to the river bank. Determine the width of the river.

Examples – Right Triangle Trigonometry

1 Use the information in the diagram to calculate the height of the mountain, PS.

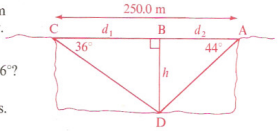


Examples – Right Triangle Trigonometry

2 Use the information in the diagram to calculate the depth of the valley.

(a) In the diagram, why is $AB + BC = h \cot 44° + h \cot 36°$?

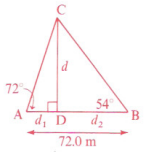
(b) Use the expression in (a) to calculate the depth, h, in metres. ($AB + BC = 250.0$ m.)



Examples – Right Triangle Trigonometry

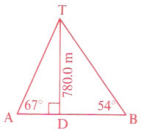
3 An engineer wishes to find the distance across a canyon. She takes a sighting from A and then a sighting from B to a point C on the opposite side of the canyon. The measurements are given on the diagram.

Find distance d across the canyon.



Examples – Right Triangle Trigonometry

4 The angles of elevation from a point A and a point B to the top of a mountain 780.0 m high are 67° and 54° as shown. Based on the information in the diagram, how long would a tunnel be from A to B?



Examples – Right Triangle Trigonometry

5 A forest ranger in a tower 128.0 m high sights two fires in the same line of sight with angles of depression 42° and 61°. How far apart are the fires?

6 From a window 26.0 m above the ground, the angle of elevation of the top of a building is 39°, while the angle of depression to the bottom of the building is 29°. How high is the building?

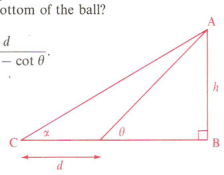
7 A helicopter, directly above a building, sights a position, A, on the ground at an angle of depression of 38°. The helicopter then rises vertically above the building, a distance of d, in metres, and sights position A, now at an angle of depression of 52°. If point A is 352.0 m from the building, how far has the helicopter risen?

Examples – Right Triangle Trigonometry

8 The angle of elevation of the top of a building from a point, A, 56.0 m from the building is 58°. A flagpole is on top of the building. The angle of elevation from point A to the top of the flagpole is 62°. What is the length of the flagpole?

9 Two spotlights are placed 10.0 m apart on the same line of sight. The blue spotlight makes an angle of elevation of 45° and hits the bottom of a mirrored ball. The white spotlight makes an angle of elevation of 70° and hits the same area. What is the height of the bottom of the ball?

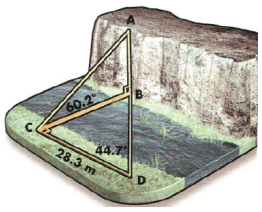
10 For the diagram, prove that $h = \dfrac{d}{\cot \alpha - \cot \theta}$.



Examples – Right Triangle Trigonometry

B

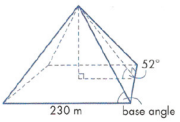
13. Surveying A surveyor measured the height of a vertical rock face by determining the measurements shown. If the surveyor's theodolite had a height of 1.7 m, find the height of the rock face, AB, to the nearest tenth of a metre.



Examples – Right Triangle Trigonometry

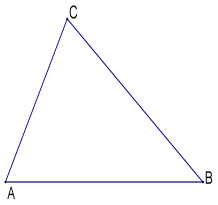
C

19. Great Pyramid The Great Pyramid of Khufu has a square base with a side length of about 230 m. The four triangular faces of the pyramid are congruent and isosceles. The altitude of each triangular face makes an angle of 52° with the base. Find the measure of each base angle of the triangular faces, to the nearest degree.



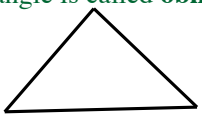
(A) Review of the Sine Law

- If we have a non right triangle, we cannot use the primary trig ratios, so we must explore new trigonometric relationships.
- One such relationship is called the Sine Law which states the following:

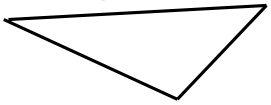


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{OR} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

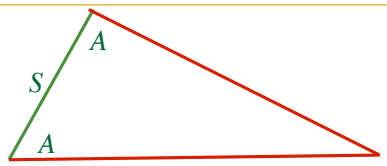
If none of the angles of a triangle is a right angle, the triangle is called **oblique.**



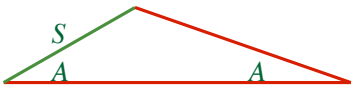
All angles are acute



Two acute angles, one obtuse angle

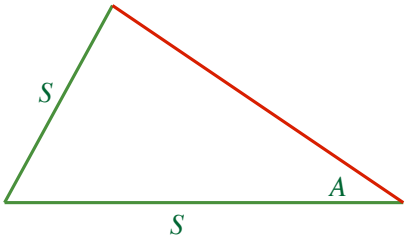


ASA

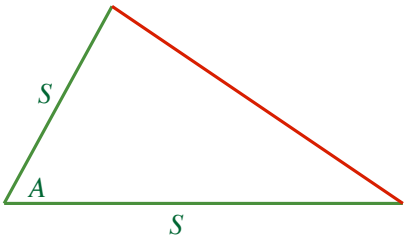


SAA

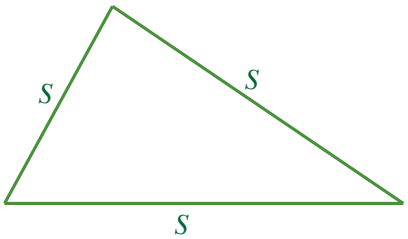
CASE 1: ASA or SAA



CASE 2: SSA



CASE 3: SAS



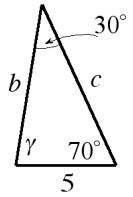
CASE 4: SSS

Sine Law - Summary

- The law of Sines is used to solve triangles in which we have ASA, SAA and also SSA
- The law of Cosines is used to solve triangles in which we have SAS and SSS

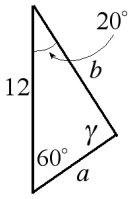
Examples

Solve the triangle: $\alpha = 30^\circ, \beta = 70^\circ, a = 5$ (SAA)



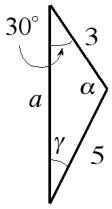
Examples

Solve the triangle: $\alpha = 20^\circ, \beta = 60^\circ, c = 12$ (ASA)



Examples

Solve the triangle: $b = 5,\ c = 3,\ \beta = 30^\circ$. (SSA).

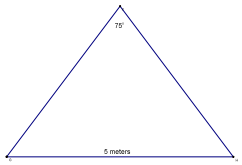


(D) Examples Sine Law

- We can use these new trigonometric relationships in solving for unknown sides and angles in acute triangles:
- ex 4. Find *A* in ABC if *a* = 10.4, *c* = 12.8 and *C* = 75°
- ex 5. Find *a* in ABC if *A* = 84°, *B* = 36°, and *b* = 3.9
- ex 6. Solve EFG if E = 82°, *e* = 11.8, and *F* = 25°
- There is one limitation on the Sine Law, in that it can only be applied if a side and its opposite angle is known. If not, the Sine Law cannot be used.

(D) Examples Sine Law

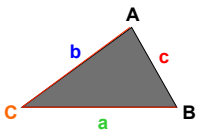
- Mark is a landscaper who is creating a triangular planting garden. The homeowner wants the garden to have two equal sides and contain an angle of 75°. Also, the longest side of the garden must be exactly 5 m.
  - **(a)** How long is the plastic edging that Mark needs to surround the garden?
  - **(b)** Determine the area of the garden.



(C) Law of Cosines:

Have: two sides, included angle

Solve for: missing side



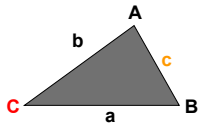
$$c^2 = a^2 + b^2 - 2ab\cos C$$

(missing side)² = (one side)² + (other side)² – 2 (one side)(other side) cos(included angle)

(C) Law of Cosines:

Have: three sides

Solve for: missing angle



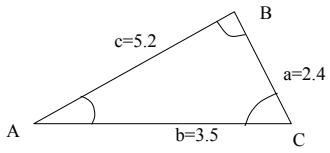
Missing Angle

Side Opposite Missing Angle

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

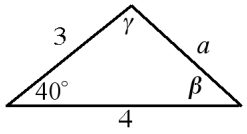
(D) Cosine Law - Examples

- Solve this triangle



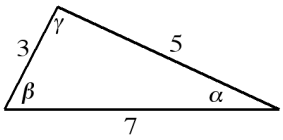
(D) Cosine Law - Examples

Solve the triangle: $b = 3, c = 4, \alpha = 40^\circ$ (SAS)



(D) Cosine Law - Examples

Solve the triangle: $a = 3, b = 5, c = 7$ (SSS)



(D) Examples Cosine Law

- We can use these new trigonometric relationships in solving for unknown sides and angles in acute triangles:
- ex 1. Find *c* in CDE if *C* = 56°, *d* = 4.7 and *e* = 8.5
- ex 2. Find *G* in GHJ if *h* = 5.9, *g* = 9.2 and *j* = 8.1
- ex 3. Solve CDE if *D* = 49°, *e* = 3.7 and *c* = 5.1

Further Mixed Practice Opportunities

- Nelson 10 textbook, Chap 6.1 ➔ http://mrsantowski.tripod.com/2010Math2Honors/Resources/NelsonS61p499.pdf
- Nelson 10 textbook, Chap 6.2 ➔ http://mrsantowski.tripod.com/2010MathSLY1/Assessments/M11SB515.pdf