Lesson 27 – Graphs of Trigonometric Functions

IB Math HL

Lesson Objectives

- 1. Relate the features of sinusoidal curves to modeling periodic phenomenon
- 2. Transformations of sinusoidal functions and their features

(A) Key Terms

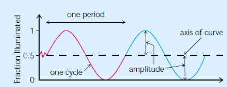
- Define the following key terms that relate to trigonometric functions:
- (a) period
- (b) amplitude
- (c) axis of the curve (or equilibrium axis)
- (d) phase shift or phase angle

(A) Key Terms

Key Ideas

- Repeating data forms a periodic function.
- A periodic function has a self-repeating graph.
- The **cycle** of a graph is the smallest complete repeating pattern of the graph.
- The length of one cycle is called the **period**.
- The horizontal line that is halfway between the maximum and minimum values of a periodic curve is called the **axis of the curve**.
- The equation of the axis of the curve is

$$y = \frac{\text{maximum value} + \text{minimum value}}{2}$$



- The magnitude of the vertical distance from the axis of the curve to either the maximum or minimum value is called the **amplitude** of the function. The amplitude, a, is calculated as

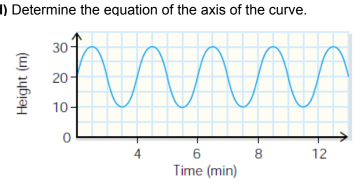
$$a = \frac{\text{maximum value} - \text{minimum value}}{2}$$

The General Sinusoidal Equation

- In the equation f(x) = ***a***sin(***k***(x+***c***)) + ***d***, explain what:
- a represents?
- k represents?
- c represents?
- d represents?

Modeling Periodic Phenomenon

- The graph shows John's height above the ground as a function of time as he rides a Ferris wheel.
  - **(a)** State the maximum and minimum height of the ride.
  - **(b)** How long does the Ferris wheel take to make one complete revolution?
  - **(c)** What is the amplitude of the curve? How does this relate to the Ferris wheel?
  - **(d)** Determine the equation of the axis of the curve.



Modeling Periodic Phenomenon

- A spring bounces up and down according to the model $d(t) = 0.5 \cos 2t$, where d is the displacement in centimetres from the rest position and t is the time in seconds. The model does not consider the effects of gravity.
  - **(a)** Draw the graph.
  - **(b)** Explain why the function models periodic behaviour.
  - **(c)** What is the period and what does it represent in the context of this question?
  - **(d)** What is the amplitude and what does it represent in the context of this question?

Modeling Periodic Phenomenon

- A small windmill has its center 6 m above the ground and the blades are 2 m in length. In a steady wind, one blade makes a rotation in 12 sec. Use the point P as a reference point on a blade that started at the highest point above the ground.
- (a) Determine an equation of the function that relates the height of a tip of a blade, h in meters, above the ground at a time t.
- (b) What is the height of the point P at the tip of a blade at 5s? 40s?
- (c) At what time is the point P exactly 7 m above the ground?

Analysis of Sinusoidal Graphs

- (a) Determine the maximums and zeroes of the function f(x) = -2sin(2x-60°)+1
- (b) Determine the equation of the asymptotes and zeroes of the function $g(\theta) = \tan\left(\frac{\theta}{4} + \pi^2\right)$
- (c) Determine the period and the equation of the axis of the curve of $y = 5 - \frac{\sqrt{2}}{\pi}\cos\left(\frac{3}{\sqrt{\pi}}x\right)$

Analysis of Systems of Trig Functions

- (a) Sketch y = sin(2x-π), given the domain of [0,2π]
- (b) Sketch y = cos(x), given the same domain
- (c) Provide a solution to the equation sin(2x-π) = cos(x)

Working with Graphs of the Secondary Trig Functions

- (a) Sketch 2 cycles of the graph of y = sin(x)
- (b) Hence, sketch y = csc(x) & label key features (asymptotes, max/min, x- & y-int)
- (c) Hence, sketch 2 cycles of y = 2csc(2x) – 4 and label key features (asymptotes, max/min, x- & y-int.

Working with Graphs of the Secondary Trig Functions

- (a) Sketch a graph of two periods of f(x) = 2 + cos(x)
- (b) Sketch a graph of two periods of g(x) = -sec(x)
- (c) How many solutions would you predict for the system f(x) = g(x)? Find the exact values of these solutions

Advanced Sinusoidals

- (a) Use your GDC to graph f(x) = |sin(x)| on [-2π,4π]
- (b) State the 6 maximums of y = f(x)
- (c) Now graph g(x) = $e^{-0.1x}f(x)$
- (d) Determine the 6 maximums of y = g(x)
- (e) Now graph y = $e^{-0.1x}$
- (f) Explain what y = $e^{-0.1x}$ does to the sinusoidal function.
- (g) Explain what dampened simple harmonic motion is

Advanced Sinusoidals

- Predict the domains of the following composite functions

$$(a)\ f(x)=\sqrt{\ln(\sec(x))}$$
$$(b)\ g(x)=\cot(\ln(x))$$
$$(c)\ h(x)=\frac{\cot(x)}{1-2\sin^2(x)}$$

Sinusoidals: f(x) = Asin(x) + Bcos(x)

- (a) Graph the function $f(x)=\sqrt{3}\sin x+\cos x$
- (b) This function can be rewritten in the form $g(x)=K\sin(x+\alpha)$. Determine the values of K and α.
- (c) Determine the value of $\sin(\alpha)$ and $\cos(\alpha)$

Sinusoidals: f(x) = Asin(x) + Bcos(x)

- (a) Graph the function $f(x)=2\sin x-2\cos x$
- (b) This function can be rewritten in the form $g(x)=K\sin(x+\alpha)$. Determine the values of K and α.
- (c) Determine the value of $\sin(\alpha)$ and $\cos(\alpha)$

Sinusoidals: f(x) = Asin(x) + Bcos(x)

The function $f(x)=A\sin x+B\cos x$ can be rewritten in the form $g(x)=K\sin(x+\alpha)$

Predict the values of K and α, based upon the connections you established in your previous work on the previous 2 slides

Test your conjecture by rewriting $f(x)=5\sin x+12\cos x$ as $g(x)=A\sin(x+\alpha)$

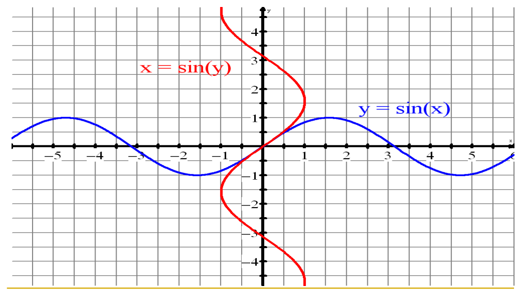
Review of Inverse Functions

- Q ➔ What is the significance of a function being 1:1 in the context of inverse functions?
- Q ➔ Is f(x) = x^2 a 1:1 function?
- Q ➔ How did we "resolve" this issue with the function f(x) = x^2?

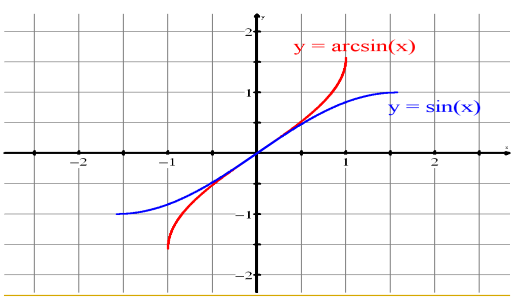
Restricted Domains

- (a) How would you restrict the domain of g(x) = sin(x) so that its inverse is a function? Use your calculator to justify your choice. (numerically & graphically)
- (b) How would you restrict the domain of g(x) = cos(x) so that its inverse is a function? Use your calculator to justify your choice. (numerically & graphically)
- (c) How would you restrict the domain of g(x) = tan(x) so that its inverse is a function? Use your calculator to justify your choice. (numerically & graphically)

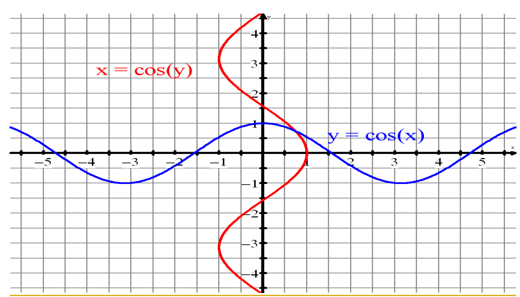
Graphs of Trig Inverse Functions: x = sin(y)



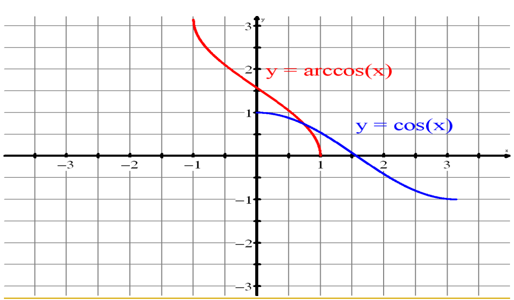
Graphs of Trig Inverse Functions: y = $\sin^{-1}(x)$



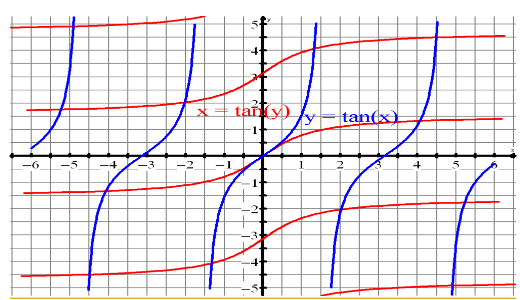
Graphs of Trig Inverse Functions: x = cos(y)



Graphs of Trig Inverse Functions: y = $\cos^{-1}(x)$



Graphs of Trig Inverse Functions: x = tan(y)



Graphs of Trig Inverse Functions: $y = \tan^{-1}(x)$