

### A. Lesson Context

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> <li>What is meant by the term FUNCTIONS and how do we work with them?</li> <li>mastery with working with basic terminology of functions</li> <li>understanding basics of function concepts and apply them to parent functions</li> </ul>		
CONTEXT of this LESSON:	Where we've been  In IM2 & IM3, you practiced with function notations and function representations	<b>Where we are</b>  <b>What are the key terms associated with studying and analyzing functions</b>	Where we are heading  How do we apply the concept of "functions" to all the functions in HL Math

### B. Function FACTS (Skills Review/Foreshadow Focus)

You will be given a graph  $f(x)$  = everything under the sun. You will use the graph to answer the following questions to the best of your group's ability. This exercise is a "skills inventory", but also a "what do you do if you're stuck" exercise.

The following questions relate to evaluations & working with & understanding function notation

- Evaluate the following:  $f(2)$  ;  $f(-5)$  ;  $f(1)$  ;  $g(12)$  ;  $f(4)$  ;  $f(-6)$  ;  $f(2) \times g(2)$  ;  $\frac{f(0)}{g(0)}$  ;  $\frac{g(0)}{f(0)}$
- Evaluate  $|f(7)|$  ;  $g(|-4|)$  ;  $|g(-4)|$  ; is  $|g(-4)| = g(4)$ ? ; is  $g(|-4|) = g(4)$  ?
- Evaluate  $f \circ g(-1)$  ;  $f \circ f(-4)$  ;  $g \circ f(11)$  ;  $g^{-1}(2)$  ;  $f^{-1}(-3)$
- Solve the following:  $f(x) = 7$  ;  $g(x) = -5$  ;  $f(x) = -3$  ;  $f(x) = 5$  ;  $g(x) = -1$
- state the graphical and algebraic significance of  $f(0)$  as well as  $g(0)$
- State the domain and range of  $g(x)$  and also for  $f(x)$
- is  $f(-4.75)$  positive or negative? Explain how you determined this.
- is  $g(-4.75)$  positive or negative? Explain how you determined this.
- state the graphical and algebraic significance of  $f(x) = 0$  as well as  $g(x) = 0$
- For what values of  $x$  is  $f(x) > 0$ ? Explain how you determined this.
- Solve  $g(x) < 0$ .
- How often does the line  $y = -1$  intersect  $y = f(x)$ ? Intersect  $y = g(x)$
- How often does the line  $x = -1$  intersect  $y = f(x)$ ? Intersect  $y = g(x)$
- Interpret the meaning of the state  $f(x) = g(x)$  then solve the equation  $f(x) = g(x)$ .
- Interpret the meaning of the state  $f(x) < g(x)$  then solve the inequality  $f(x) > g(x)$ .
- Calculate the value of the difference quotient  $\frac{f(7) - f(4)}{7 - 4}$  as well as  $\frac{f(9) - f(7)}{9 - 7}$  and explain the sig. of the DQ.
- Determine the average rate of change of  $y = g(x)$  between  $x = 4$  and  $x = 9$

## HL Math - Lesson 5: Review of Functions | Unit 2 – Working with Functions

Now let's work on other function concepts that relate to characteristics of functions, specifically  $y = f(x)$  now.

- (a) On what interval is  $y = f(x)$  increasing, given the restricted domain of  $\{x \in \mathbb{R} \mid -6 < x \leq 7\}$ ?
- (b) On what interval is  $y = f(x)$  decreasing, given the restricted domain of  $(-6, 7]$ ?
- (c) Where are the local maximums & minimums of  $y = f(x)$ , given the restricted domain of  $\{x \in \mathbb{R} \mid -6 < x \leq 7\}$ ?
- (d) Given the restricted domain of  $\{x \in \mathbb{R} \mid -10 < x \leq 4\}$ , on what interval is  $y = f(x)$  concave up? Concave down?
- (e) Where are the roots of  $y = f(x)$ ?
- (f) Does  $y = f(x)$  appear to have any asymptotes? If so, where?
- (g) What does the concept of discontinuities mean, given that I have created  $y = f(x)$  to be a discontinuous function.
- (h) What is a jump discontinuity? Where does  $f(x)$  have a "jump" discontinuity?
- (i) What is an infinite discontinuity? Where does  $f(x)$  have an infinite discontinuity?
- (j)  $f \circ h(x) = x + 2$ , what would the graph of  $y = f \circ h(x)$  look like? Why?
- (k) If  $h(x) = x + 2$ , what would the graph of  $y = h \circ f(x)$  look like? Why?
- (l) What would the graph of  $y = -f(x)$  look like? Why?
- (m) What would the graph of  $y = f(-x)$  look like? Why?
- (n) Explain how the graph of  $y = g(x)$  changes if you are asked to graph  $y = |f(x)|$ .
- (o) To determine the end behavior of the function, what does the function "do" as  $x \rightarrow +\infty$  and what does the function "do" as  $x \rightarrow -\infty$ ?
- (p) What does the term "bounded" mean and explain if/how it applies to  $y = f(x)$  & to  $y = g(x)$ .
- (q) Evaluate  $\lim_{x \rightarrow -5^+} f(x)$  &  $\lim_{x \rightarrow -5^-} f(x)$  &  $\lim_{x \rightarrow -5} f(x)$  &  $f(-5)$ .
- (r) Evaluate  $\lim_{x \rightarrow -6^+} f(x)$  &  $\lim_{x \rightarrow -6^-} f(x)$  &  $\lim_{x \rightarrow -6} f(x)$  &  $f(-6)$ .
- (s) Graph the inverse relation for  $y = g(x)$ .
- (t) Classify  $y = f(x)$  &  $y = g(x)$  as being either: (i) one to one, (ii) one to many, (iii) many to one, or (iv) many to many.
- (u) Which function(s) have/has symmetries: (i)  $f(x)$  only, (ii)  $g(x)$  only, (iii) both  $f(x)$  and  $g(x)$ , (iv) neither  $f(x)$  nor  $g(x)$ .