Lesson 92 – Conditional Probability

HL2 - Santowski

Conditional Probability

- Conditional Probability contains a condition that may limit the sample space for an event.
- You can write a conditional probability using the notation

$$P(B|A)$$

- This reads "the probability of event B, given event A"

Conditional Probability

The table shows the results of a class survey.
Find *P*(own a pet | female)

Do you own a pet?

	yes	no
female	8	6
male	5	7

Conditional Probability

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Find *P*(own a pet | female)

Do you own a pet?

	yes	no
female	8	6
male	5	7

14 females;
13 males

The condition female limits the sample space to 14 possible outcomes.

Of the 14 females, 8 own a pet.

Therefore, *P*(own a pet | female) equals $\frac{8}{14}$.

Conditional Probability

The table shows the results of a class survey.
Find *P*(wash the dishes | male)

Did you wash the dishes last night?

	yes	no
female	7	6
male	7	8

Conditional Probability

The table shows the results of a class survey.
Find *P*(wash the dishes | male)

Did you wash the dishes last night?

	yes	no
female	7	6
male	7	8

13 females;
15 males

The condition male limits the sample space to 15 possible outcomes.

Of the 15 males, 7 did the dishes.

Therefore, *P*(wash the dishes | male) $\frac{7}{15}$

Let's Try One

Using the data in the table, find the probability that a sample of not recycled waste was plastic. P(plastic | non-recycled)

The given condition limits the sample space to non-recycled waste.

A favorable outcome is non-recycled plastic.

Material	Recycled	Not Recycled
Paper	34.9	48.9
Metal	6.5	10.1
Glass	2.9	9.1
Plastic	1.1	20.4
Other	15.3	67.8

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$$P(\text{plastic} \mid \text{non-recycled}) = \frac{20.4}{48.9 + 10.1 + 9.1 + 20.4 + 67.8} = \frac{20.4}{156.3} \approx 0.13$$

The probability that the non-recycled waste was plastic is about 13%.

Conditional Probability Formula

- For any two events A and B from a sample space with P(A) does not equal zero

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Conditional Probability

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Relate: $P(\boxed{\text{male}}) = 58\%$
$P(\boxed{\text{male}} \text{ and } \boxed{\text{jogs}}) = 20\%$

Define: Let $\boxed{A}$ = male.
Let $\boxed{B}$ = jogs.

Write: $P(\boxed{A} | \boxed{B}) = \dfrac{P(\boxed{A} \text{ and } \boxed{B})}{P(\boxed{A})}$

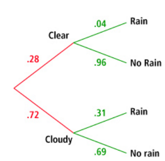
$= \dfrac{0.2}{0.58}$ Substitute 0.2 for $P(A$ and $B)$ and 0.58 for $P(A)$.

≈ 0.344 Simplify.

The probability that a male respondent jogs is about 34%.

Using Tree Diagrams

Jim created the tree diagram after examining years of weather observations in his hometown. The diagram shows the probability of whether a day will begin clear or cloudy, and then the probability of rain on days that begin clear and cloudy.



a. Find the probability that a day will start out clear, and then will rain.

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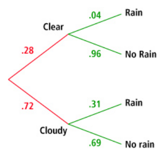


a. Find the probability that a day will start out clear, and then will rain.

The path containing clear and rain represents days that start out clear and then will rain.

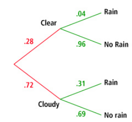
P(clear and rain) = *P*(rain | clear) • *P*(clear)
= 0.04 • 0.28
= 0.011

The probability that a day will start out clear and then rain is about 1%.

Conditional Probability

(continued)

b. Find the probability that it will not rain on any given day.



Conditional Probability

(continued)

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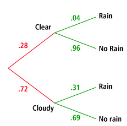

The paths containing clear and no rain and cloudy and no rain both represent a day when it will not rain. Find the probability for both paths and add them.

P(clear and no rain) + *P*(cloudy and no rain) =
P(clear) • *P*(no rain | clear) + *P*(cloudy) • *P*(no rain | cloudy)
= 0.28(.96) + .72(.69)
= 0.7656

The probability that it will not rain on any given day is about 77%.

Let's Try One

Pg 688

- A survey of Pleasanton Teenagers was given.
  - 60% of the responders have 1 sibling; 20% have 2 or more siblings
  - Of the responders with 0 siblings, 90% have their own room
  - Of the respondents with 1 sibling, 20% do not have their own room
  - Of the respondents with 2 siblings, 50% have their own room

Create a tree diagram and determine

A) *P*(own room | 0 siblings)
B) *P*(share room | 1 sibling)

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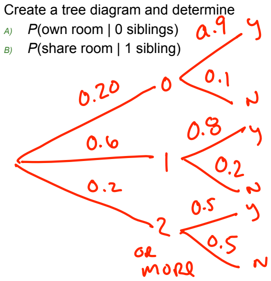

A) (0.2)(0.9) = 18%

B) (0.6)(0.2) = 12%

11.3 – Conditional Probability – Events Involving "And"

Conditional Probability

The probability of an event based on the fact that some other event has occurred, will occur, or is occurring.

The probability of event B occurring given that event A has occurred is usually stated as "***the conditional probability of B, given A***; ***P(B/A)***

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \text{ and } B)}{P(A)}$$

11.3 – Conditional Probability – Events Involving "And"

Conditional Probability

Example:

A number from the sample space S = {2, 3, 4, 5, 6, 7, 8, 9} is randomly selected. Given the defined events A and B,

A: selected number is odd, and

B: selected number is a multiple of 3

find the following probabilities.

a) P(B) b) P(A and B) c) P(B/A)

a) B = {3, 6, 9} P(B) = 3/8

b) P(A and B) = P({3, 5, 7, 9} ∩ {3, 6, 9})

= P({3, 9}) = 2/8 = 1/4

c) Probability of B given A has occurred:

$$P(B/A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{1/4}{4/8} = 1/2$$

11.3 – Conditional Probability – Events Involving "And"

Conditional Probability

Example:

Given a family with two children, find the probability that both are boys, given that at least one is a boy.

Conditional Probability $P(B/A) = \dfrac{P(A \text{ and } B)}{P(A)}$

S= {gg, gb, bg, bb}

A = at least one boy A = {gb, bg, bb}

B = both are boys B = {bb}

P(A and B) = P({gb, bg, bb} ∩ {bb}) = P({bb}) = 1/4

P(A) = P({gb, bg, bb}) = 3/4

$$\frac{P(A \text{ and } B)}{P(A)} = \frac{1/4}{3/4} = 1/3$$

11.3 – Conditional Probability – Events Involving "And"

Independent Events

Two events are ***Independent*** if the occurrence of one of them has no effect on the probability of the other.

P(B/A) = P(B)

or

P(A/B) = P(A)

11.3 – Conditional Probability – Events Involving "And"

Independent Events

Example:

A single card is randomly selected from a standard 52-card deck. Given the defined events A and B,

A: the selected card is an ace, B: the selected card is red,

find the following probabilities.

a) P(B) b) P(A and B) c) P(B/A)

a) $P(B) = \dfrac{26}{52} = 1/2$

b) P(A and B) = P({Ah, Ad, Ac, As} ∩ {all red}) = P({Ah, Ad}) = 2/52

c) $P(B/A) = \dfrac{P(A \text{ and } B)}{P(A)} = \dfrac{2/52}{4/52} = 1/2$

Events A and B are independent as P(B) = P(B/A).

11.3 – Conditional Probability – Events Involving "And"

Multiplication Rule of Probability - Events Involving "And"

If A and B are any two events then

P(A and B) = P(A) · P(B/A)

If A and B are independent events then

P(A and B) = P(A) · P(B)

Example:

A jar contains 4 red marbles, 3 blue marbles, and 2 yellow marbles. What is the probability that a red marble is selected and then a blue one *without* replacement?

P(Red and Blue) = P(Red) · P(Blue/Red)

= 4/9 · 3/8

= 12/72

= 1/8 = 0.1667

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Multiplication Rule of Probability - Events Involving "And"

Example:

A jar contains 4 red marbles, 3 blue marbles, and 2 yellow marbles. What is the probability that a red marble is selected and then a blue one *with* replacement?

P(Red and Blue) = P(Red) · P(Blue)

= 4/9 · 3/9

= 12/81

= 4/27 = 0.148