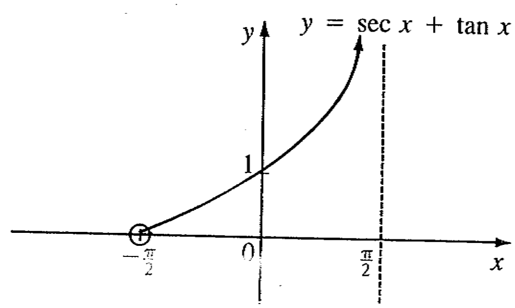
$$\lim_{x \to -\frac{\pi}{2}^+} (\sec x + \tan x) = \lim_{x \to -\frac{\pi}{2}^+} \frac{1 + \sin x}{\cos x}$$
$$= \lim_{x \to -\frac{\pi}{2}^+} \frac{1 - \sin^2 x}{\cos x(1 - \sin x)}$$
$$= \lim_{x \to -\frac{\pi}{2}^+} \frac{\cos x}{1 - \sin x}$$
$$= \frac{0}{1 - (-1)}$$
$$= 0$$

$$\lim_{x \to \frac{\pi}{2}^-} (\sec x + \tan x) = \lim_{x \to \frac{\pi}{2}^-} \frac{1 + \sin x}{\cos x}$$
$$= \infty$$

Therefore $x = \dfrac{\pi}{2}$ is a vertical asymptote.

The function considered in Examples 5 and 6 is graphed in the following diagram.



EXERCISE 7.3

B 1. Find the derivative of each of the following.

(a) $y = 3 \tan 2x$
(b) $y = \frac{1}{3} \cot 9x$
(c) $y = 12 \sec \frac{1}{4}x$
(d) $y = -\frac{1}{4} \csc(-8x)$
(e) $y = \tan x^2$
(f) $y = \tan^2 x$
(g) $y = \sec \sqrt[3]{x}$
(h) $y = x^2 \csc x$
(i) $y = \cot^3(1 - 2x)^2$
(j) $y = \sec^2 x - \tan^2 x$
(k) $y = \dfrac{1}{\sqrt{(\sec 2x - 1)^3}}$
(l) $y = \dfrac{x^2 \tan x}{\sec x}$
(m) $y = 2x(\sqrt{x} - \cot x)$
(n) $y = \sin(\tan x)$
(o) $y = \tan^2(\cos x)$
(p) $y = [\tan(x^2 - x)^{-2}]^{-3}$

2. Find $\frac{dy}{dx}$.
   (a) $\tan x + \sec y - y = 0$ (b) $\tan 2x = \cos 3y$
   (c) $\cot(x + y) + \cot x + \cot y = 0$
   (d) $y^2 - \csc(xy) = 0$
   (e) $x^2 + \sec\left(\frac{x}{y}\right) = 0$
   (f) $y^2 = \sin(\tan y) + x^2$

3. Find the equations of the tangent lines.
   (a) $y = \cot^2 x$ when $x = \frac{\pi}{4}$ (b) $y = \sin x \tan \frac{x}{2}$ when $x = \frac{\pi}{3}$
   (c) $y = \csc 2x$ when $x = -\frac{\pi}{8}$
   (d) $y = \sec x + \csc x$ when $x = \frac{3\pi}{4}$

4. Prove that $y = \sec x + \tan x$ is always increasing on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

5. Find the vertical asymptotes.
   (a) $y = \csc x - \cot x$, $0 < x < \pi$
   (b) $y = \sin x - \tan x$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$

6. Find the critical numbers, intervals of increase and decrease, and maximum and minimum values of $y = \csc x - \cot x$ on $(0, \pi)$.

7. Determine the concavity of $y = \sin x - \tan x$ on $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

8. Use the procedure described in Chapter 5 to sketch $y = x \tan x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

9. Prove the following.
   (a) $\frac{d}{dx} \sec x = \sec x \tan x$ (b) $\frac{d}{dx} \cot x = -\csc^2 x$

C 10. If $f(x) = \cot 2x$, $0 \leqslant x \leqslant 2\pi$ find all values of x for which $f(x) = f''(x)$.

11. If $x^2 + \tan^2 y = \sec^2 y - y$ find the values of x for which $\frac{dy}{dx} = \frac{dx}{dy}$.

12. If $f(x) = \sqrt{\sec^3(\sqrt[4]{x})}$ find $f'(x)$.

13. If $x = \cos 3t$ and $y = \sin^2 3t$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.