curve is always

ints at which the
to determine the

(x)

$-2\pi, -\frac{3}{2}\pi)$

n $\left(-\frac{3}{2}\pi, -\frac{1}{2}\pi\right)$

$\left(-\frac{1}{2}\pi, \frac{1}{2}\pi\right)$

n $\left(\frac{1}{2}\pi, \frac{3}{2}\pi\right)$

$\left(\frac{3}{2}\pi, 2\pi\right)$

points of inflection.

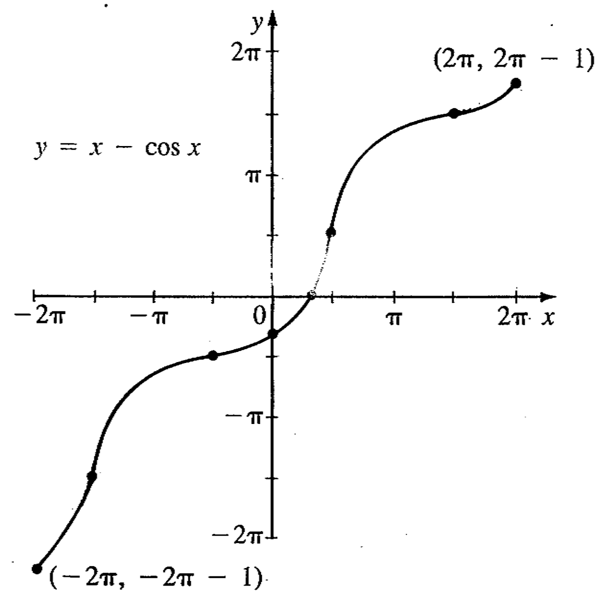
$-\frac{3}{2}\pi, -\frac{3}{2}\pi)$

$-\frac{1}{2}\pi, -\frac{1}{2}\pi)$

$\pi)$

$\pi)$

G. ***Sketch of the graph.***



EXERCISE 7.2

B 1. Find the derivative of y with respect to x in each of the following.

(a) $y = \cos(-4x)$

(b) $y = \sin(3x + 2\pi)$

(c) $y = 4\sin(-2x^2 - 3)$

(d) $y = -\frac{1}{2}\cos(4 + 2x)$

(e) $y = \sin x^2$

(f) $y = -\cos x^2$

(g) $y = \sin^{-2}(x^3)$

(h) $y = \cos(x^2 - 2)^2$

(i) $y = 3\sin^4(2 - x)^{-1}$

(j) $y = x\cos x$

(k) $y = \dfrac{x}{\sin x}$

(l) $y = \dfrac{\sin x}{1 + \cos x}$

(m) $y = (1 + \cos^2 x)^6$

(n) $y = \sin \dfrac{1}{x}$

(o) $y = \sin(\cos x)$

(p) $y = \cos^3(\sin x)$

(q) $y = x\cos\dfrac{1}{x}$

(r) $y = \dfrac{\sin^2 x}{\cos x}$

(s) $y = \dfrac{1 + \sin x}{1 - \sin 2x}$

(t) $y = \sin^3 x + \cos^3 x$

(u) $y = \cos^2\left(\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}\right)$

2. Find $\frac{dy}{dx}$ in each of the following.

(a) $\sin y = \cos 2x$ (b) $x \cos y = \sin(x + y)$

(c) $\sin y + y = \cos x + x$ (d) $\sin(\cos x) = \cos(\sin y)$

(e) $\sin x \cos y + \cos x \sin y = 1$

(f) $\sin x + \cos 2x = 2xy$

3. Find an equation of the tangent line to the given curve at the given point.

(a) $y = 2 \sin x$ at $\left(\frac{\pi}{6}, 1\right)$ (b) $y = \frac{\sin x}{\cos x}$ at $\left(\frac{\pi}{4}, 1\right)$

(c) $y = \frac{1}{\cos x} - 2 \cos x$ at $\left(\frac{\pi}{3}, 1\right)$

(d) $y = \frac{\cos^2 x}{\sin^2 x}$ at $\left(\frac{\pi}{4}, 1\right)$

(e) $y = \sin x + \cos 2x$ at $\left(\frac{\pi}{6}, 1\right)$

(f) $y = \cos(\cos x)$ at $x = \frac{\pi}{2}$

4. Find the critical numbers, the intervals of increase and decrease, and any maximum or minimum values.

(a) $y = \sin^2 x, \ -\pi \leqslant x \leqslant \pi$

(b) $y = \cos x - \sin x, \ -\pi \leqslant x \leqslant \pi$

5. Determine the concavity and find the points of inflection.

(a) $y = 2 \cos x + \sin 2x, \ 0 \leqslant x \leqslant 2\pi$

(b) $y = 4 \sin^2 x - 1, \ -\pi \leqslant x \leqslant \pi$

6. Use the procedure of Example 5 to sketch the graph of each of the following.

(a) $y = x + \sin x, \ 0 \leqslant x \leqslant 2\pi$ (b) $y = x \cos x, \ 0 \leqslant x \leqslant \pi$

7. If $f(x) = \sin x \cos 3x$, evaluate $f''\left(\frac{\pi}{3}\right)$.

8. Use Newton's method to find all roots of the given equation correct to 6 decimal places.

(a) $\cos x - x = 0$ (b) $2 \sin x = 2 - x$ (c) $\sin x = \frac{x}{2}$

C **9.** Use the results of this section to find the derivative of $y = \tan x$ and $y = \csc x$.

10. If $\sin y + \cos x = 1$ find $\frac{d^2y}{dx^2}$.

11. Find $\dfrac{dy}{dx}$ in each of the following.

(a) $y = \dfrac{1}{\sin(x - \sin x)}$ (b) $y = \sqrt{\sin\sqrt{x}}$

(c) $y = \sqrt[3]{x \cos x}$

(d) $y = \cos^3(\cos x) + \sin^2(\cos x)$

(e) $y = \sqrt{\cos(\sin^2 x)}$

12. Find an equation for the tangent line to the curve $x \sin 2y = y \cos 2x$ at the point $\left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$

13. Find the derivative of y with respect to x if $x + \tan(xy) = \sin y + \cos x$

PROBLEMS PLUS

If $f(x) = x \sin x$, find $f^{(100)}(0)$.

7.3 DERIVATIVES OF OTHER TRIGONOMETRIC FUNCTIONS

The trigonometric identities allow us to express the remaining trigonometric functions in terms of sine or cosine or both. We can then generate the derivatives of the remaining trigonometric functions using the Quotient and Chain Rules in conjunction with our differentiation formulas for sine and cosine.

$$\frac{d}{dx} \tan x = \sec^2 x$$

Proof

A basic identity transforms

$$y = \tan x$$

into $$y = \frac{\sin x}{\cos x}$$