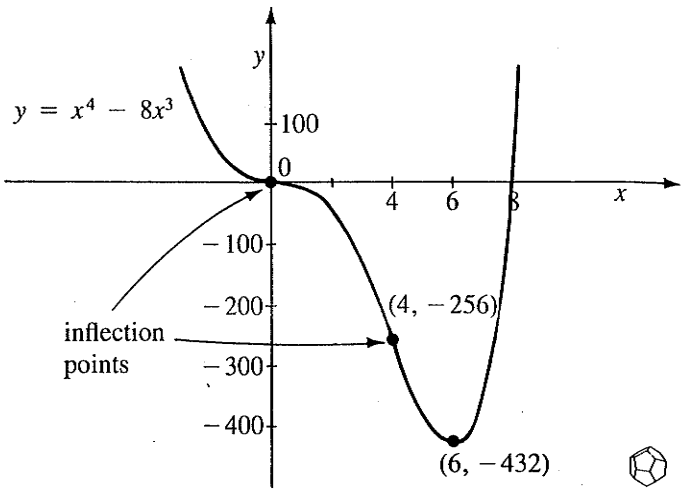


Note: Example 2 illustrates the fact that the Second Derivative Test gives no information when $f''(c) = 0$. It also fails when $f''(c)$ does not exist. For instance, in Example 4 in Section 5.3 the function has a local maximum value when $x = -3$, but $f''(-3)$ does not exist and so the Second Derivative Test does not apply. In such cases we must use the First Derivative Test. In fact, the First Derivative Test has the added advantage that we need not calculate the second derivative.

EXERCISE 5.4

B 1. Use the Second Derivative Test to find the local maximum and minimum values of each function, wherever possible.

(a) $f(x) = 3x^2 - 4x + 13$ (b) $f(x) = 2 + 6x - 6x^2$

(c) $g(x) = 2x^3 - 48x - 17$ (d) $g(x) = 1 + 3x^2 - 2x^3$

(e) $h(x) = x^3 - 9x^2 + 24x - 10$ (f) $h(x) = x^4 - x^3$

(g) $F(x) = 3x^4 - 16x^3 + 18x^2 + 1$ (h) $F(x) = 2 + 5x - x^5$

(i) $G(x) = (1 - 3x^2 + x^3)^5$ (j) $G(x) = x^2 + \dfrac{16}{x}$

2. Use any method to find the local maximum and minimum values of each function.

(a) $f(x) = x^4 - 6x^2 + 10$ (b) $f(x) = x\sqrt{x - 1}$

(c) $g(x) = \dfrac{x}{x^2 + 9}$ (d) $g(x) = \dfrac{x}{(2x - 3)^2}$

(e) $f(t) = \dfrac{t^2}{2t + 5}$ (f) $f(t) = t + 3t^{\frac{2}{3}}$

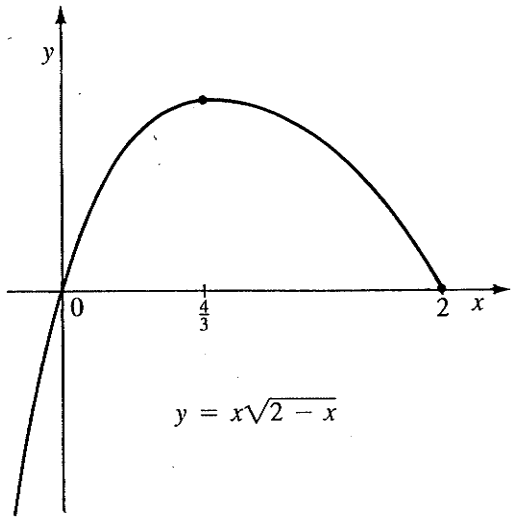
3. Find the local maximum and minimum values of each function. Use this information, together with concavity, to sketch the curve.

(a) $y = x - x^3$ (b) $y = x^4 - 3x^3 + 3x^2 - x + 1$

(c) $y = 3x^5 - 25x^3 + 60x$ (d) $y = x\sqrt{10 + x}$

Note that the domain of f is $(-\infty, 2]$ and $f''(x) < 0$ on this domain. (The denominator is positive since it is a power of a square root.) Thus, f is concave downward on $(-\infty, 2)$.

H. ***Sketch of the Curve.***



EXERCISE 5.5

B Discuss the curve in each question under the headings A. Domain; B. Intercepts; C. Symmetry; D. Asymptotes; E. Intervals of Increase or Decrease; F. Local Maximum and Minimum Values; G. Concavity and Points of Inflection; and H. Sketch of the Curve.

1. $y = 3x^5 - 10x^3 + 45x$

2. $y = (x^2 - 1)^3$

3. $y = \dfrac{x - 4}{x + 4}$

4. $y = \dfrac{x^2}{x^2 + 3}$

5. $y = \dfrac{x}{x^2 - 1}$

6. $y = \dfrac{x}{(x - 1)^2}$

7. $y = \dfrac{1}{x^3 - x}$

8. $y = \dfrac{x^2 - 1}{x^3}$

9. $y = x\sqrt{1 - x^2}$

10. $y = \dfrac{x}{\sqrt{x^2 - 4}}$

11. $y = \dfrac{\sqrt{x}}{\sqrt{x} + 1}$

12. $y = x - \sqrt[3]{x}$