which is not defined when $x = 0$. (But note that $f(x)$ is defined everywhere.) Also $f'(x) = 0$ when $x = 1$. So the critical numbers are 0 and 1.

The intervals of increase and decrease are obtained in the following chart.

Interval	$\sqrt[3]{x}$	$\sqrt[3]{x} - 1$	$f'(x)$	f
$x < 0$	$-$	$-$	$+$	increasing on $(-\infty, 0)$
$0 < x < 1$	$+$	$-$	$-$	decreasing on $(0, 1)$
$x > 1$	$+$	$+$	$+$	increasing on $(1, \infty)$

From the chart we see that the derivative changes from positive to negative at 0 and from negative to positive at 1. Thus, by the First Derivative Test,

$$f(0) = 0 \quad \text{is a local maximum}$$
$$f(1) = -1 \quad \text{is a local minimum}$$

In certain circumstances, the First Derivative Test can be used to find an *absolute* maximum or minimum.

First Derivative Test for Absolute Extreme Values

Let c be a critical number of a continuous function f defined on an interval.

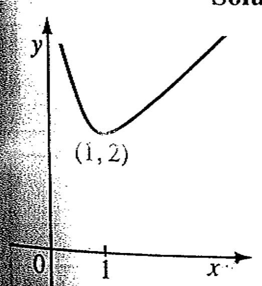
1. If $f'(x)$ is positive for all $x < c$ and $f'(x)$ is negative for all $x > c$, then $f(c)$ is the absolute maximum value.
2. If $f'(x)$ is negative for all $x < c$ and $f'(x)$ is positive for all $x > c$, then $f(c)$ is the absolute minimum value.

Example 4 Find the absolute minimum value of the function

$$f(x) = x + \frac{1}{x}, \; x > 0.$$

Solution The derivative is

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$$



Thus $f'(x) = 0$ when $x^2 = 1$, that is, $x = 1$ (since $x > 0$). Also $f'(x) > 0$ when $x^2 > 1$, that is, $x > 1$. Similarly, $f'(x) < 0$ when $0 < x < 1$.

Thus, by the First Derivative Test, the absolute minimum value of f is $f(1) = 2$. The graph of f illustrates this fact.

EXERCISE 4.3

B **1.** Find the local maximum and minimum values of f.

(a) $f(x) = 3x^2 - 4x + 13$ (b) $f(x) = x^3 - 12x - 5$

(c) $f(x) = 2 + 5x - x^5$ (d) $f(x) = x^4 - x^3$

2. Find the critical numbers, intervals of increase and decrease, and local maximum values of the function. Then use this information to sketch the graph of f.

(a) $f(x) = 2 + 6x - 6x^2$

(b) $f(x) = x^3 - 9x^2 + 24x - 10$

(c) $g(x) = 1 + 3x^2 - 2x^3$

(d) $g(x) = 3x^4 - 16x^3 + 18x^2 + 1$

(e) $h(x) = x^4 - 8x^2 + 6$

(f) $h(x) = 3x^5 - 5x^3$

3. Find the local maximum and minimum values of f.

(a) $f(x) = 2x^{\frac{2}{3}}(3 - 4x^{\frac{1}{3}})$ (b) $f(x) = \dfrac{x^2}{x^2 - 1}$

(c) $f(x) = x\sqrt{4 - x}$ (d) $f(x) = x\sqrt{1 - x^2}$

4. Find the absolute maximum or minimum value of the function.

(a) $f(x) = 27 + x - x^2$ (b) $f(x) = 3 - \dfrac{1}{\sqrt{x^2 + 1}}$

(c) $g(x) = \dfrac{x^2 - 1}{x^2 + 1}$ (d) $g(x) = \dfrac{x^2 - x + 1}{x^2 + 1}, x \geqslant 0$

C **5.** Sketch the graph of a function f that satisfies all of the following conditions.

(a) $f(2) = 3, f(5) = 6$

(b) $f'(2) = f'(5) = 0$

(c) $f'(x) \geqslant 0$ for $x < 5$

(d) $f'(x) < 0$ for $x > 5$

6. Find the local maximum and minimum values of the function f defined by

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ 2x^3 - 15x^2 + 36x & \text{if } 0 \leqslant x \leqslant 4 \\ 216 - x & \text{if } x > 4 \end{cases}$$

PROBLEMS PLUS

Find the absolute maximum value of the function

$$f(x) = \frac{1}{1 + |x|} + \frac{1}{1 + |x - 2|}$$