Example 7 Find the absolute maximum and minimum values of the function

$$g(x) = x^{\frac{2}{3}}(5 + x), \qquad -5 \leq x \leq 1$$

Solution We could differentiate this function using the Product Rule, but it is perhaps simpler to rewrite the function first.

$$g(x) = x^{\frac{2}{3}}(5 + x) = 5x^{\frac{2}{3}} + x^{\frac{5}{3}}$$
$$g'(x) = \tfrac{10}{3}x^{-\frac{1}{3}} + \tfrac{5}{3}x^{\frac{2}{3}} = \frac{10 + 5x}{3x^{\frac{1}{3}}} = \frac{5(2 + x)}{3\sqrt[3]{x}}$$

This expression shows that $g'(x) = 0$ when $2 + x = 0$, that is, $x = -2$, and $g'(x)$ does not exist when $x = 0$. So the critical numbers are -2 and 0, and

$$g(0) = 0$$
$$g(-2) = (-2)^{\frac{2}{3}}(3) = 3(4^{\frac{1}{3}}) \doteq 4.8$$

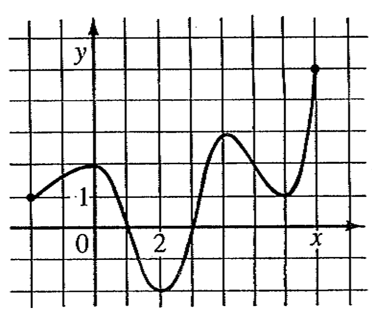
At the endpoints of the given interval $[-5, 1]$ we have

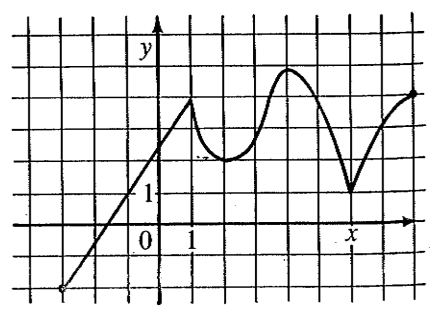
$$g(-5) = 0 \qquad g(1) = 6$$

We compare these values and see that the absolute maximum value is $g(1) = 6$ and the absolute minimum is $g(0) = g(-5) = 0$.

EXERCISE 4.2

A **1.** For the functions whose graphs are given, state
(a) the absolute maximum value,
(b) the absolute minimum value,
(c) the local maximum values,
(d) the local minimum values.

(i)


(ii)


B 2. Sketch the graph of each function and use it to state the absolute and local maximum and minimum values of the function.

(a) $f(x) = 3x - 1, x > -1$
(b) $g(x) = 3x - 1, x \geqslant -1$
(c) $f(x) = x^2 + 1$
(d) $y = x^2 + 1, -1 < x < 2$
(e) $y = x^2 + 1, -1 \leqslant x \leqslant 2$
(f) $y = 2 - x^3$
(g) $y = |x - 2| - 1$
(h) $f(x) = \begin{cases} 2 - x & \text{if } x < 1 \\ x & \text{if } x \geqslant 1 \end{cases}$

3. Find the critical numbers of the given functions.

(a) $f(x) = 17 - 6x + 12x^2$
(b) $f(x) = x^3 - 3x + 2$
(c) $g(x) = x^4 - 4x^3 - 8x^2 - 1$
(d) $g(x) = 3x^4 - 16x^3 + 6x^2 + 72x + 8$
(e) $y = 2x^3 + 3x^2 - 6x + 3$
(f) $y = x^3 + x^2 + x + 1$
(g) $y = |x + 6|$
(h) $y = \sqrt[3]{x}$
(i) $y = x - \sqrt{x}$
(j) $y = x\sqrt{x - 1}$
(k) $y = \dfrac{t}{t + 1}$
(l) $y = \dfrac{t}{t^2 + 1}$

4. Find the absolute maximum value and absolute minimum value of the function.

(a) $f(x) = 2x^2 - 8x + 1, 0 \leqslant x \leqslant 3$
(b) $f(x) = 3 + 2(x + 1)^2, -3 \leqslant x \leqslant 2$
(c) $f(x) = 2x^3 - 3x^2, -2 \leqslant x \leqslant 2$
(d) $f(x) = 2x^3 - 3x^2 - 36x + 62, -3 \leqslant x \leqslant 4$
(e) $f(x) = x^4 - 2x^2 + 16, -3 \leqslant x \leqslant 2$
(f) $f(x) = x^5 + 3x^3 + x, -1 \leqslant x \leqslant 2$
(g) $g(x) = x^2 + \dfrac{16}{x}, 1 \leqslant x \leqslant 4$
(h) $f(x) = 3x^{\frac{2}{3}} - 2x, 1 \leqslant x \leqslant 3$
(i) $f(x) = (x^2 - 9)^{\frac{2}{3}}, -6 \leqslant x \leqslant 6$
(j) $f(x) = |2x - 1| - 1, 0 \leqslant x \leqslant 2$

C 5. Show that the function $y = x^{21} + x^{11} + 13x$ does not have a local maximum or a local minimum.

6. Find the value of k if the function $y = x^2 + kx + 72$ has a local minimum at $x = 4$.

7. Find the values of a and b if the function $y = 2x^3 + ax^2 + bx + 36$ has a local maximum when $x = -4$ and a local minimum when $x = 5$.

8. (a) Use Newton's method to find the critical numbers of the function $f(x) = 2x^5 - 5x^2 - 20x + 12$ correct to three decimal places.

(b) Find the absolute minimum value of the function $f(x) = 2x^5 - 5x^2 - 20x + 12, -1 \leqslant x \leqslant 2$, correct to two decimal places.