Another method of solving the quadratic inequality in Example 2 is to take cases as in Solution 1 of Example 2 in the Review and Preview to this chapter. A third method would be to graph the parabola $y = (x + 1)(x + 3)$ and observe that it lies above the x-axis when $x < -3$ or $x > -1$. For more complicated functions, however, the chart method is usually simplest, as in the following example.

Example 3 Find the intervals of increase and decrease for the function $g(x) = x^4 - 4x^3 - 8x^2 - 1$.

Solution

$$\begin{aligned} g'(x) &= 4x^3 - 12x^2 - 16x \\ &= 4x(x^2 - 3x - 4) \\ &= 4x(x + 1)(x - 4) \end{aligned}$$

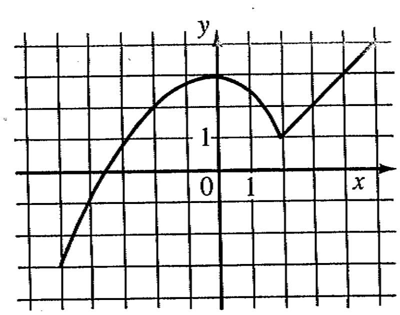
This expression is 0 when $x = 0, -1$, and 4. As in Example 2, we indicate the signs of the factors and the conclusion about g in a chart.

Interval	$4x$	$x + 1$	$x - 4$	$g'(x)$	g
$x < -1$	−	−	−	−	decreasing on $(-\infty, -1)$
$-1 < x < 0$	−	+	−	+	increasing on $(-1, 0)$
$0 < x < 4$	+	+	−	−	decreasing on $(0, 4)$
$x > 4$	+	+	+	+	increasing on $(4, \infty)$

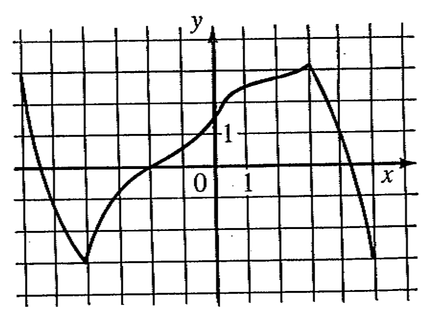
EXERCISE 4.1

A 1. State the intervals of increase or decrease for the functions whose graphs are given.

(a)



(b)



B 2. Find the intervals on which the following functions are increasing.
(a) $f(x) = 12 + x - x^2$ (b) $f(x) = x^4$
(c) $g(x) = x^3 - 3x + 2$ (d) $g(x) = 2x^3 - 3x^2$
(e) $y = 3x^4 + 4x^3 - 12x^2 + 7$ (f) $y = x^5 + 8x^3 + x$

3. Find the intervals on which the following functions are decreasing.
(a) $f(x) = x^2 + x^3$
(b) $g(x) = 2x^3 - 3x^2 - 36x + 62$
(c) $h(x) = (1 - x^2)^2$
(d) $F(x) = 4x + x^4$

4.2 MAXIM

4. Find the intervals of increase and decrease for the following functions.

(a) $f(x) = 3x^2 - 18x + 1$

(b) $f(x) = 2x^3 - 9x^2 - 60x + 82$

(c) $g(x) = x^4 - 2x^2 + 16$

(d) $g(x) = 3x^4 - 16x^3 + 6x^2 + 72x + 8$

(e) $h(x) = x^3(x - 1)^4$

(f) $h(x) = \dfrac{x - 1}{x + 1}$

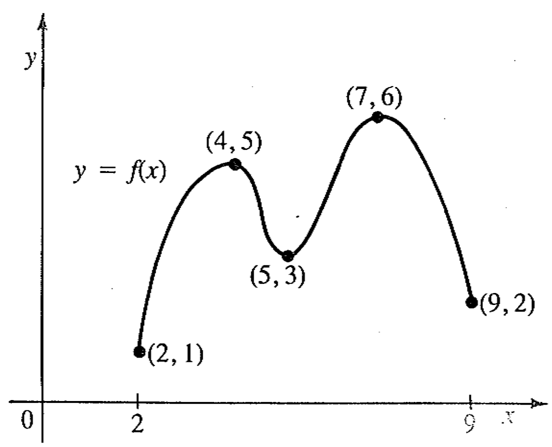
(g) $y = x\sqrt{4 - x}$

(h) $y = (x^2 - 9)^{\frac{2}{3}}$

C **5.** Where is the function $y = 12x^5 + 15x^4 - 20x^3 + 27$ decreasing?

4.2 MAXIMUM AND MINIMUM VALUES

The graph of a function f is shown. Notice that the highest point on the graph is $(7, 6)$ and so the largest value taken on by the function is $f(7) = 6$. We say that f has an *absolute maximum* at 7 and the maximum value is $f(7) = 6$. The lowest point on the graph is $(2, 1)$, so the smallest value of the function is $f(2) = 1$. We say that f has an *absolute minimum* at 2 and the minimum value is $f(2) = 1$.



In general, a function f has an **absolute maximum** at c if $f(c) \geqslant f(x)$ for all x in the domain of f, and the number $f(c)$ is called the **maximum value** of f. The function has an **absolute minimum** at c if $f(c) \leqslant f(x)$ for all x in the domain, and the number $f(c)$ is called the **minimum value** of f. The **extreme values** of f are the maximum and minimum values.