EXERCISE 2.7

B 1. Use implicit differentiation to find $\frac{dy}{dx}$.

(a) $x^2 - y^2 = 1$
(b) $x^3 + y^3 = 6$
(c) $xy = 4$
(d) $x^2 + xy + y^2 = 1$
(e) $x^3 + y^3 = 6xy$
(f) $2xy^2 - y^3 = x^2$
(g) $\sqrt{x} + \sqrt{y} = 1$
(h) $\frac{2x}{x + y} = y$

2. Find the slope of the tangent line to the curve at the given point.

(a) $x^2 + 4y^2 = 5$, $(1, -1)$
(b) $x^4 + y^4 = 17$, $(2, 1)$
(c) $x^2 + x^3y^2 - y^3 = 13$, $(1, -2)$
(d) $y^2 = 2xy - 3$, $(2, 3)$
(e) $\sqrt{x + y} + \sqrt{xy} = 4$, $(2, 2)$
(f) $\frac{1}{x} + \frac{1}{y} = 1$, $\left(\frac{3}{2}, 3\right)$

3. Find the equation of the tangent line to the curve at the given point.

(a) $2x^2 - y^2 = 1$, $(-1, -1)$
(b) $x^3 + y^3 = 9$, $(2, 1)$
(c) $y^5 + x^2y^3 = 10$, $(-3, 1)$
(d) $(x + y)^3 = x^3 + y^3$, $(-1, 1)$

4. (a) Use implicit differentiation to find the slope of the tangent line to the ellipse $9x^2 + 4y^2 = 36$ at the point $\left(\sqrt{2}, \frac{3}{2}\sqrt{2}\right)$.

(b) Find the slope in part (a) by first solving for y explicitly as a function of x.

(c) Find the equation of the tangent line.

(d) Sketch the ellipse and the tangent line.

5. (a) Find an equation of the tangent line to the circle $x^2 + y^2 + 2x - 4y - 20 = 0$ at the point $(2, -2)$.

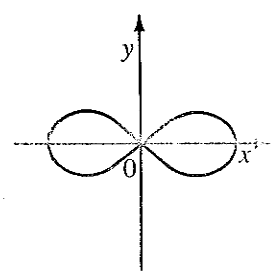
(b) Sketch the circle and the tangent line.

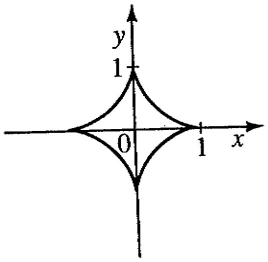
6. The curve with equation $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ is called a *lemniscate* and is shown in the figure.

(a) Find y'.

(b) Find the equation of the tangent line to the lemniscate at the point $(-3, 1)$.

(c) Find the points on the lemniscate where the tangent line is horizontal.





7. The curve with equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ is called an *astroid* and is shown in the figure.
   (a) Find y'.
   (b) Find the equation of the tangent line to the astroid at the point $\left(\frac{1}{8}, \frac{3\sqrt{3}}{8}\right)$.
   (c) Find the points on the astroid where the tangent line has slope 1.

8. Use implicit differentiation to show that an equation of the tangent line to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point (x_0, y_0) is

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$

C 9. Suppose f is a function such that $x[f(x)]^3 + x^2 f(x) = 3$ and $f(2) = 1$. Find $f'(2)$.

10. Use implicit differentiation to show that any tangent line at a point P to a circle with centre C is perpendicular to the radius CP.

11. Use implicit differentiation to show that, whenever a hyperbola with equation $x^2 - y^2 = k$ intersects a hyperbola with equation $xy = c$, the tangent lines at the points of intersection are perpendicular.

2.8 HIGHER DERIVATIVES

Since the derivative of a function f is itself a function f', we can take its derivative $(f')'$. The result is a function called the **second derivative** of f and denoted by f''.

If $y = f(x)$ and we use Leibniz notation, then

$$f''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

and we abbreviate this as

$$\frac{d^2y}{dx^2}$$

If we use D-notation, the symbol D^2 indicates that the operation of differentiation is performed twice. Thus we have the following notations for the second derivative:

$$y'' = f''(x) = \frac{d^2y}{dx^2} = D^2f(x) = D_x^2 f(x)$$