

Differentiation Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = c \frac{d}{dx}f(x)$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x))\frac{d}{dx}g(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Power and Chain Rules: $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$

PROBLEMS PLUS

Graph the function $f(x) = |x - 2| + |x - 5|$. Where is f discontinuous? Where is it not differentiable?

EXERCISE 2.6

B 1. Find the derivatives of the following functions.

(a) $F(x) = (5 - 3x)^7$

(b) $F(x) = (2x^2 + 1)^{20}$

(c) $G(x) = (x^3 + x^2 - 2)^{\frac{3}{4}}$

(d) $G(x) = \sqrt{x^4 - x + 1}$

(e) $y = \sqrt[4]{x^2 + x}$

(f) $y = (1 + 3x + 4x^2)^{-3}$

(g) $y = \frac{1}{(x^3 + 2x^2 + 1)^2}$

(h) $y = \frac{4}{\sqrt{9 - x^2}}$

(i) $y = (1 + 2\sqrt{x})^6$

(j) $y = \sqrt{x + \sqrt{x}}$

(k) $y = x - \sqrt[5]{1 + x^5 - 6x^{10}}$

(l) $y = x^2 + (x^2 - 1)^5$

2. If $y = u^4 + 5u^2$, where $u = x^5 + 2x^2 + 1$, find $\frac{dy}{dx}$. Leave your answer in terms of u and x .

3. Find $\left. \frac{dy}{dx} \right|_{x=4}$ if $y = u^2 - 2u^5$ and $u = x - \sqrt{x}$.

4. Find $\left. \frac{dy}{dt} \right|_{t=1}$ if $y = \sqrt{1 + r^2}$ and $r = \frac{t + 1}{2t + 1}$.

5. Find $\left. \frac{ds}{dt} \right|_{t=4}$ if $s = v + \frac{50}{v}$ and $v = 3t - \sqrt{t}$.

6. Differentiate:

(a) $F(x) = x\sqrt{x^2 + 1}$ (b) $F(x) = (2x + 1)(4x - 1)^5$

(c) $G(x) = (x^2 - 1)^4(2 - 3x)$

(d) $G(x) = (x^4 - x + 1)^2(x^2 - 2)^3$

(e) $F(x) = \frac{x}{\sqrt{2x + 3}}$

(f) $f(t) = \frac{(1 + 2t)^5}{(3t^2 - 5)^2}$

(g) $g(x) = \left(\frac{x + 2}{x - 2}\right)^3$

(h) $h(t) = \left(\frac{t^2 + 1}{t + 1}\right)^{10}$

(i) $y = \sqrt{\frac{x^2 - 1}{x^2 + 1}}$

(j) $y = \frac{(2x + 3)^3}{\sqrt{4x - 7}}$

(k) $y = 3\sqrt{x}(2x + 1)^5 + \sqrt{4x - 3}$

(l) $y = \sqrt{1 + \sqrt[3]{x}}$

(m) $y = (t + \sqrt[3]{t + t^2})^{20}$

(n) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

7. Find the equation of the tangent line to the curve $y = (x^2 - 3)^8$ at the point $(2, 1)$.

8. Find the equation of the tangent line to the curve $y = \frac{1}{\sqrt{20 - x^4}}$ at the point $(2, \frac{1}{2})$.

9. If $F(x) = f(g(x))$, where $g(2) = 4$, $g'(2) = 3$, and $f'(4) = 5$, find $F'(2)$.

10. If $G(x) = h(p(x))$, where $h(5) = 1$, $h'(5) = 2$, $h'(1) = 3$, $p(1) = 5$, and $p'(1) = 7$, find $G'(1)$.

C 11. If f is a differentiable function, find expressions for the derivatives of the following functions.

(a) $F(x) = f(x^4)$

(b) $G(x) = [f(x)]^4$

(c) $H(x) = f(\sqrt{x})$

(d) $P(x) = \sqrt{f(x)}$

(e) $y = f(f(x))$

(f) $y = \sqrt{1 + [f(x)]^2}$

(g) $y = [f(x^2)]^2$

(h) $y = f([f(x)]^3)$

12. (a) Use the Chain Rule and the fact that $|x| = \sqrt{x^2}$ to show that

$$\frac{d}{dx} |x| = \frac{x}{|x|}$$

(b) Sketch the graphs of the function $f(x) = |x|$ and its derivative.

(c) Use the result of part (a) to differentiate the function $g(x) = x|x|$.

x)

x)

$\frac{d}{dx} f(x)$

$g(x)$

is f

$\frac{1)^{20}}{x + 1}$
 $4x^2)^{-3}$

1)^5

leave your