llows:

formula
Quotient
imes the
erivative
ator.

$(x^3 + 1)$

ɔ simplify

Example 2 Find $\frac{dy}{dx}$ if $y = \frac{\sqrt{x}}{1 + 2x}$.

Solution

$$\frac{dy}{dx} = \frac{(1 + 2x)\frac{d}{dx}\sqrt{x} - \sqrt{x}\frac{d}{dx}(1 + 2x)}{(1 + 2x)^2}$$

$$= \frac{(1 + 2x)\frac{1}{2\sqrt{x}} - \sqrt{x}(2)}{(1 + 2x)^2}$$

Now we multiply the numerator and denominator by $2\sqrt{x}$:

$$\frac{dy}{dx} = \frac{1 + 2x - (2\sqrt{x})(2\sqrt{x})}{2\sqrt{x}(1 + 2x)^2} = \frac{1 - 2x}{2\sqrt{x}(1 + 2x)^2}$$

EXERCISE 2.5

B 1. Differentiate.

(a) $f(x) = \frac{x - 1}{x + 1}$

(b) $f(x) = \frac{2x - 1}{x^2 + 1}$

(c) $g(x) = \frac{x}{x^2 + 2x - 1}$

(d) $g(x) = \frac{x^3 - 1}{x^2 + x + 1}$

(e) $y = \frac{\sqrt{x}}{x^2 + 1}$

(f) $y = \frac{\sqrt{x} + 2}{\sqrt{x} - 2}$

(g) $f(t) = \frac{2t + 1}{t^2 - 3t + 4}$

(h) $g(t) = \frac{2t^2 + 3t + 1}{t - 1}$

(i) $f(x) = \frac{1}{x^4 - x^2 + 1}$

(j) $f(x) = \frac{ax + b}{cx + d}$

(k) $f(x) = \frac{x^6}{x^5 - 10}$

(l) $f(x) = \frac{1 - \frac{1}{x}}{x + 1}$

2. Find the domain of f and compute its derivative.

(a) $f(x) = \frac{2 + x}{1 - 2x}$

(b) $f(x) = \frac{x}{x^2 - 1}$

(c) $f(x) = \frac{1}{(x + 1)(2x - 3)}$

(d) $f(x) = \frac{2x + 1}{x^2 + 2x - 3}$

(e) $f(x) = \frac{x^2 + 2x}{x^4 - 1}$

(f) $f(x) = \frac{x^2}{\sqrt{x} - 3}$

3. Find an equation of the tangent line to the curve at the given point.

(a) $y = \dfrac{x}{x-2}$, $(4, 2)$ (b) $y = \dfrac{1+3x}{2-3x}$, $(1, -4)$

(c) $y = \dfrac{1}{x^2+1}$, $\left(-2, \frac{1}{5}\right)$ (d) $y = \dfrac{x^3-1}{1+2x^2}$, $(1, 0)$

4. If $f(2) = 3, f'(2) = 5, g(2) = -1$, and $g'(2) = -4$, find $\left(\dfrac{f}{g}\right)'(2)$.

5. Show that there are no tangents to the curve $y = \dfrac{x+2}{3x+4}$ with positive slope.

6. At what points on the curve $y = \dfrac{x^2}{2x+5}$ is the tangent line horizontal?

7. Find the points on the curve $y = \dfrac{x}{x-1}$ where the tangent line is parallel to the line $x + 4y = 1$.

8. If f is a differentiable function, find expressions for the derivatives of the following functions.

(a) $y = \dfrac{1}{f(x)}$ (b) $y = \dfrac{f(x)}{x}$ (c) $y = \dfrac{x}{f(x)}$

C 9. In Section 2.2 we proved the Power Rule for positive integer exponents. Use the Quotient Rule to deduce the Power Rule for the case of negative integer exponents; that is, prove that

$$\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$$

when n is a positive integer.

2.6 THE CHAIN RULE

Although we have learned to differentiate a variety of functions, our differentiation rules still do not enable us to find the derivative of the function

$$F(x) = \sqrt{2x^2 + 3}$$

Notice that F is a composite function; it can be built up from simpler functions. If we let

$$y = f(u) = \sqrt{u} \quad \text{and} \quad u = g(x) = 2x^2 + 3$$

then $\quad f(g(x)) = f(2x^2 + 3) = \sqrt{2x^2 + 3} = F(x)$