EXERCISE 2.4

B **1.** Use the Product Rule to find the derivative. Do not simplify your answer.

(a) $f(x) = (2x - 1)(x^2 + 1)$ (b) $f(x) = x(3x - 8)$

(c) $y = x^2(1 + x - 3x^2)$ (d) $y = (x^3 + x^2 + 1)(x^2 + 2)$

(e) $f(t) = (t^4 + t^2 - 1)(t^2 - 2)$ (f) $f(t) = \sqrt[3]{t}(1 - t)$

(g) $F(y) = \sqrt{y}(y - 2\sqrt{y} + 2)$ (h) $G(y) = (y - y^2)(2y - y^{\frac{4}{3}})$

2. Use the Product Rule to differentiate each function. Simplify your answer.

(a) $y = x^3(x^2 + 2x + 3)$ (b) $y = x^{-2}(x^3 - 3x^2 + 6)$

(c) $f(x) = (1 - x^2)(2 - x^3)$ (d) $f(x) = (3x^3 + 4)(1 - 2x^3)$

(e) $f(t) = (6 + t^{-2})(8t^{10} - 5t^3)$ (f) $f(t) = (at + b)(ct^2 - d)$

(g) $g(u) = \sqrt{u}(2 - u^2 + 5u^4)$ (h) $g(v) = (v - \sqrt{v})(v^2 + \sqrt{v})$

3. Find the slope of the tangent to the given curve at the point whose x-coordinate is given.

(a) $y = (1 - 2x)(3x - 4)$, $x = 2$

(b) $y = (1 - x + x^2)(x - 2)$, $x = 1$

(c) $y = x^4(4x^3 + 2)$, $x = -1$

(d) $y = (1 + x - 2x^2)(3x^3 + x - 1)$, $x = 1$

(e) $y = x^{-5}(1 + x^{-1})$, $x = 1$

(f) $y = (2 - 3\sqrt{x})(4 - \sqrt{x})$, $x = 4$

4. If $f(x) = (6x^4 - 3x^2 + 1)(2 - x^3)$, find $f'(1)$ by two methods:

(a) by using the Product Rule;

(b) by expanding $f(x)$ first.

5. Find the equation of the tangent line to the curve $y = (2 - \sqrt{x})(1 + \sqrt{x} + 3x)$ at the point $(1, 5)$.

6. If $f(2) = 3$, $f'(2) = 5$, $g(2) = -1$, and $g'(2) = -4$, find $(fg)'(2)$.

7. If f is a differentiable function, find expressions for the derivatives of the following functions.

(a) $g(x) = xf(x)$ (b) $h(x) = \sqrt{x}f(x)$ (c) $F(x) = x^c f(x)$

8. (a) Use the Product Rule with $g = f$ to show that if f is differentiable, then

$$\frac{d}{dx}[f(x)]^2 = 2f(x)f'(x)$$

(b) Use part (a) to differentiate $y = (2 + 5x - x^3)^2$.

C **9.** (a) Use the Product Rule twice to show that if f, g, and h are differentiable, then

$$(fgh)' = f'gh + fg'h + fgh'$$

(b) Use part (a) to differentiate $y = \sqrt{x}(3x + 5)(6x^2 - 5x + 1)$.

10. (a) Taking $f = g = h$ in Question 9, show that

$$\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x)$$