Practice questions

1 A toy on an elastic string is attached to the top of a doorway. It is pulled down and released, allowing it to bounce up and down. The length of the elastic string, L centimetres, is modelled by the function $L = 110 + 25\cos(2\pi t)$, where t is time in seconds after release.

a) Find the length of the elastic string after 2 seconds.

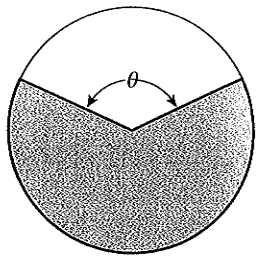
b) Find the minimum length of the string.

c) Find the first time after release that the string is 85 cm.

d) What is the period of the motion?

2 Find the exact solution(s) to the equation $2\sin^2 x - \cos x + 1 = 0$ for $0 \leqslant x \leqslant 2\pi$.

3 The diagram shows a circle of radius 6 cm.
The perimeter of the shaded sector is 25 cm.
Find the radian measure of the angle θ.



4 Consider the two functions $f(x) = \cos 4x$ and $g(x) = \cos\left(\frac{x}{2}\right)$.

a) Write down: **(i)** the minimum value of the function f
(ii) the period of g.

b) For the equation $f(x) = g(x)$, find the number of solutions in the interval $0 \leqslant x \leqslant \pi$.

5 A reflector is attached to the spoke of a bicycle wheel. As the wheel rolls along the ground, the distance, d centimetres, that the reflector is above the ground after t seconds is modelled by the function

$$d = p + q\cos\left(\frac{2\pi}{m}t\right), \text{ where } p, q \text{ and } m \text{ are constants.}$$

The distance d is at a maximum of 64 cm at $t = 0$ seconds and at $t = 0.5$ seconds, and is at a minimum of 6 cm at $t = 0.25$ seconds and at $t = 0.75$ seconds. Write down the value of:

a) p **b)** q **c)** m.

6 Find all solutions to $1 + \sin 3x = \cos(0.25x)$ such that $x \in [0, \pi]$.

7 Find all solutions to both trigonometric equations in the interval $x \in [0, 2\pi]$. Express the solutions exactly.

a) $2\cos^2 x + 5\cos x + 2 = 0$ **b)** $\sin 2x - \cos x = 0$

8 The value of x is in the interval $\frac{\pi}{2} < x < \pi$ and $\cos^2 x = \frac{8}{9}$. Without using your GDC, find the exact values for the following:

a) $\sin x$ **b)** $\cos 2x$ **c)** $\sin 2x$

9 The depth, d metres, of water in a harbour varies with the tides during each day. The first high (maximum) tide after midnight occurs at 5:00 a.m. with a depth of 5.8 m. The first low (minimum) tide occurs at 10:30 a.m. with a depth of 2.6 m.

a) Find a trigonometric function that models the depth, d, of the water t hours after midnight.

b) Find the depth of the water at 12 noon.

c) A large boat needs at least 3.5 m of water to dock in the harbour. During what time interval after 12 noon can the boat dock safely?

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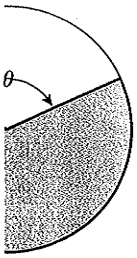

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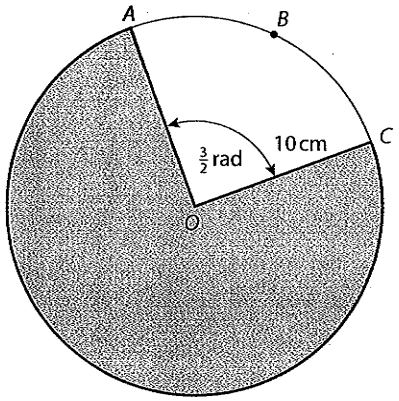
10 Solve the equation $\tan^2 x + 2\tan x - 3 = 0$ for $0 \leqslant x \leqslant \pi$. Give solutions exactly, if possible. Otherwise, give solutions to 3 significant figures.

11 The following diagram shows a circle of centre O and radius 10 cm. The arc ABC subtends an angle of $\frac{3}{2}$ radians at the centre O.

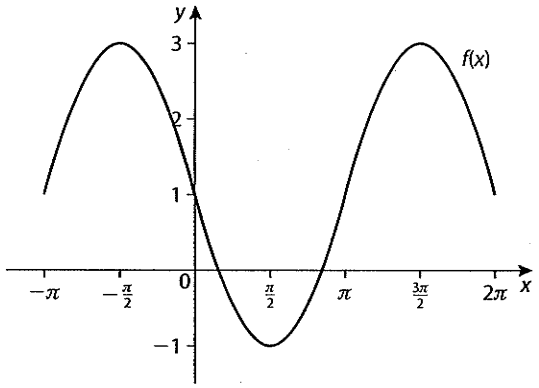
a) Find the length of the arc ACB.

b) Find the area of the shaded region.



12 Consider the function $f(x) = \frac{5}{2}\cos\left(2x - \frac{\pi}{2}\right)$. For what values of k will the equation $f(x) = k$ have no solutions?

13 A portion of the graph of $y = k + a\sin x$ is shown below. The graph passes through the points (0, 1) and $\left(\frac{3\pi}{2}, 3\right)$. Find the value of k and a.



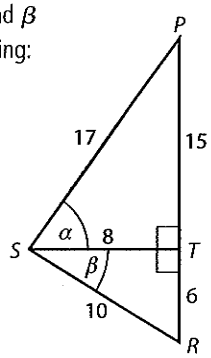
14 The angle α satisfies the equation $2\tan^2\alpha - 5\sec\alpha - 10 = 0$ where α is in the second quadrant. Find the **exact** value of $\sec\alpha$.

15 Triangles PTS and RTS are right-angled at T with angles α and β as shown in the diagram. Find the exact values of the following:

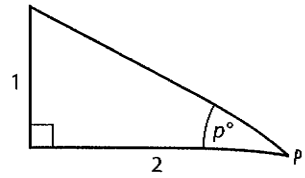
a) $\sin(\alpha + \beta)$

b) $\cos(\alpha + \beta)$

c) $\tan(\alpha + \beta)$



16 The diagram shows a right triangle with legs of length 1 unit and 2 units as shown. The angle at vertex P has a degree measure of $p°$. Find the exact values of $\sin 2p°$ and $\sin 3p°$.



17 The obtuse angle B is such that $\tan B = -\frac{5}{12}$. Find the values of

a) $\sin B$ **b)** $\cos B$ **c)** $\sin 2B$ **d)** $\cos 2B$

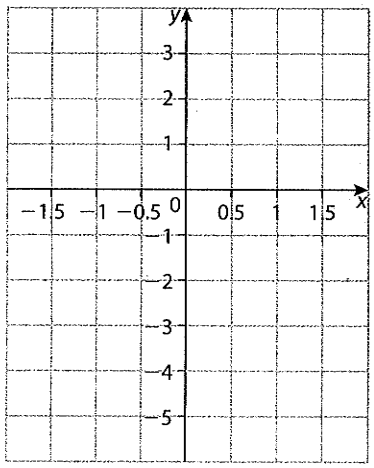
18 Given that $\tan 2\theta = \frac{3}{4}$, find the possible values of $\tan \theta$.

19 If $\sin(x - \alpha) = k \sin(x + \alpha)$ express $\tan x$ in terms of k and α.

20 Solve $\tan^2 2\theta = 1$, in the interval $-\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$.

21 Let f be the function $f(x) = x \arccos x + \frac{1}{2}x$ for $-1 \leqslant x \leqslant 1$ and g the function $g(x) = \cos 2x$ for $-1 \leqslant x \leqslant 1$.

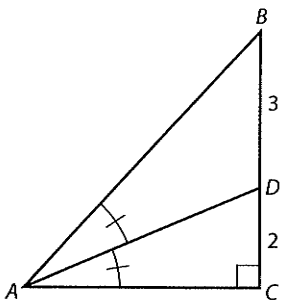
a) On the grid below, sketch the graph of f and of g.

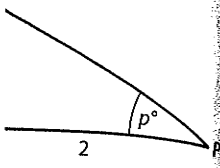


b) Write down the solution of the equation $f(x) = g(x)$.

c) Write down the range of g.

22 Let ABC be a right-angled triangle, where $\hat{C} = 90°$. The line (AD) bisects $B\hat{A}C$, $BD = 3$, and $DC = 2$, as shown in the diagram. Find $D\hat{A}C$.





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d) $\cos 2B$

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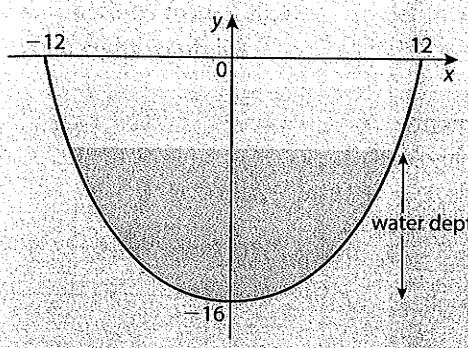
bisects $B\hat{A}C$, $BD = 3$,

23 The diagram below shows the boundary of the cross section of a water channel.



The equation that represents this boundary is $y = 16\sec\left(\frac{\pi x}{36}\right) - 32$ where x and y are both measured in cm. The top of the channel is level with the ground and has a width of 24 cm. The maximum depth of the channel is 16 cm. Find the width of the water surface in the channel when the water depth is 10 cm. Give your answer in the form $a \arccos b$, where $a, b \in \mathbb{R}$.