AP Calculus AB Test: Limits & Continuity 2009-2010

Name: ____

Directions: For ALL problems on this test you may use a graphing calculator to verify your answers and help with calculations; however, YOU MUST SHOW ALL YOUR WORK FOR CREDIT. With the exception of questions 11-18, all problems on this test are worth 5 points each.

Please put all work & solutions on a separate sheet of paper and PUT A BOX AROUND YOUR FINAL ANSWER!!

Use the function $f(x)=\dfrac{|x|(x-3)}{9-x^2}$ to answer questions 1-3 that follow.

#1. Evaluate $\lim\limits_{x\to 3} f(x)$.

#2. Determine all vertical asymptotes (if any) of $f(x)$.

#3. Find all the removable discontinuities (if any) of $f(x)$.

#4. Determine $\lim\limits_{\theta\to 0}\dfrac{\csc 3\theta}{\cot\theta}$.

#5. Determine the constants c and k that make the following function continuous.

$$f(x)=\begin{cases} x+2c & ,x<-2 \\ 3cx+k & ,\ -2\le x\le 1 \\ 3x-2k & ,\ x>1 \end{cases}$$

#6. What is the value of $\lim\limits_{x\to -1}\dfrac{x^2-3x-4}{x^2-1}$?

#7. Evaluate the limit, if it exists: $\lim\limits_{x\to 9}\dfrac{\sqrt{x-5}-2}{x-9}$.

#8. Evaluate the limit, if it exists: $\lim\limits_{x\to 2}\dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$.

#9. Find the limit (if it exists) : $\lim\limits_{x\to 6}\dfrac{x^4-4x^3-7x^2-31x+6}{x^3-x^2-33x+18}$

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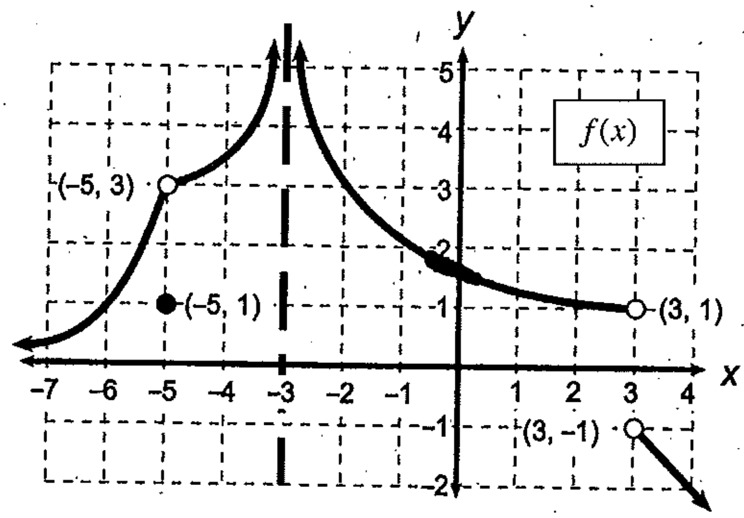
#10. Find the limit (if it exists) : $\lim_{x\to\infty}\frac{3x^5+7x^3-5x^2+1}{2x^5+2x^2-8}$ You must use Calculus.

Use the graph of $f(x)$ below to answer questions 11 – 20 that follow.



#11. $f(-5)=$

#12. $f(3)=$

#13. $\lim_{x\to -3} f(x)=$

#14. $\lim_{x\to 3^-} f(x)=$

#15. $\lim_{x\to 3^+} f(x)=$

#16. $\lim_{x\to 3} f(x)=$

#17. $\lim_{x\to -\infty} f(x)=$

#18. $\lim_{x\to \infty} f(x)=$

*************** Numbers 11-18 are worth 2 points each ***************

#19. Is $f(x)$ continuous at $x=3$? Justify your answer using Calculus.

#20. Is $f(x)$ continuous at $x=-2$? Justify your answer using Calculus.

Bonus: (worth 5 points) YOU MUST SHOW YOUR WORK!

Find ALL vertical asymptotes for the function $f(x)=\frac{1}{2\sin^2 x-\sin x-1}$

in the interval $0\le x\le 2\pi$. EXPRESS YOUR ANSWERS IN RADIANS!

Answer Key - Limits Test 2009-2010

$f(x) = \dfrac{|x|(x-3)}{9-x^2}$

1) $\lim_{x \to 3} \dfrac{|x|\cancel{(x-3)}^{-1}}{\cancel{(3-x)}(3+x)} = \lim_{x \to 3} \dfrac{-|x|}{3+x} = \dfrac{-3}{3+3} = \boxed{\dfrac{-1}{2}}$

2) Vertical asymptote @ $x = -3$ b/c $(3+x)$ did not cancel.

3) removable discontinuity @ $x = 3$ b/c $(3-x)$ canceled, leaving a hole.

4) $\lim_{\theta \to 0} \dfrac{\csc 3\theta}{\cot \theta} = \lim_{\theta \to 0} \dfrac{\frac{1}{\sin 3\theta}}{\frac{\cos\theta}{\sin\theta}} = \lim_{\theta \to 0} \dfrac{1}{\sin 3\theta} \cdot \dfrac{\sin\theta}{\cos\theta}$

$= \lim_{\theta \to 0} \dfrac{\sin\theta}{\sin 3\theta} \cdot \dfrac{1}{\cos\theta} = \dfrac{1}{3} \cdot 1 = \boxed{\dfrac{1}{3}}$

5) @ $x = -2$

$x + 2c = 3cx + K$

$-2 + 2c = -6c + K$

$-2 = -8c + K$

@ $x = 1$

$3cx + K = 3x - 2K$

$3c + K = 3 - 2K$

$3c + 3K = 3$

$-3(-2 = -8c + K) \rightarrow 6 = 24c - 3K$

$3 = 3c + 3K$ $\qquad + \; 3 = 3c + 3K$

$9 = 27c$

$\boxed{c = \dfrac{1}{3}}$

$3 = 3\left(\dfrac{1}{3}\right) + 3K$

$3 = 1 + 3K$ $\qquad \boxed{\dfrac{2}{3} = K}$

6) $\lim_{x\to -1} \frac{x^2-3x-4}{x^2-1} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x-4)}{\cancel{(x+1)}(x-1)}$

$= \lim_{x\to -1} \frac{(x-4)}{(x-1)} = \frac{-1-4}{-1-1} = \frac{-5}{-2} = \boxed{\frac{5}{2}}$

7) $\lim_{x\to 9} \left(\frac{\sqrt{x-5}-2}{x-9}\right)\left(\frac{\sqrt{x-5}+2}{\sqrt{x-5}+2}\right) =$

$\lim_{x\to 9} \frac{\cancel{x-5-4}}{\cancel{(x-9)}(\sqrt{x-5}+2)} = \lim_{x\to 9} \frac{1}{\sqrt{x-5}+2}$

$= \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

8) $\lim_{x\to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} = \lim_{x\to 2} \frac{\frac{2-x}{2x}}{x-2}$

$= \lim_{x\to 2} \frac{-\cancel{(2-x)}}{2x} \cdot \frac{1}{\cancel{(x-2)}} = \lim_{x\to 2} \frac{-1}{2x} = \boxed{-\frac{1}{4}}$

9) $$\lim_{x\to 6} \frac{x^4-4x^3-7x^2-31x+6}{x^3-x^2-33x+18}$$

$$\begin{array}{r|rrrrr} \underline{6} & 1 & -4 & -7 & -31 & 6 \\ + & \downarrow & 6 & 12 & 30 & -6 \\ \hline & 1 & 2 & 5 & -1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} \underline{6} & 1 & -1 & -33 & 18 \\ + & \downarrow & 6 & 30 & -18 \\ \hline & 1 & 5 & -3 & 0 \end{array}$$

$$= \lim_{x\to 6} \frac{\cancel{(x-6)}(x^3+2x^2+5x-1)}{\cancel{(x-6)}(x^2+5x-3)}$$

$$= \frac{216+72+30-1}{36+30-3} = \boxed{\frac{317}{63}}$$

10) $$\lim_{x\to\infty} \frac{3x^5+7x^3-5x^2+1}{2x^5+2x^2-8} = \lim_{x\to\infty} \frac{3+\cancel{\frac{7}{x^2}}-\cancel{\frac{5}{x^3}}+\cancel{\frac{1}{x^5}}}{2+\cancel{\frac{2}{x^3}}-\cancel{\frac{8}{x^5}}}$$

$$= \boxed{\frac{3}{2}}$$

11) $f(-5) = \boxed{1}$

12) $f(3) = \boxed{DNE}$

13) $\lim_{x \to -3} f(x) = \boxed{\infty \text{ or } DNE}$

14) $\lim_{x \to 3^-} f(x) = \boxed{1}$

15) $\lim_{x \to 3^+} f(x) = \boxed{-1}$

16) $\lim_{x \to 3} f(x) \boxed{DNE}$

17) $\lim_{x \to -\infty} f(x) = \boxed{0}$

18) $\lim_{x \to \infty} f(x) = \boxed{-\infty}$

19) $f(x)$ is discontinuous @ $x = 3$ b/c $\lim_{x \to 3} f(x)$ DNE. or $f(3)$ DNE

20) $f(x)$ is continuous @ $x = -2$ b/c

1) $f(-2) = 3$
2) $\lim_{x \to -2} f(x) = 3$
3) $f(-2) = \lim_{x \to -2} f(x)$

Bonus

$$f(x) = \frac{1}{2\sin^2 x - \sin x - 1} = \frac{1}{(2\sin x + 1)(\sin x - 1)}$$

VA: $2\sin x + 1 = 0$

$\sin x = -\frac{1}{2} \Rightarrow \boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

$\sin x - 1 = 0$

$\sin x = 1$

$\boxed{x = \frac{\pi}{2}}$