7 The sum of the first and third terms of a geometric sequence is 40 while the sum of its second and fourth terms is 96. Find the sixth term of the sequence.

8 The sum of three successive terms of a geometric sequence is $\frac{35}{2}$, while their product is 125. Find the three terms.

9 The population in a town of 40 000 increases at 3% per annum. Estimate the town's population after 10 years.

10 Following new government funding it is expected that the unemployed workforce will decrease by 1.2% per month. Initially there are 120000 people unemployed. How large an unemployed workforce can the government expect to report in 8 months time.

11 The cost of erecting the ground floor of a building is \$44 000, for erecting the first floor it costs \$46 200, to erect the second floor costs \$48 510 and so on.
Using this cost structure:

a How much will it cost to erect the 5th floor?

b What will be the total cost of erecting a building with six floors?

8.2.2 Geometric series

When the terms of a geometric sequence are added, the result is a **geometric series.**
For example:
The sequence 3, 6, 12, 24, 48, ... gives rise to the series: 3 + 6 + 12 + 24 + 48 + ...

and, the sequence $24, -16, 10\frac{2}{3}, -7\frac{1}{9}, \ldots$ leads to the series $24 - 16 + 10\frac{2}{3} - 7\frac{1}{9} + \ldots$

Geometric series can be summed using the formula that is derived by first multiplying the series by r:

$$S_n = a + ar + ar^2 + ar^3 + \ldots\ldots\ldots + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

$$r \times S_n = ar + ar^2 + ar^3 + \ldots\ldots\ldots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

$$S_n - r \times S_n = a - ar^n \quad \text{(subtracting the second equation from the first)}$$

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

This formula can also be written as: $S_n = \frac{a(r^n-1)}{r-1}, r \neq 1$. It is usual to use the version of the formula that gives a positive value for the denominator. And so, we have:

The sum of the first n terms of a geometric series, S_n, where $r \neq 1$ is given by

$$S_n = \frac{a(1-r^n)}{1-r}, |r| < 1 \quad \text{or} \quad S_n = \frac{a(r^n-1)}{r-1}, |r| > 1$$

Example 8.18

Sum the following series to the number of terms indicated.

a $2+4+8+16+\ldots$ 9 terms

b $5-15+45-135+\ldots$ 7 terms

c $24+18+\frac{27}{2}+\frac{81}{8}+\ldots$ 12 terms

d $20-30+45-67.5+\ldots$ 10 terms

Solution

a In this case $a=2$, $r=2$ and $n=9$.

Because $r=2$ it is more convenient to use:

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$S_9 = \frac{2(2^9-1)}{2-1}$$

$$= 1022$$

Using this version of the formula gives positive values for the numerator and denominator. The other version is correct but gives negative numerator and denominator and hence the same answer.

b $a=5$, $r=-3$ and $n=7$.

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_7 = \frac{5(1-(-3)^7)}{1-(-3)}$$

$$= 2735$$

or

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$S_7 = \frac{5((-3)^7-1)}{(-3)-1}$$

$$= 2735$$

c $a=24$, $r=0.75$ and $n=12$.

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{12} = \frac{24\left(1-\left(\frac{3}{4}\right)^{12}\right)}{1-\left(\frac{3}{4}\right)}$$ This version gives the positive values.

$$= 92.95907$$

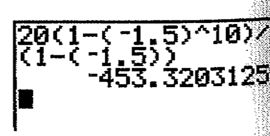
d $a=20$, $r=-1.5$ and $n=10$.

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{10} = \frac{20(1-(-1.5)^{10})}{1-(-1.5)}$$

$$= -453.32031$$

When using a calculator to evaluate such expressions, it is advisable to use brackets to ensure that correct answers are obtained. For both the graphics and scientific calculator, the negative common ratio must be entered using the +/– or (–) key.



Other questions that may be asked in examinations could involve using both formulae. A second possibility is that you may be asked to apply sequence and series theory to some simple problems.

Example 8.19

The second term of a geometric series is –30 and the sum of the first two terms is –15. Find the first term and the common ratio.

Solution

From the given information we have: $u_2 = -30 \therefore ar = -30 \quad (1)$

$$S_2 = -15 \therefore \frac{a(r^2-1)}{r-1} = -15 \quad (2)$$

The result is a pair of simultaneous equations in the two unknowns. The best method of solution is substitution:

From (1): $a = \frac{-30}{r}$. Substituting into (2): $\frac{\frac{-30}{r}(r^2-1)}{r-1} = -15 \Leftrightarrow \frac{(-30)(r^2-1)}{r(r-1)} = -15$

$$\therefore \frac{-30(r+1)(r-1)}{r(r-1)} = -15$$
$$\Leftrightarrow -30(r+1) = -15r$$
$$\Leftrightarrow -30r - 30 = -15r$$
$$\Leftrightarrow r = -2$$
$$\therefore a = \frac{-30}{r} = \frac{-30}{-2} = 15$$

The series is 15 – 30 + 60 – 120 + 240 – ... which meets the conditions set out in the question.

Example 8.20

A family decide to save some money in an account that pays 9% annual compound interest calculated at the end of each year. They put \$2500 into the account at the beginning of each year. All interest is added to the account and no withdrawals are made. How much money will they have in the account on the day after they have made their tenth payment?

Solution

The problem is best looked at from the last payment of \$2500 which has just been made and which has not earned any interest.

The previous payment has earned one lot of 9% interest and so is now worth 2500×1.09.

The previous payment has earned two years' worth of compound interest and is worth 2500×1.09^2.

This process can be continued for all the other payments and the various amounts of interest that each has earned. They form a geometric series:

Last payment First payment

$$2500 + 2500 \times 1.09 + 2500 \times 1.09^2 + \ldots\ldots + 2500 \times 1.09^9$$

The total amount saved can be calculated using the series formula:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{2500(1.09^{10} - 1)}{1.09 - 1}$$

$$= 37982.32$$

The family will save about $37 982.

Exercise 8.2.2

1 Find the common ratios of these geometric sequences:

a $7, 21, 63, 189, \ldots$

b $12, 4, \frac{4}{3}, \frac{4}{9}, \ldots$

c $1, -1, 1, -1, 1, \ldots$

d $9, -3, 1, -\frac{1}{3}, \frac{1}{9}, \ldots$

e $64, 80, 100, 125, \ldots$

f $27, -18, 12, -8, \ldots$

2 Find the term indicated for each of these geometric sequences.

a $11, 33, 99, 297, \ldots$ 10th term.

b $1, 0.2, 0.04, 0.008, \ldots.$ 5th term.

c $9, -6, 4, -\frac{8}{3}, \ldots$ 9th term.

d $21, 9, \frac{27}{7}, \frac{81}{49}, \ldots$ 6th term.

e $-\frac{1}{3}, -\frac{1}{4}, -\frac{3}{16}, -\frac{9}{64}, \ldots$ 6th term.

3 Find the number of terms in each of these geometric sequences and the sum of the numbers in each sequence:

a $4, 12, 36, \ldots, 236196$

b $11, -22, 44, \ldots, 704$

c $100, -10, 1, \ldots, -10^{-5}$

d $48, 36, 27, \ldots, \frac{6561}{1024}$

e $\frac{1}{8}, -\frac{9}{32}, \frac{81}{128}, \ldots, \frac{6561}{2048}$

f $100, 10, 1, \ldots, 10^{-10}$

4 Write the following in expanded form and evaluate.

a $\sum_{k=1}^{7} \left(\frac{1}{2}\right)^k$

b $\sum_{i=1}^{6} 2^{i-4}$

c $\sum_{j=1}^{4} \left(\frac{2}{3}\right)^j$

d $\sum_{s=1}^{4} (-3)^s$

e $\sum_{t=1}^{6} 2^{-t}$

5 The third term of a geometric sequence is 36 and the tenth term is 78732. Find the first term in the sequence and the sum of these terms.

6 A bank account offers 9% interest compounded annually. If $750 is invested in this account, find the amount in the account at the end of the twelfth year.

7 When a ball is dropped onto a flat floor, it bounces to 65% of the height from which it was dropped. If the ball is dropped from 80 cm, find the height of the fifth bounce.

8 A computer loses 30% of its value each year.

a Write a formula for the value of the computer after n years.

b How many years will it be before the value of the computer falls below 10% of its original value?

9 A geometric sequence has a first term of 7 and a common ratio of 1.1. How many terms must be taken before the value of the term exceeds 1000?

10 A colony of algae increases in size by 15% per week. If 10 grams of the algae are placed in a lake, find the weight of algae that will be present in the lake after 12 weeks. The lake will be considered 'seriously polluted' when there is in excess of 10 000 grams of algae in the lake. How long will it be before the lake becomes seriously polluted?

11 A geometric series has nine terms, a common ratio of 2 and a sum of 3577. Find the first term.

12 A geometric series has a third term of 12, a common ratio of $-\frac{1}{2}$ and a sum of $32\frac{1}{16}$. Find the number of terms in the series.

13 A geometric series has a first term of 1000, seven terms and a sum of $671\frac{7}{8}$. Find the common ratio.

14 A geometric series has a third term of 300, and a sixth term of 37500. Find the common ratio and the sum of the first fourteen terms (in scientific form correct to two significant figures).

15 A $10 000 loan is offered on the following terms: 12% annual interest on the outstanding debt calculated monthly. The required monthly repayment is $270. How much will still be owing after nine months.

16 As a prize for inventing the game of chess, its originator is said to have asked for one grain of wheat to be placed on the first square of the board, 2 on the second, 4 on the third, 8 on the fourth and so on until each of the 64 squares had been covered. How much wheat would have been the prize?

8.2.3 Combined arithmetic and geometric progressions

There will be occasions on which questions will be asked that relate to both arithmetic and geometric sequences and series.

Example 8.21

A geometric sequence has the same first term as an arithmetic sequence. The third term of the geometric sequence is the same as the tenth term of the arithmetic sequence with both being 48. The tenth term of the arithmetic sequence is four times the second term of the geometric sequence. Find the common difference of the arithmetic sequence and the common ratio of the geometric sequence.

Solution

When solving these sorts of question, write the data as equations, noting that a is the same for both sequences. Let u_n denote the general term of the arithmetic sequence and v_n the general term of the geometric sequence.

We then have:

$$u_{10} = a + 9d = 48,\ v_3 = ar^2 = 48,$$

i.e. $a+9d = ar^2 = 48$ (1)

$u_{10} = 4v_2 \Rightarrow a+9d = 4ar$ (2)

(1) represents the information 'The third term of the geometric sequence is the same as the tenth term of the arithmetic sequence with both being 48'.

(2) represents 'The tenth term of the arithmetic sequence is four times the second term of the geometric sequence'.

There are three equations here and more than one way of solving them. One of the simplest is:

From (1) $a+9d = 48$ and so substituting into (2): $48 = 4ar \Leftrightarrow ar = 12$ (3)

Also from (1) we have: $ar^2 = 48 \Leftrightarrow (ar)r = 48$ (4)

Substituting (3) into (4): $12r = 48 \Leftrightarrow r = 4$

Substituting result into (1): $a \times 16 = 48 \Leftrightarrow a = 3$

Substituting result into (1): $3+9d = 48 \Leftrightarrow d = 5$

The common ratio is 4 and the common difference is 5.

It is worth checking that the sequences are as specified:

Geometric sequence: 3, 12, 48

Arithmetic sequence: 3, 8, 13, 18, 23, 28, 33, 38, 43, 48

Exercise 8.2.3

1 Consider the following sequences:

Arithmetic: 100, 110, 120, 130, ...

Geometric: 1, 2, 4, 8, 16, ...

Prove that:

The terms of the geometric sequence will exceed the terms of the arithmetic sequence after the 8th term.

The sum of the terms of the geometric sequence will exceed the sum of the terms of the arithmetic after the 10th term.

2 An arithmetic series has a first term of 2 and a fifth term of 30. A geometric series has a common ratio of −0.5. The sum of the first two terms of the geometric series is the same as the second term of the arithmetic series. What is the first term of the geometric series?

3 An arithmetic series has a first term of −4 and a common difference of 1. A geometric series has a first term of 8 and a common ratio of 0.5. After how many terms does the sum of the arithmetic series exceed the sum of the geometric series?

4 The first and second terms of an arithmetic and a geometric series are the same and are equal to 12. The sum of the first two terms of the arithmetic series is four times the first term of the geometric series. Find the first term of each series, if the A.P. has $d = 4$.

5 Bo-Youn and Ken are to begin a savings program. Bo-Youn saves \$1 in the first week \$2 in the second week, \$4 in the third and so on, in geometric progression. Ken saves \$10 in the first week, \$15 in the second week, \$20 in the third and so on, in arithmetic progression.
After how many weeks will Bo-Youn have saved more than Ken?

6 Ari and Chai begin a training program. In the first week Chai will run 10 km, in the second he will run 11 km and in the third 12 km, and so on, in arithmetic progression. Ari will run 5 km in the first week and will increase his distance by 20% in each succeeding week.

a When does Ari's weekly distance first exceed Chai's?

b When does Ari's total distance first exceed Chai's?

7 The Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, … in which each term is the sum of the previous two terms is neither arithmetic nor geometric. However, after the eighth term (21) the sequence becomes approximately geometric. If we assume that the sequence is geometric:

a What is the common ratio of the sequence (to four significant figures)?

b Assuming that the Fibonacci sequence can be approximated by the geometric sequence after the eighth term, what is the approximate sum of the first 24 terms of the Fibonacci sequence?

8.2.4 Convergent series

If a geometric series has a common ratio between −1 and 1, the terms get smaller and smaller as n increases.

The sum of these terms is still given by the formula: $S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$

For $-1 < r < 1, r^n \to 0$ as $n \to \infty \Rightarrow S_n = \frac{a}{1-r}$.

If $|r| < 1$, the infinite sequence has a sum given by $S_\infty = \frac{a}{1-r}$.

This means that if the common ratio of a geometric series is between −1 and 1, the sum of the series will approach a value of $\frac{a}{1-r}$ as the number of terms of the series becomes large, i.e. the **series is convergent**.

Example 8.22

Find the sum to infinity of the series:

a $16 + 8 + 4 + 2 + 1 + \ldots$ **b** $9 - 6 + 4 - \frac{8}{3} + \frac{16}{9} - \ldots$

Solution

a $16 + 8 + 4 + 2 + 1 + \ldots$

In this case $a = 16, r = \frac{1}{2} \Rightarrow S_\infty = \frac{a}{1-r} = \frac{16}{1-\frac{1}{2}} = 32$

b $9 - 6 + 4 - \frac{8}{3} + \frac{16}{9} -$

$a = 9, r = -\frac{2}{3} \Rightarrow S_\infty = \frac{a}{1-r} = \frac{9}{1-\left(-\frac{2}{3}\right)} = 5.4$

There are many applications for convergent geometric series. The following examples illustrate two of these.

Example 8.23

Use an infinite series to express the recurring decimal $0.\dot{4}6\dot{2}$ as rational number.

Solution

$0.\dot{4}6\dot{2}$ can be expressed as the series: $0.462 + 0.000462 + 0.000000462 + \ldots$

or $\frac{462}{1000} + \frac{462}{1000000} + \frac{462}{1000000000} + \ldots$

This is a geometric series with $a = \frac{462}{1000}, r = \frac{1}{1000}$

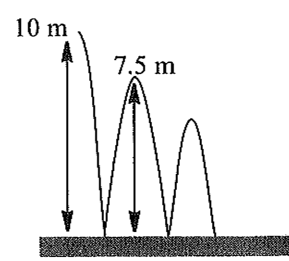
It follows that $S_\infty = \frac{a}{1-r} = \frac{\frac{462}{1000}}{1-\frac{1}{1000}} = \frac{\frac{462}{1000}}{\frac{999}{1000}} = \frac{462}{999}$

Example 8.24

A ball is dropped from a height of 10 metres. On each bounce the ball bounces to three-quarters of the height of the previous bounce. Find the distance travelled by the ball before it comes to rest (if it does not move sideways).

Solution

The ball bounces in a vertical line and does not move sideways. On each bounce after the drop, the ball moves both up and down and so travels twice the distance of the height of the bounce.



Distance $= 10 + 15 + 15 \times \frac{3}{4} + 15 \times \left(\frac{3}{4}\right)^2 + \ldots$

All but the first term of this series are geometric $a = 15, r = \frac{3}{4}$

Distance $= 10 + S_\infty = 10 + \frac{15}{1-\frac{3}{4}} = 70$ m

Exercise 8.2.4

1 Evaluate:

a $27 + 9 + 3 + \frac{1}{3} + \ldots$

b $1 - \frac{3}{10} + \frac{9}{100} - \frac{27}{1000} + \ldots$

c $500 + 450 + 405 + 364.5 + \ldots$

d $3 - 0.3 + 0.03 - 0.003 + 0.0003 - \ldots$

2 Use geometric series to express the recurring decimal $23.232323\ldots$ as a mixed number.

Biologists estimate that there are 1000 trout in a lake. If none are caught, the population will increase at 10% per year. If more than 10% are caught, the population will fall. As an approximation, assume that if 25% of the fish are caught per year, the population will fall by 15% per year. Estimate the total catch before the lake is 'fished out'. If the catch rate is reduced to 15%, what is the total catch in this case? Comment on these results.

4 Find the sum to infinity of the sequence 45, −30, 20, . . .

5 The second term of a geometric sequence is 12 while the sum to infinity is 64. Find the first three terms of this sequence.

6 Express the following as rational numbers.

a $0.3\dot{6}$ b $0.\dot{3}\dot{7}$ c $2.1\dot{2}$

7 A swinging pendulum covers 32 centimetres in its first swing, 24 cm on its second swing, 18 cm on its third swing and so on. What is the total distance this pendulum swings before coming to rest?

8 The sum to infinity of a geometric sequence is $\frac{27}{2}$ while the sum of the first three terms is 13. Find the sum of the first 5 terms.

9 Find the sum to infinity of the sequence $1+\sqrt{3}, 1, \frac{1}{\sqrt{3}+1}, \ldots$

10 a Find: i $\sum_{i=0}^{n}(-t)^i,\ |t|<1$ ii $\sum_{i=0}^{\infty}(-t)^i,\ |t|<1$.

b i Hence, show that $\ln(1+x) = x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\ldots,\ |x|<1$

ii Using the above result, show that $\ln 2 = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$.

11 a Find: i $\sum_{i=0}^{n}(-t^2)^i,\ |t|<1$ ii $\sum_{i=0}^{\infty}(-t^2)^i,\ |t|<1$.

b i Hence, show that $\arctan x = x-\frac{1}{3}x^3+\frac{1}{5}x^5-\frac{1}{7}x^7+\ldots,\ |x|<1$

ii Using the above result, show that $\frac{\pi}{4} = 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$.

Exercise 8.2.5 Miscellaneous questions

1 $2k+2$, $5k+1$ and $10k+2$ are three successive terms of a geometric sequence. Find the value(s) of k.

2 Evaluate $\dfrac{1+2+3+\ldots+10}{1+\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{512}}$.

3 Find a number which, when added to each of 2, 6 and 13, gives three numbers in geometric sequence.

4 Find the fractional equivalent of:

a $2.3\dot{8}$ b $4.\dot{6}\dot{2}$ c $0.41717\ldots$

5 Find the sum of all integers between 200 and 400 that are divisible by 6.

6 Find the sum of the first 50 terms of an arithmetic progression given that the 15th term is 34 and the sum of the first 8 terms is 20.

7 Find the value of p so that $p + 5$, $4p + 3$ and $8p - 2$ will form successive terms of an arithmetic progression.

8 For the series defined by $S_n = 3n^2 - 11n$, find t_n and hence show that the sequence is arithmetic.

9 How many terms of the series $6 + 3 + \frac{3}{2} + \ldots$ must be taken to give a sum of $11\frac{13}{16}$?

10 If $1 + 2x + 4x^2 + \ldots = \frac{3}{4}$, find the value of x.

11 Logs of wood are stacked in a pile so that there are 15 logs on the top row, 16 on the next row, 17 on the next and so on. There are 246 logs in total.

a How many rows are there?

b How many logs are there in the bottom row?

12 The lengths of the sides of a right-angled triangle form the terms of an arithmetic sequence. If the hypotenuse is 15 cm in length, what is the length of the other two sides?

13 The sum of the first 8 terms of a geometric series is 17 times the sum of its first four terms. Find the common ratio.

14 Three numbers a, b and c whose sum is 15 are successive terms of a G.P, and b, a, c are successive terms of an A.P. Find a, b and c.

15 The sum of the first n terms of an arithmetic series is given by $S_n = \frac{n(3n+1)}{2}$.

a Calculate S_1 and S_2.

b Find the first three terms of this series.

c Find an expression for the nth term.

16 An ant walks along a straight path. After travelling 1 metre it stops, turns through an angle of 90° in an anticlockwise direction and sets off in a straight line covering a distance of half a metre. Again, the ant turns through an angle of 90° in an anticlockwise direction and sets off in a straight line covering a quarter of a metre. The ant continues in this manner indefinitely.

a How many turns will the ant have made after covering a distance of $\frac{63}{32}$ metres?

b How far will the ant eventually travel?

13 a 145 **b** 390 **c** −1845
14 b $3n-2$

Exercise 8.2.1

1 a $r = 2, u_5 = 48, u_n = 3 \times 2^{n-1}$ **b** $r = \frac{1}{3}, u_5 = \frac{1}{27}, u_n = 3 \times \left(\frac{1}{3}\right)^{n-1}$
c $r = \frac{1}{5}, u_5 = \frac{2}{625}, u_n = 2 \times \left(\frac{1}{5}\right)^{n-1}$ **d** $r = -4, u_5 = -256, u_n = -1 \times (-4)^{n-1}$
e $r = \frac{1}{b}, u_5 = \frac{a}{b^3}, u_n = ab \times \left(\frac{1}{b}\right)^{n-1}$ **f** $r = \frac{b}{a}, u_5 = \frac{b^4}{a^2}, u_n = a^2 \times \left(\frac{b}{a}\right)^{n-1}$
2 a ±12 **b** $\frac{\pm\sqrt{5}}{2}$
3 a ±96 **b** 15th
4 a $u_n = 10 \times \left(\frac{5}{6}\right)^{n-1}$ **b** $\frac{15625}{3888} \cong 4.02$ **c** $n = 5$ 4 times
5 $-2, \frac{4}{3}$
6 a i \$4096 **ii** \$2097.15 **b** 6.2 years
7 $\left(u_n = \frac{1000}{169} \times \left(\frac{12}{5}\right)^{n-1}\right), \frac{1990656}{4225} \cong 471.16$
8 2.5, 5, 10 or 10, 5, 2.5
9 53757
10 108 952
11 a \$56156 **b** \$299284

Exercise 8.2.2

1 a 3 **b** $\frac{1}{3}$ **c** −1 **d** $-\frac{1}{3}$ **e** 1.25 **f** $-\frac{2}{3}$
2 a 216513 **b** 1.6384×10^{-10} **c** $\frac{256}{729}$ **d** $\frac{729}{2401}$ **e** $-\frac{81}{1024}$
3 a 11; 354292 **b** 7; 473 **c** 8; 90.90909 **d** 8; 172.778 **e** 5; 2.256
f 13; 111.1111111111
4 a $\frac{127}{128}$ **b** $\frac{63}{8}$ **c** $\frac{130}{81}$ **d** 60 **e** $\frac{63}{64}$
5 4; 118096
6 \$2109.50
7 9.28 cm
8 a $V_n = V_0 \times 0.7^n$ **b** 7
9 54
10 53.5 gms; 50 weeks.
11 7
12 9
13 −0.5, −0.7797

14 $r = 5$, 1.8×10^{10}
15 \$8407.35
16 1.8×10^{19} or about 200 billion tonnes.

Exercise 8.2.3

1 Term 9 AP = 180, GP = 256. Sum to 11 terms AP = 1650, GP = 2047.
2 18
3 12
4 7, 12
5 8 weeks Ken \$220 and Bo-Youn \$255)
6 a week 8 **b** week 12
7 a 1.618 **b** 121379 (~121400, depends on rounding errors)

Exercise 8.2.4

1 a $\frac{81}{2}$ **b** $\frac{10}{13}$ **c** 5000 **d** $\frac{30}{11}$
2 $23\frac{23}{99}$
3 6667 fish. (**Note:** $t_{43} < 1$. If we use $n = 43$ then ans is 6660 fish); 20 000 fish.
Overfishing means that fewer fish are caught in the long run.
4 27
5 48, 12, 3 or 16, 12, 9
6 a $\frac{11}{30}$ **b** $\frac{37}{99}$ **c** $\frac{191}{90}$
7 128 cm
8 $\frac{121}{9}$
9 $2 + \frac{4}{3}\sqrt{3}$
10 $\frac{1-(-t)^n}{1+t}$ $\frac{1}{1+t}$
11 $\frac{1-(-t^2)^n}{1+t^2}$ $\frac{1}{1+t^2}$

Exercise 8.2.5 Miscellaneous questions

1 3, −0.2
2 $\frac{2560}{93}$
3 $\frac{10}{3}$
4 a $\frac{43}{18}$ **b** $\frac{458}{99}$ **c** $\frac{413}{990}$
5 9900
6 3275
7 3