2 a Show that $f : x \mapsto \frac{1}{x}$ has an inverse function for all $x \neq 0$.

b Find f^{-1} algebraically and show that f is a self-inverse function.

3 Show that $f : x \mapsto \frac{3x-8}{x-3}$, $x \neq 3$ is a self-inverse function by:

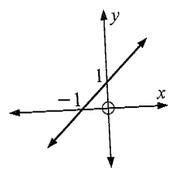
a reference to its graph b using algebra.

4 The '**horizontal line test**' says that:
for a function to have an inverse function, no horizontal line can cut it more than once.

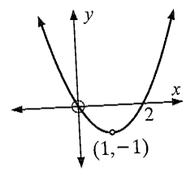
a Explain why this is a valid test for the existence of an inverse function.

b Which of the following functions have an inverse function?

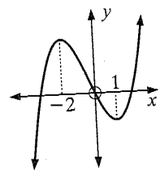
i



ii



iii



c For the functions in b which do not have an inverse, specify domains as wide as possible where each function does have an inverse.

5 Consider $f : x \mapsto x^2$ where $x \leqslant 0$.

a Find $f^{-1}(x)$.

b Sketch $y = f(x)$, $y = x$ and $y = f^{-1}(x)$ on the same set of axes.

6 a Explain why $f : x \mapsto x^2 - 4x + 3$ is a function but does not have an inverse function.

b Explain why f for $x \geqslant 2$ has an inverse function.

c Show that the inverse function of the function in b is $f^{-1}(x) = 2 + \sqrt{1+x}$.

d If the domain of f is restricted to $x \geqslant 2$, state the domain and range of

i f ii f^{-1}.

e Show that $f \circ f^{-1} = f^{-1} \circ f = x$, the identity function.

7 Given $f : x \mapsto (x+1)^2 + 3$ where $x \geqslant -1$:

a find the defining equation of f^{-1}

b sketch, using technology, the graphs of $y = f(x)$, $y = x$ and $y = f^{-1}(x)$

c state the domain and range of i f ii f^{-1}.

8 Consider the functions $f : x \mapsto 2x + 5$ and $g : x \mapsto \frac{8-x}{2}$.

a Find $g^{-1}(-1)$. b Solve for x if $(f \circ g^{-1})(x) = 9$.

9 Given $f : x \mapsto 5^x$ and $g : x \mapsto \sqrt{x}$:

a find i $f(2)$ ii $g^{-1}(4)$ b solve the equation $(g^{-1} \circ f)(x) = 25$.

10 Given $f : x \mapsto 2x$ and $g : x \mapsto 4x - 3$ show that $(f^{-1} \circ g^{-1})(x) = (g \circ f)^{-1}(x)$.

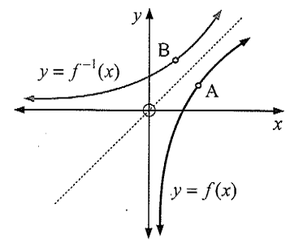
11 Which of these functions is a self-inverse function, i.e., $f^{-1}(x) = f(x)$?

a $f(x) = 2x$ b $f(x) = x$ c $f(x) = -x$ d $f(x) = \frac{2}{x}$ e $f(x) = -\frac{6}{x}$

12 Show that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$ for:

a $f(x) = 3x + 1$ b $f(x) = \frac{x+3}{4}$ c $f(x) = \sqrt{x}$

13



a B is the image of A under a reflection in the line $y = x$.
If A is $(x, f(x))$, what are the coordinates of B under the reflection?

b Substitute your result from a into $y = f^{-1}(x)$. What result do you obtain?

c Explain how to establish that $f(f^{-1}(x)) = x$ also.

REVIEW SET 1A

1 If $f(x) = 2x - x^2$ find: **a** $f(2)$ **b** $f(-3)$ **c** $f(-\frac{1}{2})$

2 For each of the following graphs determine:

i the range and domain **ii** the x and y-intercepts
iii whether it is a function **iv** if it has an inverse function

a

b

3 Find a, b and c if $f(0) = 5$, $f(-2) = 21$ and $f(3) = -4$ and $f(x) = ax^2 + bx + c$.

4 Draw a sign diagram for:

a $(3x+2)(4-x)$ **b** $\frac{x-3}{x^2+4x+4}$

5 If $f(x) = 2x - 3$ and $g(x) = x^2 + 2$, find: **a** $f(g(x))$ **b** $g(f(x))$

6 Solve for x:

a $\frac{x(x+8)}{x+2} \leqslant 5$ **b** $\frac{3}{x-1} > \frac{5}{2x+1}$

7 Consider $x \mapsto 2x - 7$.

a On the same set of axes graph $y = x$, f and f^{-1}.

b Find $f^{-1}(x)$ using variable interchange.

c Show that $f \circ f^{-1} = f^{-1} \circ f = x$, the identity function.

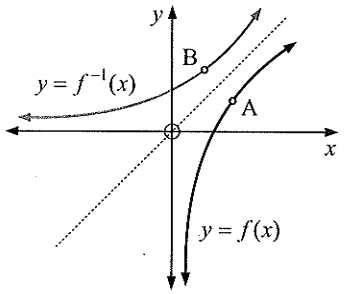
11 Which of these functions is a self-inverse function, i.e., $f^{-1}(x) = f(x)$?

a $f(x) = 2x$ b $f(x) = x$ c $f(x) = -x$ d $f(x) = \frac{2}{x}$ e $f(x) = -\frac{6}{x}$

12 Show that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$ for:

a $f(x) = 3x + 1$ b $f(x) = \frac{x+3}{4}$ c $f(x) = \sqrt{x}$

13



a B is the image of A under a reflection in the line $y = x$.
If A is $(x, f(x))$, what are the coordinates of B under the reflection?

b Substitute your result from a into $y = f^{-1}(x)$. What result do you obtain?

c Explain how to establish that $f(f^{-1}(x)) = x$ also.

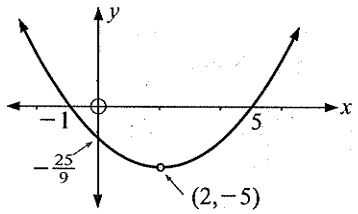
REVIEW SET 1A

1 If $f(x) = 2x - x^2$ find: **a** $f(2)$ **b** $f(-3)$ **c** $f(-\frac{1}{2})$

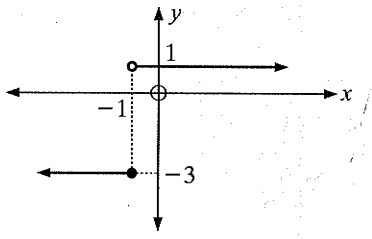
2 For each of the following graphs determine:

i the range and domain **ii** the x and y-intercepts
iii whether it is a function **iv** if it has an inverse function

a



b



3 Find a, b and c if $f(0) = 5$, $f(-2) = 21$ and $f(3) = -4$ and $f(x) = ax^2 + bx + c$.

4 Draw a sign diagram for:

a $(3x + 2)(4 - x)$ **b** $\frac{x-3}{x^2+4x+4}$

5 If $f(x) = 2x - 3$ and $g(x) = x^2 + 2$, find: **a** $f(g(x))$ **b** $g(f(x))$

6 Solve for x:

a $\frac{x(x+8)}{x+2} \leqslant 5$ **b** $\frac{3}{x-1} > \frac{5}{2x+1}$

7 Consider $x \mapsto 2x - 7$.

a On the same set of axes graph $y = x$, f and f^{-1}.
b Find $f^{-1}(x)$ using variable interchange.
c Show that $f \circ f^{-1} = f^{-1} \circ f = x$, the identity function.

8 Solve for x:

a $\left|\dfrac{2x+1}{x-2}\right| = 3$ **b** $|3x-2| \geqslant |2x+3|$

9 For $f : x \mapsto \dfrac{4x+1}{x^2+x-6}$

a determine the asymptotes

b discuss the behaviour of the function as it approaches its asymptotes

c sketch the graph.

10 Given $f : x \mapsto 3x+6$ and $h : x \mapsto \dfrac{x}{3}$, show that $(f^{-1} \circ h^{-1})(x) = (h \circ f)^{-1}(x)$.

REVIEW SET 1B

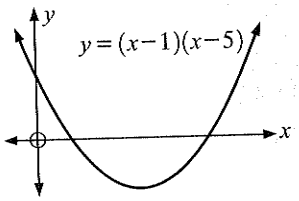
1 If $g(x) = x^2 - 3x$, find in simplest form **a** $g(x+1)$ **b** $g(x^2-2)$

2 For each of the following functions $f(x)$ find $f^{-1}(x)$:

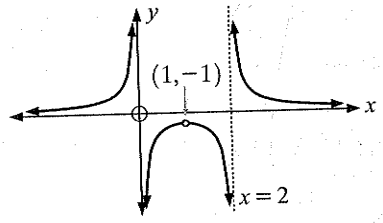
a $f(x) = 7 - 4x$ **b** $f(x) = \dfrac{3+2x}{5}$

3 For each of the following graphs, find the domain and range.

a

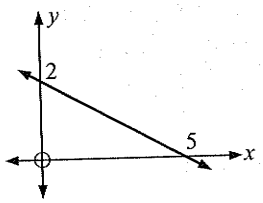


b

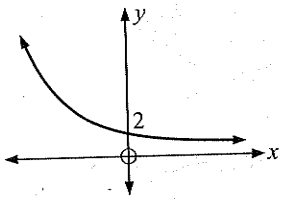


4 Copy the following graphs and draw the graph of each inverse function:

a



b



5 Draw a sign diagram for:

a $\dfrac{x^2-6x-16}{x-3}$ **b** $\dfrac{x+9}{x+5} + x$

6 Solve for x:

a $2x^2 + x \leqslant 10$ **b** $\dfrac{x^2-3x-4}{x+2} > 0$

7 Find an f and a g function given that:

a $f(g(x)) = \sqrt{1-x^2}$ **b** $g(f(x)) = \left(\dfrac{x-2}{x+1}\right)^2$

8 Solve for x:

a $|4x-2| = |x+7|$ **b** $|7-3x| \geqslant 8$

9 For $f(x) = 3 + \dfrac{3x-2}{x^2-4}$

a determine the asymptotes

b discuss the behaviour of the function as it approaches its asymptotes

c sketch the graph

d find the coordinates of all points where the function crosses its asymptotes.

10 Given $h : x \mapsto (x-4)^2 + 3, \quad x \in [4, \infty[$

a find the defining equation of h^{-1} **b** show that $h \circ h^{-1} = h^{-1} \circ h = x$

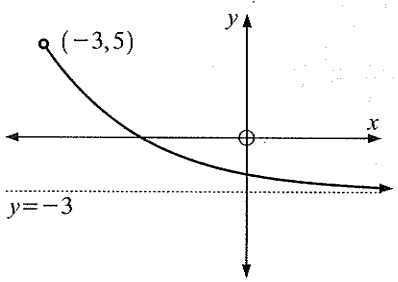
REVIEW SET 1C

1 If $h(x) = 7 - 3x$:

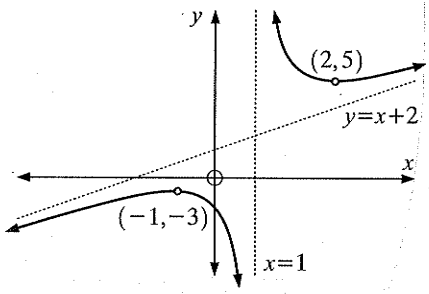
a find in simplest form $h(2x-1)$ **b** find x if $h(2x-1) = -2$

2 For each of the following graphs find the domain and range:

a



b



3 If $f(x) = 1 - 2x$ and $g(x) = \sqrt{x}$:

a find in simplest form **i** $(f \circ g)(x)$ **ii** $(g \circ f)(x)$

b What is the domain and range of $f \circ g$ and $g \circ f$?

4 Solve for x:

a $\dfrac{x^2-3}{x-2} < 6$ **b** $\dfrac{2x+1}{x-1} \geqslant \dfrac{2x+3}{x+2}$

5 Consider $f(x) = \dfrac{1}{x^2}$.

a For what value of x is $f(x)$ meaningless?

b Sketch the graph of this function using technology.

c State the domain and range of the function.

6 **a** Solve graphically: $|2x-6| > x+3$

b Graph the function $f(x) = \dfrac{x}{|x|+1}$ and hence find all values of x for which $\dfrac{x}{|x|+1} \geqslant \frac{1}{3}$

7 **a** Draw a sign diagram for $\dfrac{(x+2)(x-3)}{x-1}$.

b Hence, solve for x: $\dfrac{x^2+x-8}{x-1} < 2$

8 For $f(x) = x - 2 + \dfrac{5}{(x-1)^2}$

a determine the asymptotes

b discuss the behaviour of the function as it approaches its asymptotes

c sketch the graph.

9 Find $f^{-1}(x)$ given that $f(x)$ is: **a** $4x + 2$ **b** $\dfrac{3 - 5x}{4}$

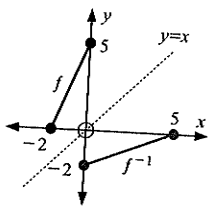
10 **a** Sketch the graph of $g : x \mapsto x^2 + 6x + 7$.

b Explain why g for $x \in]-\infty, -3]$ has an inverse function g^{-1}.

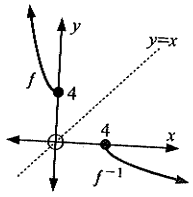
c Find algebraically, the equation of g^{-1}. **d** Sketch the graph of g^{-1}.

e Find the range of g and hence the domain and range of g^{-1}.

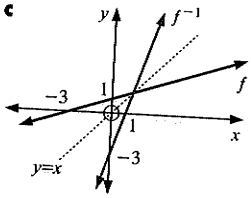
3 **a**



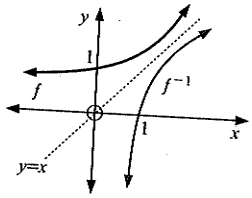
b



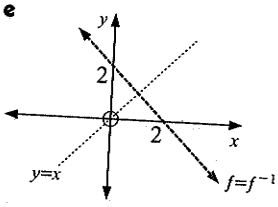
c



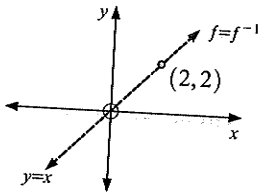
d



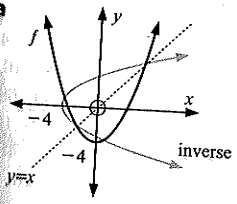
e



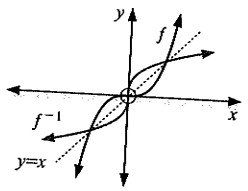
f



4 **a**



5



b No **c** Yes, it is $y = \sqrt{x+4}$

EXERCISE 1J

a $\{(2,1),(4,2),(5,3)\}$ **b** not invertible

c $\{(0,-1),(1,2),(2,0),(3,1)\}$ **d** $\{(-1,-1),(0,0),(1,1)\}$

e $f : x \mapsto \dfrac{1}{x}$, $x \neq 0$ satisfies both the vertical and horizontal line tests and $\therefore$ has an inverse function.

f $f^{-1}(x) = \dfrac{1}{x}$ and $f(x) = \dfrac{1}{x}$ i.e., $f = f^{-1}$

$\therefore$ f is a self-inverse function

g $y = \dfrac{3x-8}{x-3}$ is symmetrical about $y = x$,

$\therefore$ f is a self-inverse function.

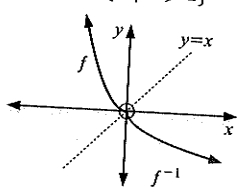
$f^{-1}(x) = \dfrac{3x-8}{x-3}$ and $f(x) = \dfrac{3x-8}{x-3}$

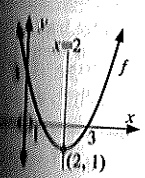
i.e., $f = f^{-1}$ $\therefore$ f is a self-inverse function

h is the only one

i **i** Domain $\{x \mid x \leqslant 1\}$ **iii** Domain $\{x \mid x \geqslant 1\}$

$f^{-1}(x) = -\sqrt{x}$ **b**





b For $x \geqslant 2$, all horizontal lines cut 0 or once only, $\therefore$ has an inverse.

A horizontal line above the vertex cuts the graph **twice**. So, it does not have an inverse.

c **Hint:** Inverse is $x = y^2 - 4y + 3$ for $y \geqslant 2$

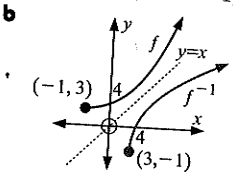
d **i** Domain is $\{x \mid x \geqslant 2\}$, Range is $\{y \mid y \geqslant -1\}$

ii Domain is $\{x \mid x \geqslant -1\}$, Range is $\{y \mid y \geqslant 2\}$

e **Hint:** Find $(f \circ f^{-1})(x)$ and $(f^{-1} \circ f)(x)$ and show that they both equal x.

7 **a** $f^{-1}(x) = \sqrt{x-3} - 1$, $x \geqslant 3$

b



c **i** Domain $\{x \mid x \geqslant -1\}$
Range $\{y \mid y \geqslant 3\}$

ii Domain $\{x \mid x \geqslant 3\}$
Range $\{y \mid y \geqslant -1\}$

8 **a** 10 **b** $x = 3$ **9** **a** **i** 25 **ii** 16 **b** $x = 1$

10 $(f^{-1} \circ g^{-1})(x) = \dfrac{x+3}{8}$ and $(g \circ f)^{-1}(x) = \dfrac{x+3}{8}$

11 **a** Is not **b** Is **c** Is **d** Is **e** Is

13 **a** B is $(f(x), x)$ **b** $x = f^{-1}(f(x)) = (f^{-1} \circ f)(x)$

c Start with B first and repeat the process used in **a** and **b**.

REVIEW SET 1A

1 **a** 0 **b** -15 **c** $-\frac{5}{4}$

2 **a** **i** Range $= \{y \mid y \geqslant -5\}$, Domain $= \{x \mid x \text{ is in } \mathbb{R}\}$

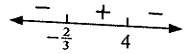
ii x-int. $-1, 5$; y-int. $-\frac{25}{9}$ **iii** is a function **iv** no

b **i** Range $= \{y \mid y = 1 \text{ or } -3\}$ Domain $= \{x \mid x \text{ is in } \mathbb{R}\}$

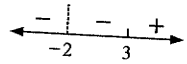
ii no x-intercepts; y-intercept 1 **iii** is a function **iv** no

3 $a = 1$, $b = -6$, $c = 5$

4 **a**



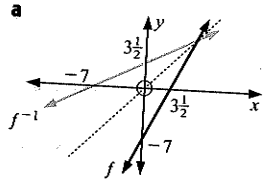
b



5 **a** $2x^2 + 1$ **b** $4x^2 - 12x + 11$

6 **a** $x \in]-\infty, -5]$ or $]-2, 2]$ **b** $x \in]-8, -\frac{1}{2}[$ or $]1, \infty[$

7 **a**



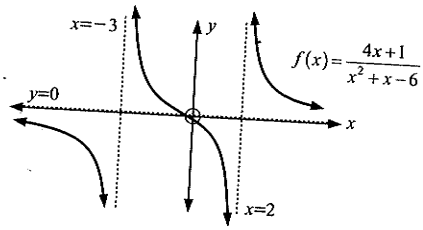
b $f^{-1}(x) = \dfrac{x+7}{2}$

8 **a** $x = 1$ or 7 **b** $x \in]-\infty, -\frac{1}{5}]$ or $[5, \infty[$

9 **a** vertical asymptotes $x = -3$, $x = 2$
horizontal asymptote $y = 0$

b as $x \to -3^-$, $y \to -\infty$
as $x \to -3^+$, $y \to \infty$
as $x \to 2^-$, $y \to -\infty$
as $x \to 2^+$, $y \to \infty$
as $x \to \infty$, $y \to 0^+$
as $x \to -\infty$, $y \to 0^-$

c



10 $(f^{-1} \circ h^{-1})(x) = x - 2$ and $(h \circ f)^{-1}(x) = x - 2$

REVIEW SET 1B

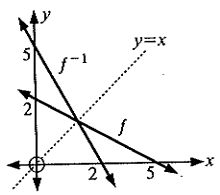
1 **a** $x^2 - x - 2$ **b** $x^4 - 7x^2 + 10$

2 **a** $f^{-1}(x) = \dfrac{7-x}{4}$ **b** $f^{-1}(x) = \dfrac{5x-3}{2}$

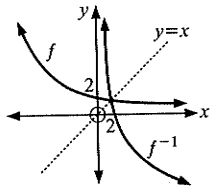
3 **a** Domain $\{x \mid x \in \mathbb{R}\}$, Range $\{y \mid y \geqslant -4\}$

b Domain $\{x \mid x \neq 0, 2\}$, Range $\{y \mid y \leqslant -1 \text{ or } y > 0\}$

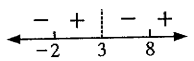
4 a



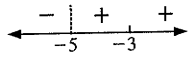
b



5 a



b



6 a $x \in [-\frac{5}{2}, 2]$ **b** $x \in]-2, -1[$ or $]4, \infty[$

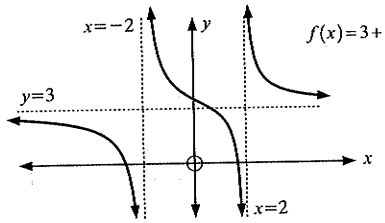
7 a $f(x) = \sqrt{x}$, $g(x) = 1 - x^2$ **b** $g(x) = x^2$, $f(x) = \dfrac{x-2}{x+1}$

8 a $x = -1$ or 3 **b** $x \in]-\infty, -\frac{1}{3}]$ or $[5, \infty[$

9 a vertical asymptotes $x = \pm 2$, horizontal asymptote $y = 3$

b as $x \to -2^-$, $y \to -\infty$ as $x \to \infty$, $y \to 3^+$
as $x \to -2^+$, $y \to \infty$ as $x \to -\infty$, $y \to 3^-$
as $x \to 2^-$, $y \to -\infty$
as $x \to 2^+$, $y \to \infty$

c



d Function crosses horizontal asymptote at $(\frac{2}{3}, 3)$.

10 a $h^{-1}(x) = 4 + \sqrt{x-3}$

REVIEW SET 1C

1 a $10 - 6x$ **b** $x = 2$

2 a Domain $\{x \mid x > -3\}$, Range $\{y \mid -3 < y < 5\}$

b Domain $\{x \mid x \neq 1\}$, Range $\{y \mid y \leqslant -3,\ y \geqslant 5\}$

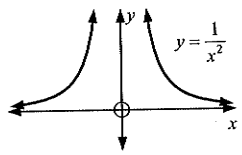
3 a i $1 - 2\sqrt{x}$ **ii** $\sqrt{1-2x}$

b For $f \circ g$, Domain $\{x \mid x \geqslant 0\}$, Range $\{y \mid y \leqslant 1\}$
For $g \circ f$, Domain $\{x \mid x \leqslant \frac{1}{2}\}$, Range $\{y \mid y \geqslant 0\}$

4 a $x \in]-\infty, 2[$ **b** $x \in]-2, -\frac{5}{4}]$ or $]1, \infty[$

5 a $x = 0$

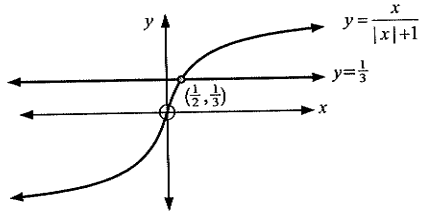
b



c Domain $= \{x \mid x \neq 0\}$,
Range $= \{y \mid y > 0\}$

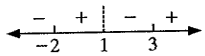
6 a $x \in]-\infty, 1[$ or $]9, \infty[$

b



$\therefore \dfrac{x}{|x|+1} \geqslant \frac{1}{3}$ for $x \in [\frac{1}{2}, \infty[$

7 a

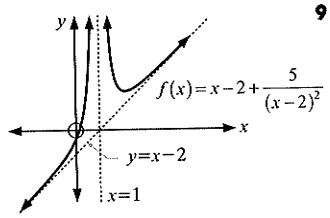


b $x \in]-\infty, -2[$ or $]1, 3[$

8 a vertical asymptote $x = 1$, oblique asymptote $y = x - 2$

b as $x \to 1^-$, $y \to \infty$ as $x \to \infty$, $y \to (x-2)^+$
as $x \to 1^+$, $y \to \infty$ as $x \to -\infty$, $y \to (x-2)^+$

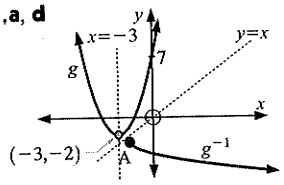
c



9 a $f^{-1}(x) = \dfrac{x-2}{4}$

b $f^{-1}(x) = \dfrac{3-4x}{5}$

10 a, d



b If $x \leqslant -3$, we have the graph to the left of $x = -3$ and any horizontal line cuts it at most once.

c $y = -3 - \sqrt{x+2}$

e Range of g $\{y \mid y \geqslant -2\}$, Domain of g^{-1} $\{x \mid x \geqslant -2\}$
Range of g^{-1} $\{y \mid y \leqslant -3\}$

EXERCISE 2A

1 a 4, 13, 22, 31, ... **b** 45, 39, 33, 27, ...
c 2, 6, 18, 54, ... **d** 96, 48, 24, 12, ...

2 a Starts at 8 and each term is 8 more than the previous term. Next two terms 40, 48.

b Starts at 2, each term is 3 more than the previous term; 14, 17

c Starts at 36, each term is 5 less than the previous term; 16, 11

d Starts at 96, each term is 7 less than the previous term; 68, 61

e Starts at 1, each term is 4 times the previous term; 256, 1024

f Starts at 2, each term is 3 times the previous term; 162, 486

g Starts at 480, each term is half the previous term; 30, 15

h Starts at 243, each term is $\frac{1}{3}$ of the previous term; 9, 3

i Starts at 50 000, each term is $\frac{1}{5}$ of the previous term; 80, 16

3 a Each term is the square of the term number; 36, 49, 64

b Each term is the cube of the term number; 125, 216, 343

c Each term is $n(n+1)$ where n is the term number; 90, 110, 132

EXERCISE 2B

1 a 2, 4, 6, 8, 10 **b** 4, 6, 8, 10, 12 **c** 1, 3, 5, 7, 9
d −1, 1, 3, 5, 7 **e** 5, 7, 9, 11, 13 **f** 15, 16, 17, 18, 21
g 4, 7, 10, 13, 16 **h** 1, 5, 9, 13, 17

2 a 2, 4, 8, 16, 32 **b** 6, 12, 24, 48, 96
c 3, $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$ **d** −2, 4, −8, 16, −32

3 17, 11, 23, −1, 47

EXERCISE 2C

1 a $u_1 = 6$, $d = 11$ **b** $u_n = 11n - 5$ **c** 610
d yes, u_{30} **e** no

2 a $u_1 = 87$, $d = -4$, **b** $u_n = 91 - 4n$ **c** 99 **d** 16

3 b $u_1 = 1$, $d = 3$ **c** 169 **d** $u_{101} = 451$

4 b $u_1 = 32$, $d = -\frac{7}{2}$ **c** −227 **d** $n = 96$

5 a $k = 17\frac{1}{2}$ **b** $k = 4$ **c** $k = 3$, $k = -1$

6 a $u_n = 6n - 1$ **b** $u_n = -\frac{3}{2}n + \frac{11}{2}$ **c** $u_n = 5n + 100$
d $u_n = -\frac{3}{2}n + \frac{1}{2}$

7 a $6\frac{1}{4}, 7\frac{1}{2}, 8\frac{3}{4}$ **b** $3\frac{5}{7}, 8\frac{3}{7}, 13\frac{1}{7}, 17\frac{6}{7}, 22\frac{5}{7}, 27\frac{4}{7}$

8 a $u_1 = 36$, $d = -\frac{2}{3}$ **b** 100 **c** 100 000

EXERCISE 2D.1

1 a $b = 18$, $c = 54$ **b** $b = 2\frac{1}{2}$, $c = 1\frac{1}{4}$ **c** $b = 3$, $c = 2\frac{1}{4}$

2 a $u_1 = 5$, $r = 2$ **b** $u_n = 5 \times 2^{n-1}$, $u_{10} = 2560$

3 a $u_1 = 12$, $r = -\frac{1}{2}$ **b** $u_n = 12 \times (-\frac{1}{2})^{n-1}$, $u_{11} = -\frac{3}{256}$

4 $u_1 = 8$, $r = -\frac{3}{4}$, $u_{10} = -0.600\,677\,49$

5 $u_1 = 8$, $r = \frac{1}{\sqrt{2}}$, $u_n = 2^{\frac{7}{2}}$ **9**