

The following questions relate to evaluations & working with & understanding function notation

(a) Evaluate the following:

$f(2); \approx 1$     $f(-5); = 0$     $f(1); \text{undefined}$     $g(12); = -7$     $f(4); = 4$     $f(-6); \text{no value}$

$f(2) \times g(2); 1 \times -5.5 = -5.5$     $\frac{f(0)}{g(0)}; \frac{0}{-5.25} = 0$     $\frac{g(0)}{f(0)} = \frac{-5.25}{0} = \text{undefined}$

(b) Evaluate

$|f(7)|; |-2| = 2$     $g(|-4|); g(4) = -5$     $|g(-4)|; |2.75| = 2.75$    is  $|g(-4)| = g(4)?$  ; NO    $is\ g(|-4|) = g(4)?$  ; YES  $y = -3$

(c) Evaluate  $f \circ g(-1); f(-5.5) = -1$     $f \circ f(-4); f(2) = 1$     $g \circ f(11); g(-2) = 3.5$     $g^{-1}(2); 0.25$     $f^{-1}(-3) = .75, 1.25, 11.25$

(d) Solve the following:

$f(x) = 7; x = -7$     $g(x) = -5; x = -1.4$     $f(x) = -3; x = 7.5$     $f(x) = 5; x = 1.25, 11.25$     $g(x) = -1; x = -5.5, 8.25$

(e) state the graphical and algebraic significance of  $f(0)$  as well as  $g(0)$

$x = 0$  which represents the y-intercepts

(f) State the domain and range of  $g(x)$  and also for  $f(x)$

$D: f(x) \{x \in \mathbb{R} \mid x \neq -6, 1\}$   
 $D: g(x) \{x \in \mathbb{R} \mid x \geq -5.75\}$     $R: f(x) \{y \in \mathbb{R} \mid y \leq 4 \text{ OR } 6 < y < 8\}$

(g) is  $f(-4.75)$  positive or negative? Explain how you determined this.

positive as  $f(x)$  is ABOVE  $x$  axis at this point

(h) is  $g(-4.75)$  positive or negative? Explain how you determined this.

negative as  $g(x)$  is below  $x$  axis at this point

(i) state the graphical and algebraic significance of  $f(x) = 0$  as well as  $g(x) = 0$

these are the x-intercepts, zeroes or roots where  $y = 0$

(j) For what values of  $x$  is  $f(x) > 0$ ? Explain how you determined this.  $\{x \in \mathbb{R} \mid x < -6 \text{ OR } -5 < x < 0 \text{ OR } 1.75 < x < 5 \text{ OR } 7.75 < x < 10.5\}$   
 anywhere the graph is ABOVE the  $x$ -axis

(k) Solve  $g(x) < 0$ .  
 $\{x \in \mathbb{R} \mid -5 < x < 9\}$

(l) How often does the line  $y = -1$  intersect  $y = f(x)$ ? Intersect  $y = g(x)$  6 times on  $f(x)$ ; twice on  $g(x)$

(m) How often does the line  $x = -1$  intersect  $y = f(x)$ ? Intersect  $y = g(x)$  once on each

intersection

(n) Interpret the meaning of the state  $f(x) = g(x)$  then solve the equation  $f(x) = g(x)$ .  $x = -5.5$  or  $1.2$  or  $9.8$   
 where  $f$  and  $g$  have an identical function value for a given  $x$  value

(o) Interpret the meaning of the state  $f(x) < g(x)$  then solve the inequality  $f(x) > g(x)$ .  
 where  $f$  is BELOW  $g$   $\{x \in \mathbb{R} \mid -5.5 < x < 1.2 \text{ OR } 1.2 < x < 9.8\}$

(p) Calculate the value of the difference quotient  $\frac{f(7) - f(4)}{7 - 4}$  as well as  $\frac{f(9) - f(7)}{9 - 7}$  and explain the sig. of the DQ.

$$\frac{-2 - 4}{7 - 4} = \frac{-6}{3} = -2$$

$$\frac{2 - -2}{9 - 7} = \frac{4}{2} = 2$$

average rate of change

(q) Determine the average rate of change of  $y = g(x)$  between  $x = 4$  and  $x = 9$

$$\frac{g(9) - g(4)}{9 - 4} = \frac{0 - -5}{9 - 4} = \frac{5}{5} = 1$$

Now let's work on other function concepts that relate to characteristics of functions, specifically  $y = f(x)$  now.

(a) On what interval is  $y = f(x)$  increasing, given the restricted domain of  $\{x \in \mathbb{R} \mid -6 < x \leq 7\}$ ?

$$\{x \in \mathbb{R} \mid -6 < x < -2.25 \text{ or } 1 < x < 4\}$$

(b) On what interval is  $y = f(x)$  decreasing, given the restricted domain of  $(-6, 7]$ ?

$$\{x \in \mathbb{R} \mid -2.25 < x < 1 \text{ or } 4 < x < 7\}$$

(c) Where are the local maximums & minimums of  $y = f(x)$ , given the restricted domain of  $\{x \in \mathbb{R} \mid -6 < x \leq 7\}$ ?

$$\text{at } x = -2.25 \text{ (local max at } (-2.25, 3.75) \text{) and } (4, 4) \text{ (local max) and } (7, -2) \text{ (local min)}$$

(d) Given the restricted domain of  $\{x \in \mathbb{R} \mid -10 < x \leq 4\}$ , on what interval is  $y = f(x)$  concave up? Concave down?

$$\text{Concave up } \{x \in \mathbb{R} \mid -6 < x < 1\} \quad \text{Concave Down } \{x \in \mathbb{R} \mid -6 < x < 1 \text{ or } 1 < x < 4\}$$

(e) Where are the roots of  $y = f(x)$ ?

$$x = -5, 0, 1.75, 6, 7.75, 10.5$$

(f) Does  $y = f(x)$  appear to have any asymptotes? If so, where?

$$x = 1 \text{ and } y = 6$$

(g) What does the concept of discontinuities mean, given that I have created  $y = f(x)$  to be a discontinuous function.

(h) What is a jump discontinuity? Where does  $f(x)$  have a "jump" discontinuity?

$$\text{at } x = -6 \text{ \& } x = 4$$

(i) What is an infinite discontinuity? Where does  $f(x)$  have an infinite discontinuity?

$$\text{at } x = 1$$

(g) What does the concept of discontinuities mean, given that I have created  $y = f(x)$  to be a discontinuous function.

(h) What is a jump discontinuity? Where does  $f(x)$  have a "jump" discontinuity?

(i) What is an infinite discontinuity? Where does  $f(x)$  have an infinite discontinuity?

(j)  $f(h(x) = x + 2)$ , what would the graph of  $y = f \circ h(x)$  look like? Why?

$F(x+2) \Rightarrow$  graph of  $F$  is translated 2 units to the Left

(k) If  $h(x) = x + 2$ , what would the graph of  $y = h \circ f(x)$  look like? Why?

$f(x) + 2 \Rightarrow$  graph of  $f$  is translated 2 units Up

(l) What would the graph of  $y = -f(x)$  look like? Why?

a reflection across the  $x$ -axis b/c all  $y$  values have their sign changed by the  $-ve$

(m) What would the graph of  $y = f(-x)$  look like? Why?

a reflection across the  $y$ -axis

(n) Explain how the graph of  $y = \frac{1}{f(x)}$  changes if you are asked to graph  $y = |f(x)|$

all parts of the graph of  $f$  that were BELOW the  $x$ -axis are now reflected across  $x$ -axis

(o) To determine the end behavior of the function, what does the function "do" as  $x \rightarrow +\infty$  and what does the function "do" as  $x \rightarrow -\infty$ ?

as  $x \rightarrow +\infty$ ,  $f(x) \rightarrow -\infty$  as the graph appears to go down

as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow 6$  as the graph levels off at  $y=6$  on the right

(p) What does the term "bounded" mean and explain if/how it applies to  $y = f(x)$  & to  $y = g(x)$

(q) Evaluate  $\lim_{x \rightarrow -5^+} f(x)$  &  $\lim_{x \rightarrow -5^-} f(x)$  &  $\lim_{x \rightarrow -5} f(x)$  &  $f(-5)$ .  
 $\lim_{x \rightarrow -5^+} f(x) = 0$        $\lim_{x \rightarrow -5^-} f(x) = 0$

(r) Evaluate  $\lim_{x \rightarrow -6^+} f(x)$  &  $\lim_{x \rightarrow -6^-} f(x)$  &  $\lim_{x \rightarrow -6} f(x)$  &  $f(-6)$

$\lim_{x \rightarrow -6} f(x) = 0$  and  $f(-6) = 0$

(s) Graph the inverse relation for  $y = g(x)$ .

(t) Classify  $y = f(x)$  &  $y = g(x)$  as being either: (i) one to one, (ii) one to many, (iii) many to one, or (iv) many to many

$f(x)$  one output  $\rightarrow$  many inputs       $g(x)$  one output to 2 inputs

(u) Which function(s) have/has symmetries: (i)  $f(x)$  only, (ii)  $g(x)$  only, (iii) both  $f(x)$  and  $g(x)$ , (iv) neither  $f(x)$  nor  $g(x)$

$$\lim_{x \rightarrow -6^+} f(x) = -2$$

$$\lim_{x \rightarrow -6^-} f(x) = 9$$

$$\lim_{x \rightarrow -6} f(x) = \text{does not exist}$$

$$f(-6) \rightarrow \text{does not exist}$$

