IM3 L7.4 – Probability of Compound Events and Venn Diagrams

(A) Lesson Objectives
    a.   Work through compound events using Venn diagrams as our problem solving strategy
    b.   Introduce the concept of conditional probability through the use of Venn diagrams

## (B) Working with Venn Diagrams

EXAMPLE #1

Numbers are written on cards, the cards are shuffled and one is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram. Complete the following.
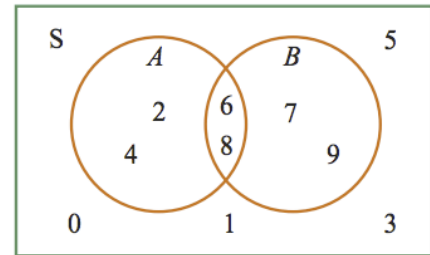
a The sample space S = {_____} (outcomes inside the rectangle)
b Event A = {_____} (outcomes inside the circle A)
c Event 'not A' = {_____} (outcomes outside the circle A)
d Event B = {_____} (outcomes inside circle B)
e Event 'not B' = {_____} (outcomes outside circle B)
f Event 'A and B' = {_____} (outcomes in the intersection of the circles)
g i Event 'A or B or both' = {_____} (outcomes within both circles, including the intersection)
    ii Is this an 'inclusive or' event or an 'exclusive or' event?
h i Event 'A or B but not both' = {_____} (outcomes within both circles, excluding the intersection)
    ii Is this an 'inclusive or' event or an 'exclusive or' event?
i Event 'neither A nor B' = {_____} (outcomes outside both circles)

EXAMPLE #2

In a class of 30 students 5 study Japanese, 13 study French, 9 study German, 2 study both Japanese and French and 5 study both French and German.
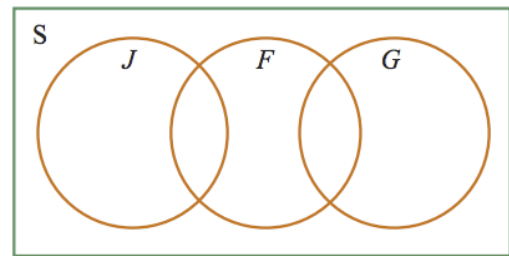
a Complete the Venn diagram where J is the event 'studies Japanese', F is the event 'studies French' and G is the event 'studies German'.
b How many students in this class:
    i do not study a language?
    ii do not study Japanese?
    iii do not study French?
    iv do not study German?
    v study Japanese or French or both?
    vi study Japanese or French but not both?
    vii study French or German or both?
    viii study French or German but not both?
    ix study Japanese or German?
    x study Japanese and French and German?
    xi study Japanese only?
    xii study French only?
    xiii study German only?
    xiv study exactly one language?
    xv study exactly two languages?
    xvi study at least one language?
    xvii study at most one language

EXAMPLE #3:

A six-sided die is rolled. The outcomes for this experiment and the events X and Y are shown in the Venn diagram.

a List the sample space.

b List the set of outcomes for event X.

c List the set of outcomes for the event 'not X'.

d List the set of outcomes for event Y.

e List the set of outcomes for the event 'not Y'.

f List the set of outcomes for event 'X and Y'.

g i List the set of outcomes for the event 'X or Y or both'.

   ii Is this an 'inclusive or' event or an 'exclusive or' event?

h i List the set of outcomes for the event 'X or Y but not both'.

   ii Is this an 'inclusive or' event or an 'exclusive or' event?

i List the outcomes that belong to neither X nor Y.
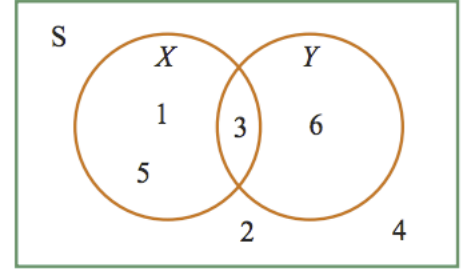
EXAMPLE #4

In a class of 30 students 25 passed the term mathematics test, 24 passed the science test and 23 students passed both tests.

a Draw a Venn diagram to represent this information using M for the event 'passed mathematics' and S for the event 'passed science'.

b How many student from this class:

   i did not pass mathematics?   ii did not pass science?

   iii passed neither mathematics nor science?   iv passed mathematics or science or both?

   v passed mathematics or science but not both? vi passed mathematics but not science?

   vii passed science but not mathematics?   viii passed at least one subject?

   ix passed at most one subject?   x failed at least one subject?

   xi failed at most one subject?

(C) **Probabilities and Venn Diagrams**

1. A group of 60 students were asked if they played field hockey (F), basketball (B) or soccer (S). The diagram below displays the results.

   How probable is it that a randomly chosen student plays::

     i. field hockey & basketball?
     ii. field hockey or basketball?
     iii. field hockey & soccer?
     iv. neither of the three sports?
     v. only 1 sport?

     vi. Basketball **given that** they play soccer?
     vii. Soccer **given that** they play field hockey?
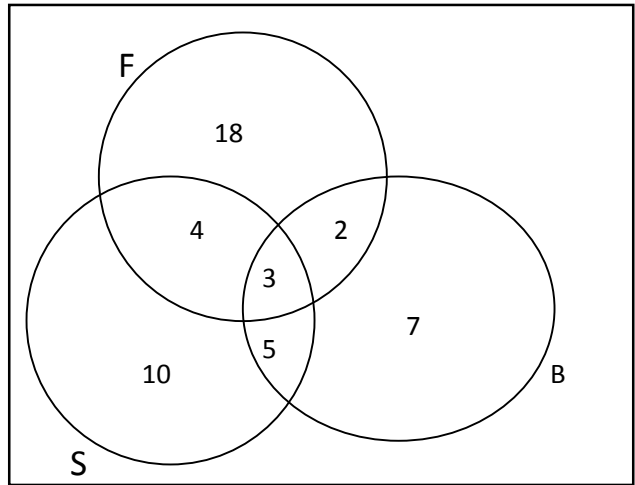
2. The Venn diagram displays the results of a survey of 100 families regarding technology in their homes. Computer (C), DVD player (D) and fax machine (F)

   How probable is it that a family has:

     i. a computer at home?
     ii. all three machines?
     iii. none of the machines in their home?
     iv. no fax machine?
     v. a computer and a VCR?
     vi. a VCR or a computer?

     vii. A computer **given that** they have a fax machine?
     viii. A DVD players **given that** they have a computer?
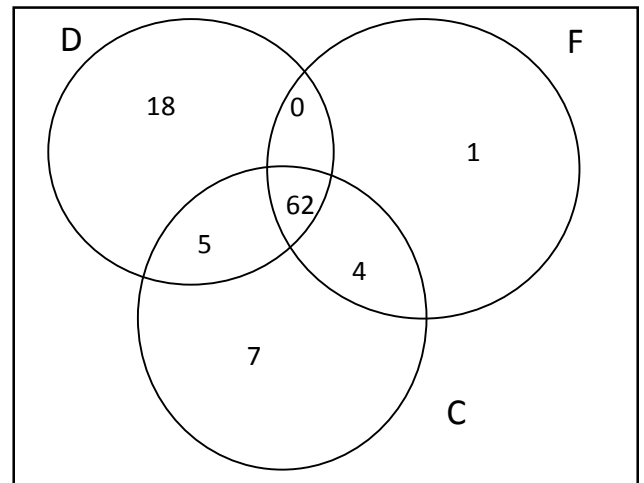
3. Use the Venn Diagram below to answer question (a) – (f).

a. How many total people are represented in the diagram?
b. How many people like country?
c. If one person is chosen at random, what is the probability that:
   i. they will like rap music?
   ii. They will like rap or country music?
   iii. They will not like rock nor country?

   iv. They will like rock **given that** they like country?
   v. They will not like country **given that** they like rock?
   vi. They will not like rap **given that** they do not like country?
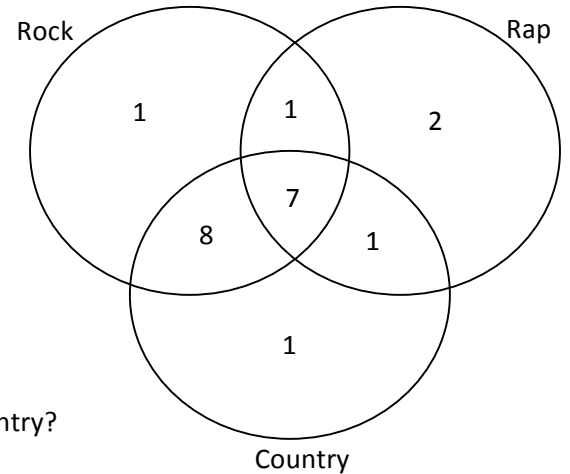
   BONUS: (RESEARCH HOW TO DETERMINE odds)

d. If one person is chosen at random, what are the odds for picking a person who likes country?
e. If one person is chosen at random, what are the odds against picking a person who likes all three types of music?
f. Odds against all three = ?

4. 100 people were asked if they liked Math, Science, or Social Studies. Everyone answered that they liked at least one. The results where that 56 like Math, 43 like Science, 35 like Social Studies, 18 like Math and Science, 10 like Science and Social Studies, 12 like Math and Social Studies and finally 6 like all three subjects. A student is chosen at random. How probable is it that:

a. they like Math only?
b. they like Science only?
c. they like Social Studies only?
d. They like social studies and Science?
e. They like math or science?

f. They like science **given that** they like math?
g. They like social sciences **given that** they do not like math

5. A survey was done to see how many people visit the beach, go camping or go to the waterslides during the summer months. It was found that 35% went camping, 57% went to the beach and 20% went to the waterslides. 15% went camping & to the beach, 8% went to the beach & to the waterslides, 5% went camping & to the waterslides and 3% went to all three. Draw a Venn diagram to display the information and find the probability that a randomly selected person:

    a. went to the beach or went to the slides.
    b. went camping or went to the beach.
    c. only went to one of the three locations.
    d. did none of the three activities.

6. Of the 28 students in a class, 12 have a part time job, 22 have a part time job or do regular volunteer work, and 4 of the students have a part time job and do regular volunteer work.
    a. Display the data in a Venn diagram.
    b. How many of the students do not have a part time job or do not volunteer regularly?
    c. How probable is it that a student does volunteer work given that they have a part time job?

7. Of 400 college students, 120 are enrolled in math, 220 are enrolled in English, and 55 are enrolled in both.
    If a student is selected at random, find the probability that
    a. the student is enrolled in mathematics.
    b. the student is enrolled in mathematics or English.
    c. the student is enrolled in either mathematics or English, but not both.

8. Three of the top Canadian paid-circulation magazines are Reader's Digest, Chatelaine, and MacLean's. A market survey has estimated the probability of a household subscribing to these magazines:

Subscription	Probability
Reader's Digest	0.6
Chatelaine	0.5
MacLean's	0.4
Reader's Digest & Chatelaine	0.2
Reader's Digest & MacLean's	0.25
Chatelaine & MacLean's	0.15
All three	0.05

    a. What is the probability that a household chosen at random
        i. subscribes to only Reader's Digest?
        ii. subscribes to neither Chatelaine nor MacLean's?
        iii. subscribes to one magazine only?
        iv. Subscribes to Readers Digest given that they subscribe to MacLeans?

9. In a survey of 2140 teachers in a certain metropolitan area conducted by a nonprofit organization regarding teacher attitudes, the following data were obtained:

    a. 900 said that lack of parental support is a problem.
    b. 890 said that abused or neglected children are problems.
    c. 680 said that malnutrition or students in poor health is a problem.
    d. 120 said that lack of parental support and abused or neglected children are problems.
    e. 110 said that lack of parental support and malnutrition or poor health are problems.
    f. 140 said that abused or neglected children and malnutrition or poor health are problems.
    g. 40 said that all three issues are problems.

    Draw a Venn diagram and then find the probability that a teacher selected at random from this group said that lack of parental support is the only problem hampering a student's schooling?

10. In a group of 35 children, 10 have blonde hair, 14 have brown eyes, and 4 have both blonde hair and brown eyes. If a child is selected at random, find the probability that the child has blonde hair or brown eyes.

11. Amber, a college senior, interviews with Acme Corp. and Mills, Inc. The probability of receiving an offer from Acme is 0.35, from Mills is 0.48, and from both is 0.15. Find the probability of receiving an offer from either Acme Corp. or Mills, Inc., but not both.

12. A survey of couples in a city found the following probabilities:

    a. The probability that the husband is employed is 0.85.
    b. The probability that the wife is employed is 0.60.
    c. The probability that both are employed is 0.55.

        A couple is selected at random. Find the probability that:
                (i) at least one of them is employed.
                (ii) neither is employed.

13. Each member of a sports club plays at least one of soccer, rugby or tennis. The following is known: 43 members play tennis, 11 play tennis & rugby, 7 play tennis & soccer, 6 play soccer & rugby, 84 play rugby or tennis, 68 play soccer or rugby and 4 play all three sports.

    a. How many members does the club have?
    b. Two members are chosen at random. How probable is it that both play rugby?
    c. Two members are chosen at random. How probable is it that both play rugby, given that neither plays tennis?

Math 12
Venn Diagrams & Basic Probability

1. In a class of 50 students, 18 take choir, 26 take concert band, and 2 students take both. How many students are not in either choir or concert band?

2. In a class of 320 students, 85 take geography and 200 take math. If 60 people are in both classes, how many people take math or geography?

3. A veterinarian surveys 26 of his customers, and finds that 14 have dogs, 10 have cats, and 5 have fish. 4 people have dogs and cats, 3 have dogs and fish, and only one person has a cat and a fish. No one surveyed has all 3 types of pets. How many patrons have none of the 3 pets?

4. What is the probability of selecting a red non-face card from a deck of cards?

5. If the sample space for student test scores is as follows: S = { 3 A's, 6 B's , 9 C's , 4 D's, 3 E's}, and one student's test is selected at random, what is the probability of selecting a B or better?

6. If the number of ways an event can happen is "A" and the number of ways an event cannot happen is "B", then P(A happening) = ?

7. A coin is flipped 3 times, and heads or tails are recorded.
a) what is the sample space?   b) List all the possible outcomes in the event "at least 2 tails"
c) what is the probability of "at least 2 tails" ?

8. If two dice are rolled at the same time, calculate the probability of getting:
a) a sum of at least 10
b) a sum of at most 10
c) a sum which is a multiple of 3

9. Each student at a music camp plays at least one of the following instruments: violin, piano or clarinet. It is known that 6 students play all three instruments, 163 play piano, 36 play piano and violin, 13 play piano and clarinet, 11 play clarinet and violin, 208 play violin or piano, and 98 play clarinet or violin. Draw a Venn diagram to find out how many students there are at the camp.

10. A survey was taken among grade 12 students. It showed that 63% took math 12, 26% took Biology 12 and 18% took both.
a) What is the probability that a student chosen at random took math 12 but not Biology 12 ?
b) What is the probability that a student chosen at random did not take math 12 or Biology 12?

11. If 75% of grade 12 students take math 12, 60% take Chemistry 12, and 50% take both Math and Chemistry, draw a Venn diagram to figure out the probability that a student chosen at random takes:
a) math but not chemistry ?   b) math or chemistry ?
c) math and chemistry ?   d) neither math nor chemistry ?

12. A counsellor is planning schedules for 300 students wanting to take a language course. 160 of the students want French, 160 want Spanish, and 110 want to take Mandarin. 50 want to take both French and Mandarin, and of these students, 30 wanted to take Spanish as well. 50 want only Mandarin, and 80 want only Spanish. How many students want only French?

13. A person was calculating the odds of winning certain prizes in a raffle. She calculated that the probability of winning prize "A" was 0.28 and the probability of winning prize "B" was 0.62. If the probability of winning both prizes was 0.15 , what is the probability that she wins neither prize?

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14. Of the 28 students in a class, 12 have a part time job, 22 have a part-time job or do regular volunteer work, and 4 of the students have a part-time job and do regular volunteer work. Draw a Venn diagram to calculate each of the following probabilities:

a) what is the probability that a student selected at random does volunteer work?

b) what is the probability that a student selected at random has a part-time job?

c) what is the probability that a student selected at random will only volunteer, but does not work part-time?

d) what is the probability that a student selected at random will not have a part-time job, and does not do regular volunteer work?

15. A study was made of 200 people on which reality TV shows they watch. The following data was determined:

- 22 people didn't watch any reality shows
- 73 people watch only American Idol
- 136 people watch American Idol
- 14 people watch only Survivor and Fear Factor
- 31 people watch only American Idol and Fear Factor
- 63 people watch Survivor
- 135 people do not watch Fear Factor.

a) Draw a Venn diagram to support this data.

b) How many people watch ONLY Fear Factor?

16. My cat plays with the mice that she catches. Suppose that in one month she "gives" me: six gray mice, twelve that had their tails missing, and 15 that were chewed on. Only one of the mice was gray, chewed on and had its tail missing. Two were gray and tailless and two were gray and chewed on. If there were a total of 24 mice left on my doorstep in one month, and all of the mice had at least one of the properties (gray, tailless, chewed on), how many of the mice were tailless and chewed on but not gray ?

Answers:

1. 8   2. 225   3. 5   4. 5/13 5. 9/25   6. $A/(A + B)$

7. a) { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}   b) {HTT, THT, TTT, TTH}   c) ½

8. a) 1/6   b) 11/12   c) 1/3   9. 218   10. a) 45%   b) 29%

11. a) 25%   b) 85%   c) 50%   d) 15%

12. 70   13. 0.25   14. a) 1/2   b) 3/7   c) 5/14   d) 3/14

15. a)   15. b) 11   16. 5

American Idol

Survivor

73

23

31

9

17

14

11

22

Fear Factor