

(A) Lesson Context

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> How do algebraically & graphically work with growth and decay applications? What are logarithms and how do we invert or undo an exponential function? How do we work with simple algebraic and graphic situations involving the use of logarithms (or inverting exponentials?) 		
CONTEXT of this LESSON:	<p>Where we've been</p> <p>We have seen algebra skills related to the parent exponential function $f(x) = AB^x$ in Lesson 1 and we've worked with Inverses in SEM 1</p>	<p>Where we are</p> <p>What are & How do work with the inverse of exponential functions?</p>	<p>Where we are heading</p> <p>How do work with the mathematically model $f(x) = AB^{k(x+c)} + d$?</p>

(B) Lesson Objectives:

- How can we summarize number patterns associated with logarithmic & exponential relationships?
- Convert between exponential & logarithmic forms of numerical expressions
- Solve simple logarithmic equations using fundamental knowledge of exponents

(C) EXPLORATION #1: Looking for PATTERNS

Consider the following logarithmic equations below → explain what is happening/going on in all these equations

$\log_5 25 = 2$

$\log_7 1 = 0$

$\log_2 8 = 3$

$\log_6 36 = 2$

$\log_5 25 = 2$

$\log_3 81 = 4$

$\log_4 64 = 3$

$\log_2 32 = 5$

$\log_5 25 = 2$

$\log_{12} 144 = 2$

$\log_4 2 = \frac{1}{2}$

$\log_{\frac{2}{3}} \left(\frac{4}{9} \right) = 2$

$\log_{125} 5 = \frac{1}{3}$

$\log_9 3 = \frac{1}{2}$

$\log_8 2 = \frac{1}{3}$

$\log_2 \frac{1}{16} = -4$

$\log_{243} 27 = \frac{3}{5}$

$\log_8 4 = \frac{2}{3}$

$\log_2 \frac{1}{8} = -3$

$\log_9 \frac{1}{81} = -2$

$\log_3 \frac{1}{27} = -3$

$\log_{\frac{3}{5}} \left(\frac{25}{9} \right) = -2$

$\log_{27} \frac{1}{3} = -\frac{1}{3}$

$\log_{128} \frac{1}{2} = -\frac{1}{7}$

(D)Application: Solving Logarithmic Equations

Given the pattern you found in Part C, evaluate & solve the following logarithmic expressions/equations

Evaluate the following logarithmic expressions

$$\log_5 125 =$$

$$\log_2 \frac{1}{16} =$$

$$\log_2 \frac{1}{128} =$$

$$4\log_9 3 =$$

$$\log_4 256 =$$

$$\log_3 \frac{1}{243} =$$

$$4\log_2 4 =$$

$$\log_2 64 =$$

$$\log_{\frac{1}{6}} 36 =$$

$$2\log_4 2 =$$

More examples at:

<http://www.mathworksheets4kids.com/logarithms/evaluating-expressions-level1-easy2.pdf> (EASY)

<http://www.mathworksheets4kids.com/logarithms/evaluating-expressions-level2-medium1.pdf> (MEDIUM)

<http://www.mathworksheets4kids.com/logarithms/evaluating-expressions-level2-hard2.pdf> (HARD)

Solve the following logarithmic equations

$$\log_x 32 = 5$$

$$\log_3 x = 3$$

$$\log_3 81 = x$$

$$\log_5 x = -2$$

$$\log_6 x = 2$$

$$\log_9 x = \frac{1}{2}$$

$$\log_5 0.04 = x$$

$$\log_2 \frac{1}{x} = 4$$

$$\log_x 2 = \frac{1}{3}$$

$$\log_x 256 = -4$$

More examples at:

<http://www.mathworksheets4kids.com/logarithms/solve-level1-easy2.pdf> (EASY)

<http://www.mathworksheets4kids.com/logarithms/solve-level1-medium2.pdf> (MEDIUM)

<http://www.mathworksheets4kids.com/logarithms/solve-level2-medium2.pdf> (HARD)

(E) Converting Forms → between Exponential & Logarithmic

Given the following examples → convert all log equations to equivalent exponential equations & vice versa (convert exponential equations into equivalent logarithmic equations)

1) $\log_{16} 256 = 2$	2) $\log_9 81 = 2$	21) $4^{\frac{1}{2}} = 2$	22) $3^5 = 243$
3) $\log_2 \frac{1}{8} = -3$	4) $\log_5 25 = 2$	23) $14^{-2} = \frac{1}{196}$	24) $18^2 = 324$
5) $\log_{20} 400 = 2$	6) $\log_{17} 289 = 2$	25) $3^3 = 27$	26) $\left(\frac{1}{6}\right)^3 = \frac{1}{216}$
7) $\log_{13} 169 = 2$	8) $\log_5 125 = 3$	27) $14^2 = 196$	28) $36^{-\frac{1}{2}} = \frac{1}{6}$
9) $\log_9 \frac{1}{81} = -2$	10) $\log_{169} 13 = \frac{1}{2}$	29) $6^3 = 216$	30) $17^2 = 289$
11) $\log_y x = \frac{2}{3}$	12) $\log_y 76 = x$		

Further Examples

<http://www.mathworksheets4kids.com/logarithms/log-exp-form-num1.pdf>

<http://www.mathworksheets4kids.com/logarithms/log-exp-form-var1.pdf>

(F) Working with our TI-84 → Evaluating Log & Exponent Expressions & Equations

Using the logbase key on the TI-84, for each of the following log expressions, (i) evaluate the expression and then write the exponential equation that would have created the given log expression.

Use a calculator to approximate each to the nearest thousandth.

635) $\log_5 33$

636) $\log_4 90$

637) $\log_4 57$

638) $\log_3 23$

639) $\log 17$

640) $\log_2 47$

641) $\log_9 14$

642) $\log_8 4.9$

643) $\log_8 1.4$

644) $\log_7 86$

645) $\log_6 53$

646) $\log_6 7.33$

647) $\log_5 2.4$

648) $\log_3 9$

649) $\log_4 43$

650) $\log_3 66$

651) $\log_2 25$

652) $\log_9 82$

653) $\log_9 48$

654) $\log_8 5.9$

655) $\log_7 1.3$

656) $\log_7 39$

657) $\log_6 5$

658) $\log_5 2.6$

659) $\log_5 7.2$

660) $\log_3 44$

661) $\log_4 78$

662) $\log_2 -8.3$

663) $\log_3 -4.8$

664) $\log -8.3$

(G) Summary

(i) Equivalence of Exponential & Logarithmic equations →

(ii) Key Terminology →

Natural Logarithms

1. Verify $\ln(1) = 0$
2. Verify $\ln(2.718281828) = 1$ approximately. Your calculator may round to 1.
3. **Experiment:** Calculate the natural logarithm of 2.7, 2.71, 2.71, 2.718, 2.7182, and so on. See how the values approach or get closer and closer to 1.
4. Evaluate $\ln(2) =$
5. Evaluate $\ln(3) =$
6. Evaluate $\ln(6) =$
7. Evaluate $\ln(2) + \ln(3) - \ln(6) =$
8. Evaluate $\ln(0.5)$
9. Evaluate $\ln(1/2)$
10. Evaluate $\ln(1/3)$
11. Evaluate $\ln(4)$
12. Evaluate $\ln(1/4) =$
13. Evaluate $\ln(1.25) + \ln(0.8)$
14. Evaluate $\ln(10)$
15. Evaluate $\ln(5)/\ln(10) = \ln(5)$ divided by $\ln(10)$.
16. Use the log button on your calculator to compute $\log(5)$. That should give the same result as $\ln(5)/\ln(10)$.
17. Evaluate $\ln(2^5) - 5 \ln(2)$

<http://www.mathworksheets4kids.com/logarithms/solving-expressions-calc-natural3.pdf>

<http://www.mathworksheets4kids.com/logarithms/solving-expressions-calc-natural2.pdf>

<http://www.mathworksheets4kids.com/logarithms/solving-expressions-calc-common3.pdf>