IM3L5.9 : Applying Sinusoidal Functions | Unit 5 – Trigonometric Functions

(A) Lesson Context

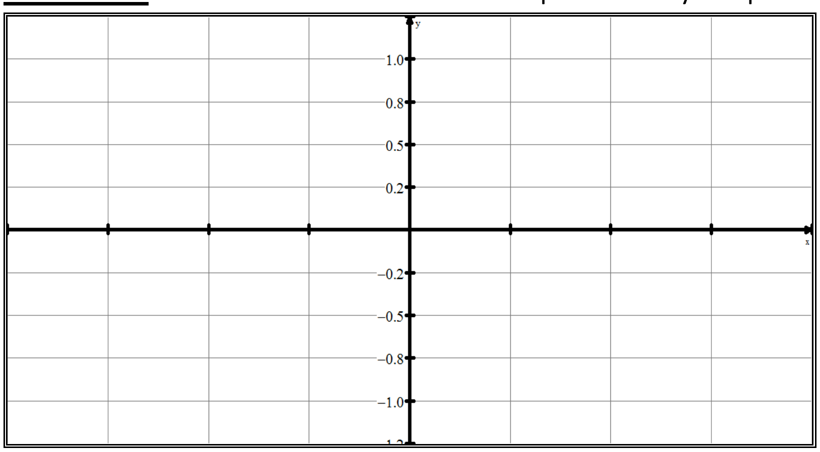
BIG PICTURE of this UNIT:	• How do work through geometry based problems, wherein triangles are used to model the problem • How do we model phenomenon that are periodic in nature		
CONTEXT of this LESSON:	Where we've been We have analyzed the sine and cosine functions and worked with the transformed sinusoidal function y = A sin k(x + C) + D	Where we are How do we apply the function f(x) = A sin k(x + C) + D to real world problems?	Where we are heading How do we mathematically model phenomenon that are periodic in nature

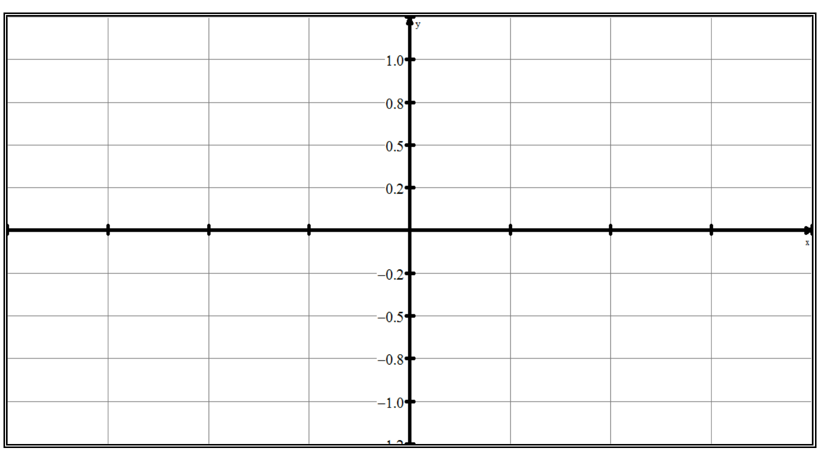
(B) ***Lesson Objectives***

a. Work with the equation $f(x) = A\sin(B(x - C)) + D$ in the context of word problems using the TI-84 and graphic approaches to answering application questions.

b. Use the graphing calculator to answer application questions involving the key terms related to periodic phenomenon (periodic, period, amplitude, axis of the curve (equilibrium axis)) and relate them back to the context of the problem/equation

(C) **Fast Five:** Basic sinusoidal functions ➔ Graph and analyze 2 periods of $f(x) = \sin(x)$ and $f(x) = \cos(x)$

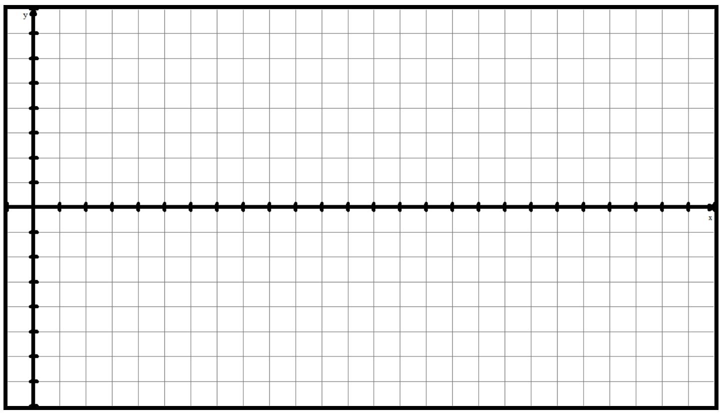




(D)Modeling Periodic Phenomenon – Using A sin K(x + C) + D or A cos K(x + C) + D

16. The depth of water in a harbour on the Bay Fundy that faces the ocean changes each hour, as shown.

Time (h)	00:00	01:00	02:00	03:00	04:00	05:00	06:00	07:00	08:00	09:00	10:00	11:00	12:00
Depth (m)	5.5	6.3	8.5	11.5	14.5	16.7	17.5	16.7	14.5	11.5	8.5	6.3	5.5

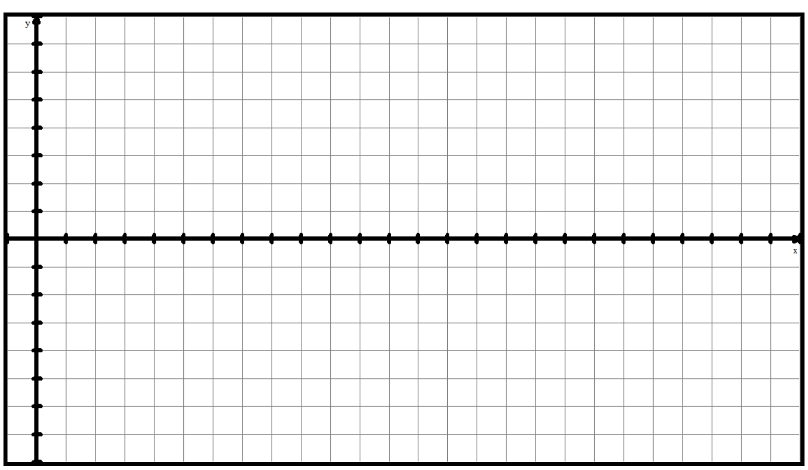


a. Complete a scatter plot (by hand & on GDC & DESMOS)
b. Determine equation
c. Use the equation to determine the depth of water at 10:30. Verify your answer using the graph.
d. When is the water 7 m deep?

15. The table shows the average monthly high temperature for one year in Kapuskasing.

Time (months)	J	F	M	A	M	J	J	A	S	O	N	D
Temperature (°C)	−18.6	−16.3	−9.1	0.4	8.5	13.8	17.0	15.4	10.3	4.4	−4.3	−14.8

Source: Environment Canada.



a. Complete a scatter plot (by hand & on GDC & DESMOS)
b. Determine equation.
c. What is the average monthly temperature for the 38th month?

(E) <u>Modeling Periodic Phenomenon – Using AsinK(x + C) + D or A cosK(x + C) + D</u>

(1) Use your graphing calculator to help you answer these questions:

Evaluate $y = \cos\theta$ for $0° \leq \theta \leq 540°$ when $y = -0.7$. Answer to the nearest degree.

Evaluate $y = \sin\theta$ for $-90° \leq \theta \leq 540°$ when $y = -0.3$. Answer to the nearest degree.

(a) Evaluate $h(t) = \cos(20t)°$ for $t = 3$.

(b) What is the value of t when $h(t) = 0.3$ for $0 \leq t \leq 18$?

(a) Evaluate $h(t) = 4\sin(30t)°$ for $t = 10$.

(b) What is the value of t when $h(t) = 3.2$ for $0 \leq t \leq 12$?

(2) Use your TI-84 to help answer these questions:

The height, h, of a basket on a water wheel at time t is given by $h(t) = \sin(6t)°$, where t is in seconds and h is in metres.

(a) How high is the basket at 14 s?

(b) When will the basket first be 0.5 m under water?

(3) Use your TI-84 to help answer these questions:

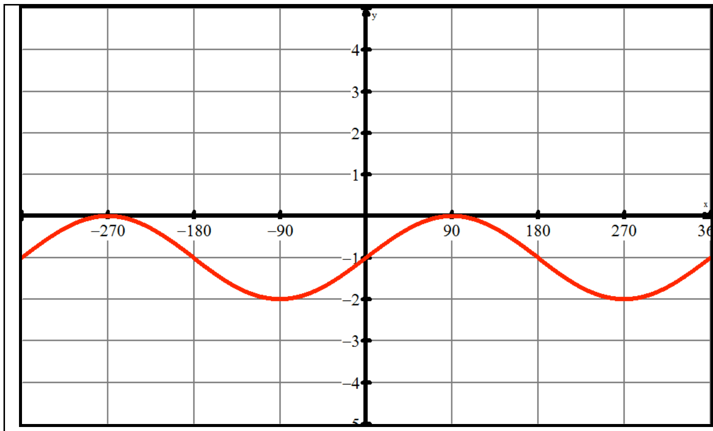
The vertical distance in metres of a rider with respect to the horizontal diameter of a Ferris wheel is modelled by $h(t) = 5\cos(18t)°$, where t is the number of seconds.

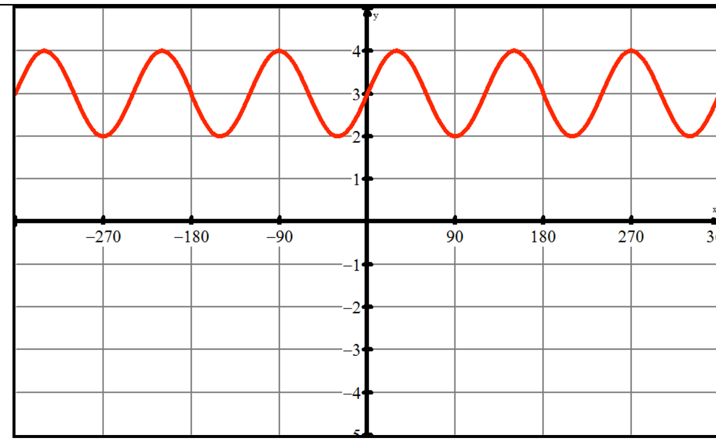
(a) To one decimal place, what is the rider's vertical distance with respect to the horizontal diameter of the wheel when $t = 8$ s? 16 s? 30 s?

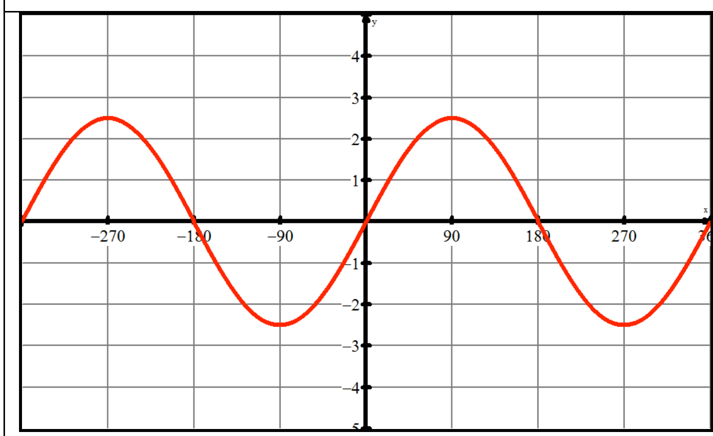
(b) When is the rider first at 4.5 m? -3.2 m?

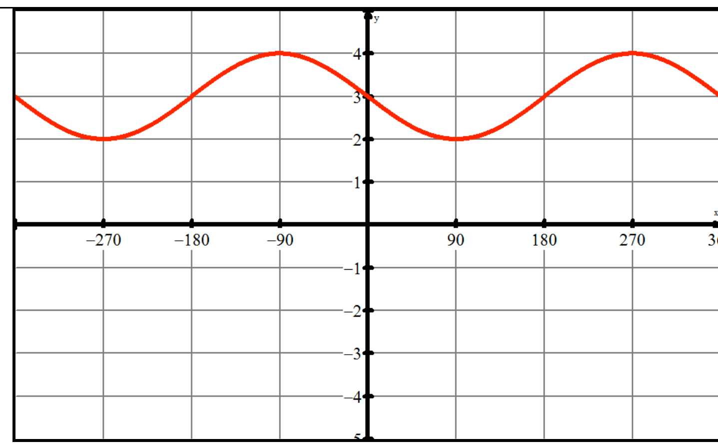
(c) When is the third time the rider is at -2.5 m?

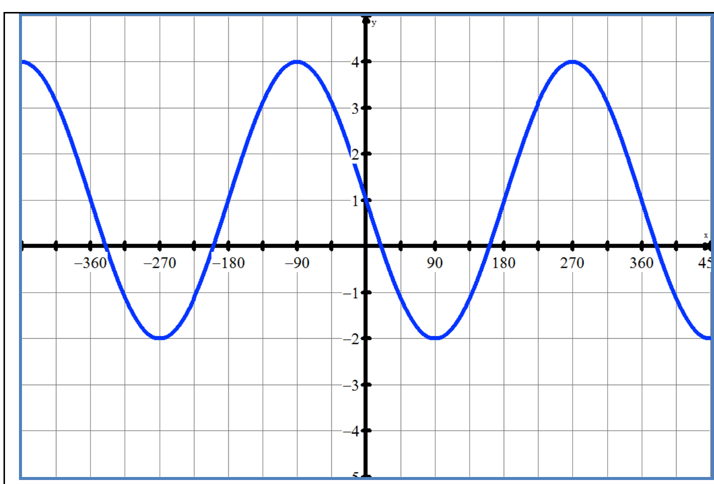
(F) Modeling Periodic Phenomenon: Practice – From Graph to Equation

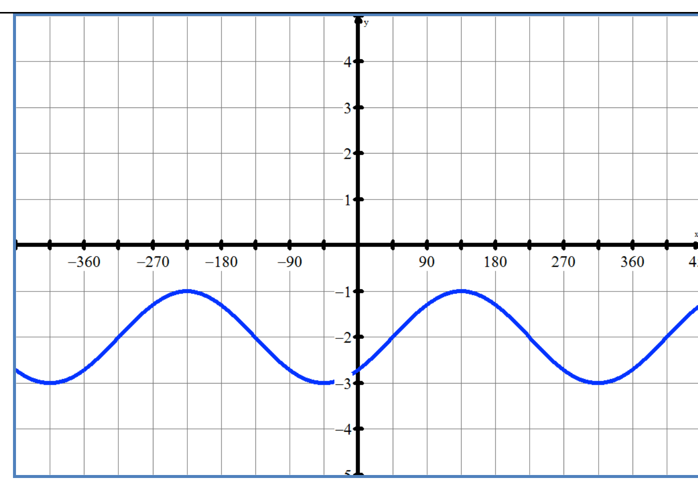


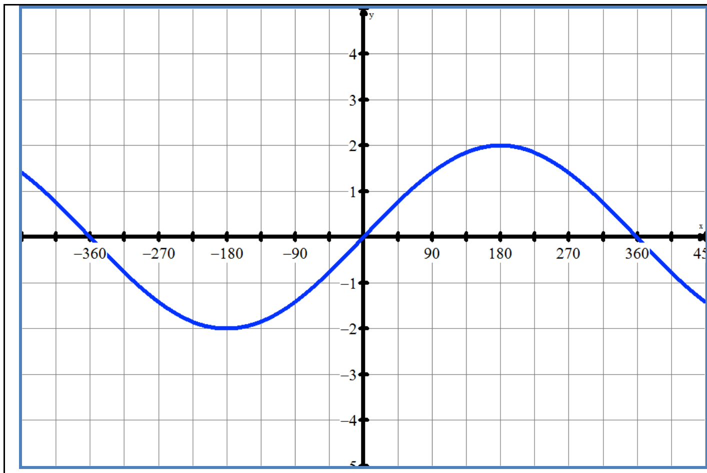
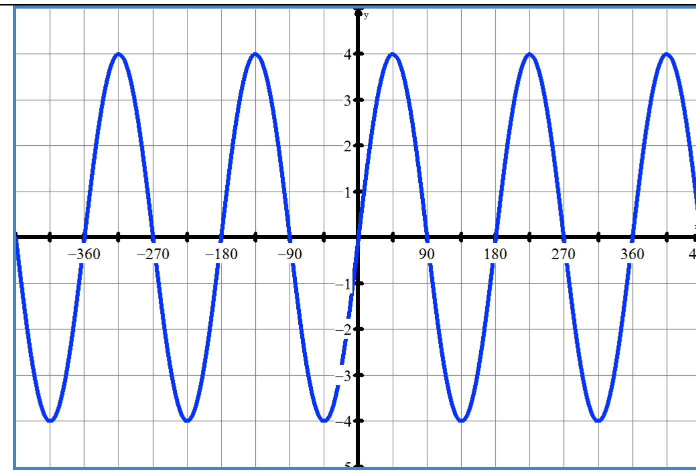
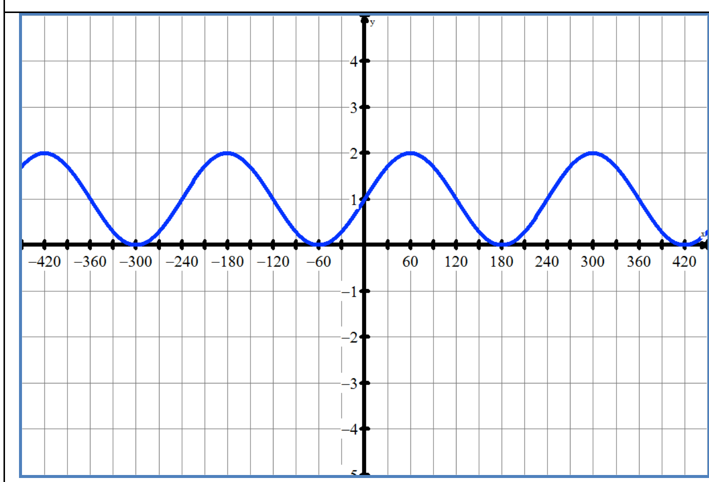
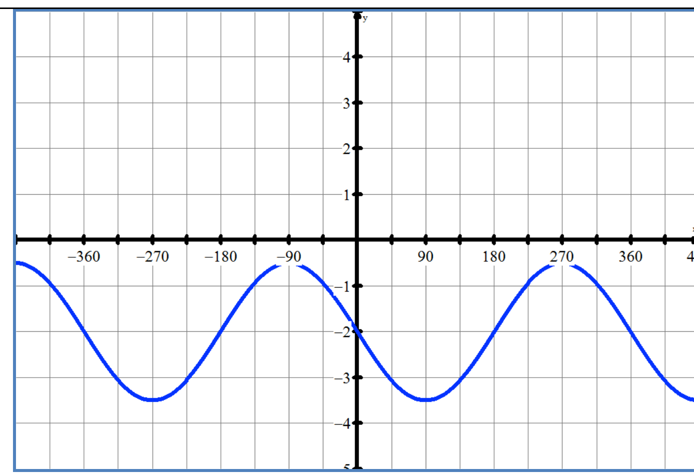




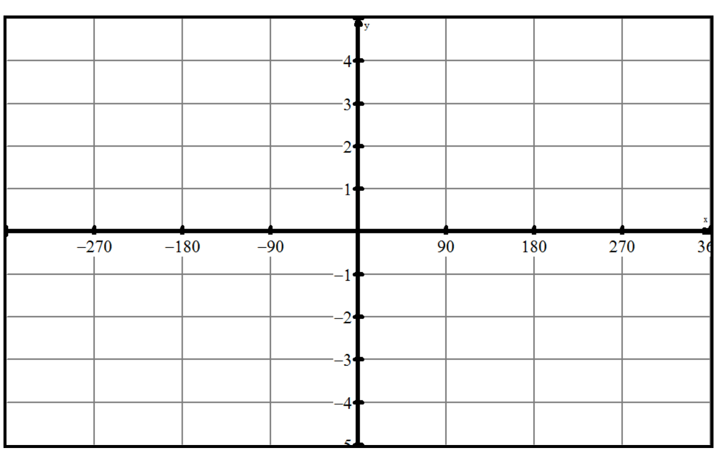
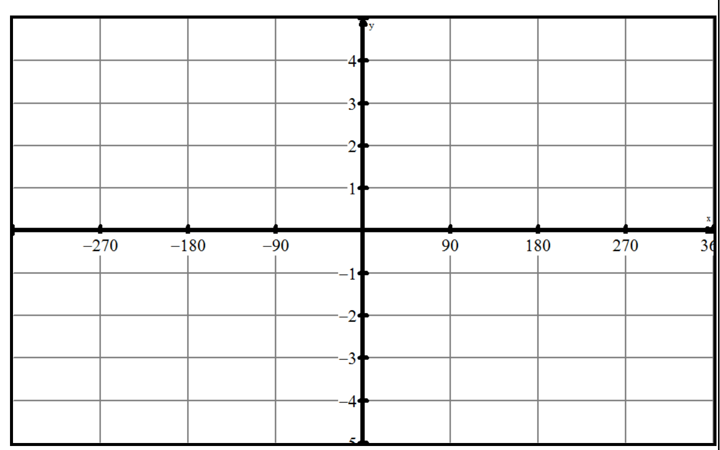




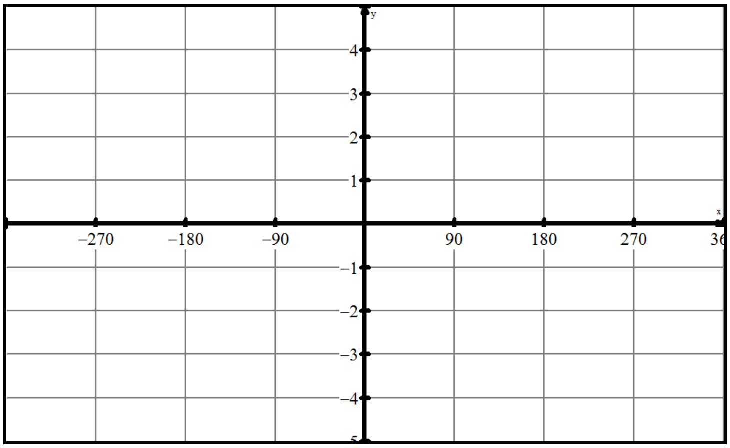


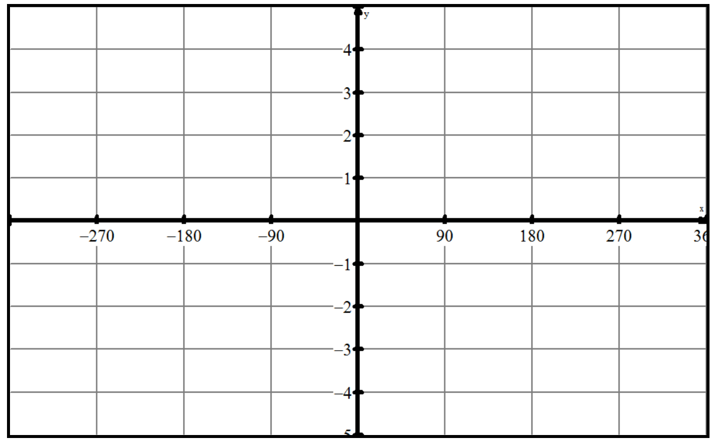
(G)Modeling Periodic Phenomenon: Practice – From Equation to Graph

f(x) = 3 sin(x) – 1 	g(x) = 3cos(2x) 

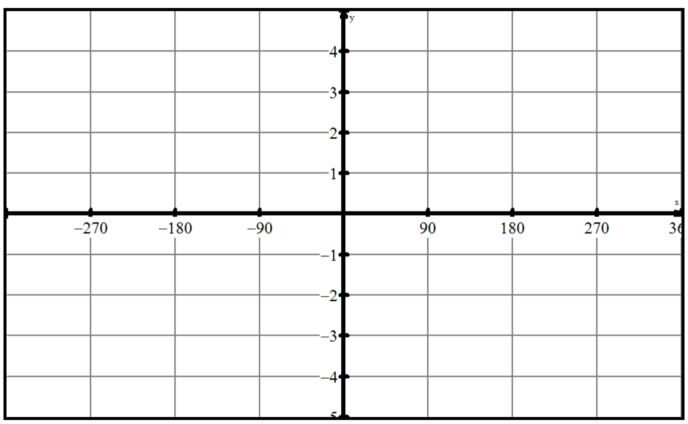
f(x) = sin(x + 45) – 2



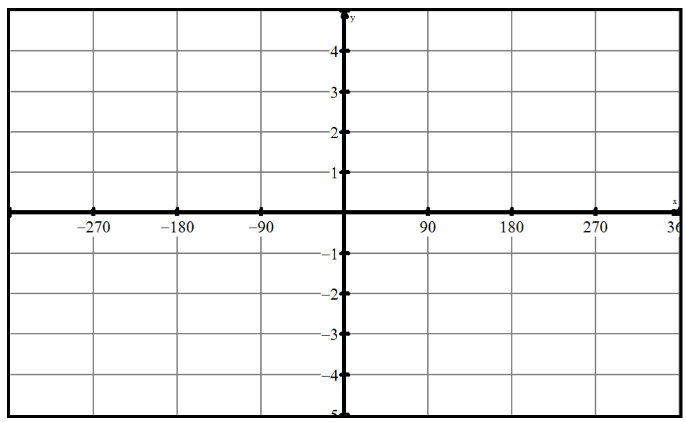
g(x) = ½ cos(x – 90)



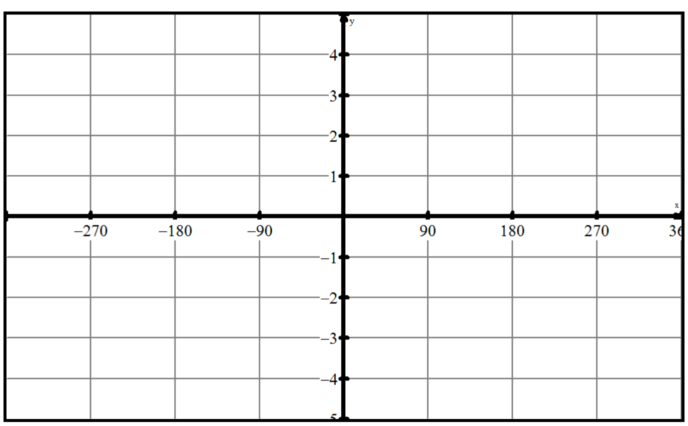
f(x) = -2 cos(x) + 2



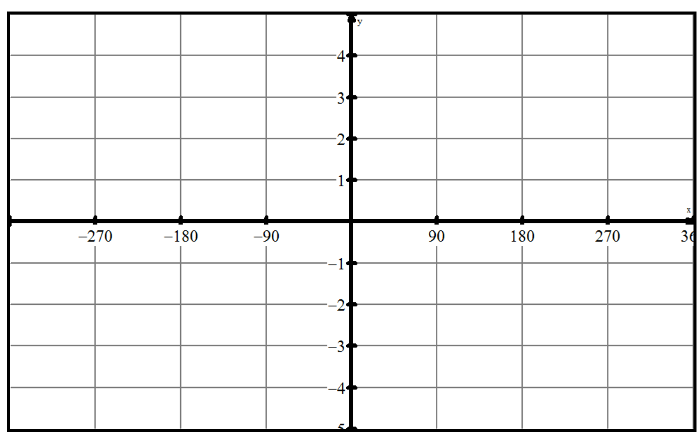
g(x) = 2sin(3x)



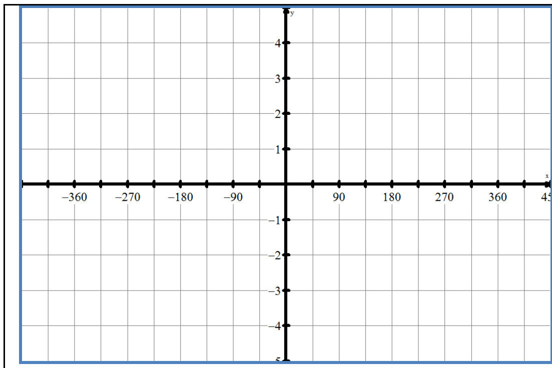
f(x) = cos (x – 60) + 3



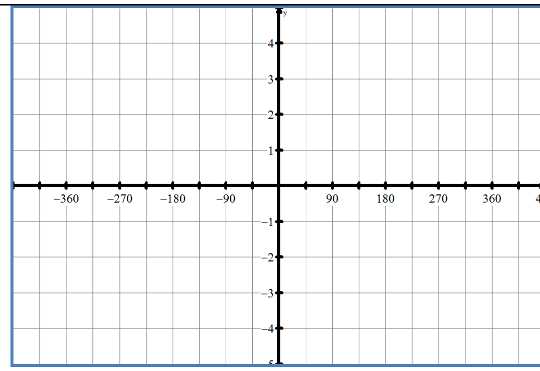
g(x) = 2 cos 2(x – 45)



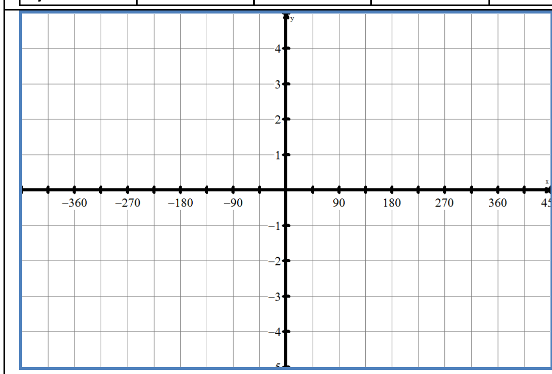
(H)<u>Modeling Periodic Phenomenon: Practice – From Data Set to Graph to Equation</u>



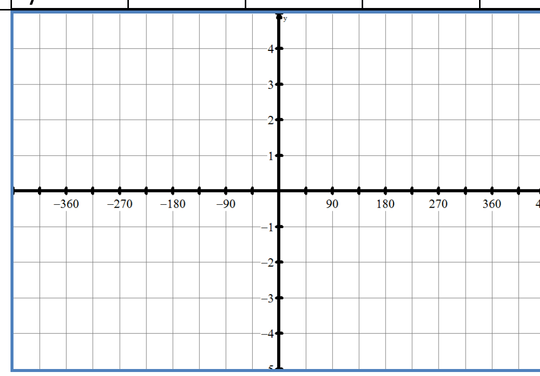
X°	0	90	180	270	360
y	-3	0	3	0	-3



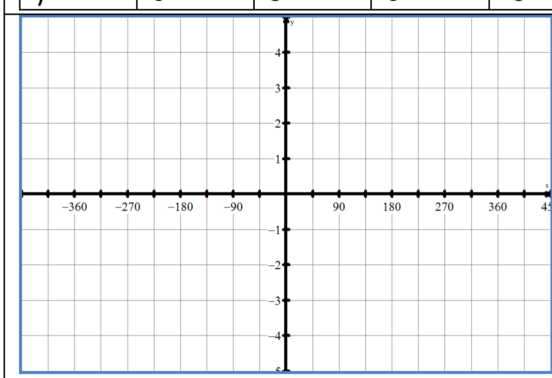
X°	45	135	225	315	405
y	3.5	2	0.5	2	3.5



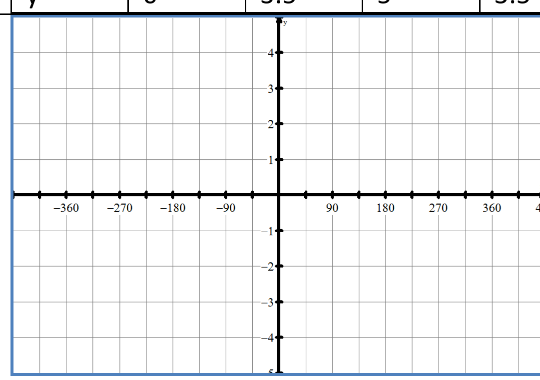
X°	0	45	90	135	180
y	0	3	0	-3	0



X°	-90	-30	30	90	150
y	0	3.5	5	3.5	0



X°	15	105	195	285	375
y	2	1	0	1	2

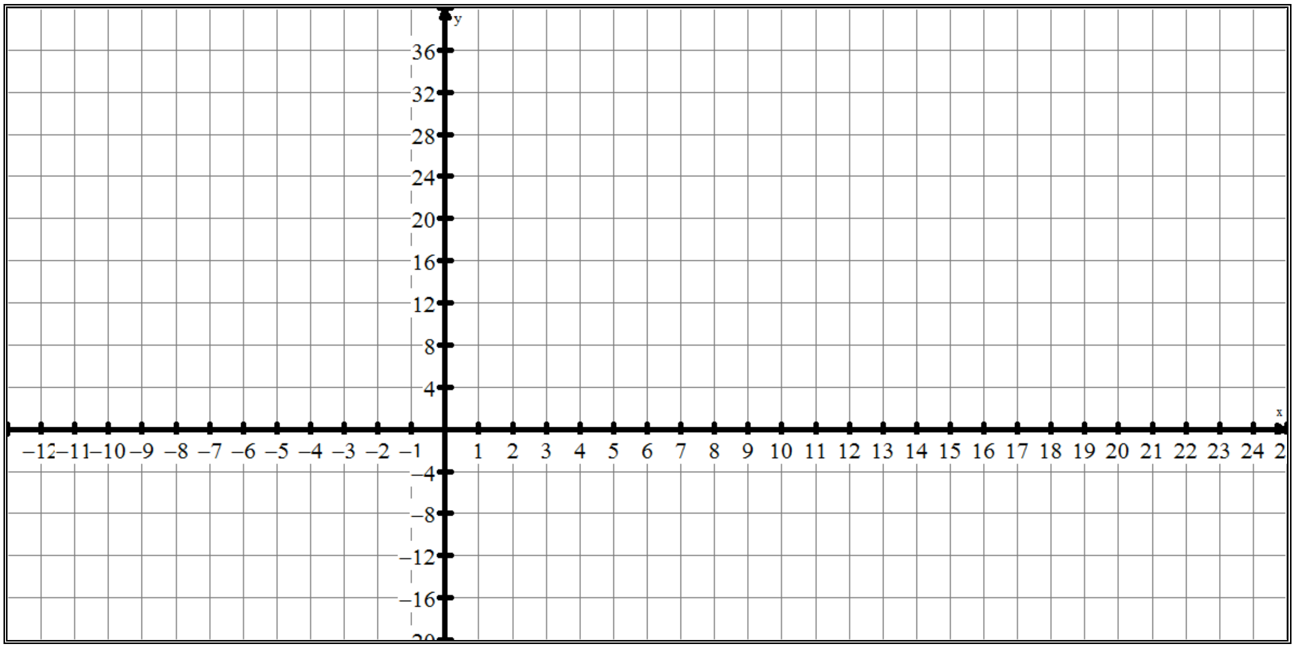


X°	-90	-45	0	45	90
y	6	-2	6	-2	6

(I) __In-Class Example to Work Through__

The average monthly temperature, *T*, in degrees Celsius in the Kawartha Lakes was modelled by $T(t) = -22\cos(30t) + 10$, where *t* represents the number of months. For *t* = 0, the month is January; for *t* = 1, the month is February, and so on.

a. Sketch the graph from your GDC.



b. What is the period? Explain the period in the context of the problem.

c. What is the amplitude? Explain the amplitude in the context of the problem.

d. What is the maximum temperature? the minimum temperature?

e. What is the range of temperatures for this model?

f. What is the annual/yearly average temperature?

g. What is the predicted temperature on April 15th?

h. Evaluate $T(18.75)$ and explain the solution in the context of the problem.

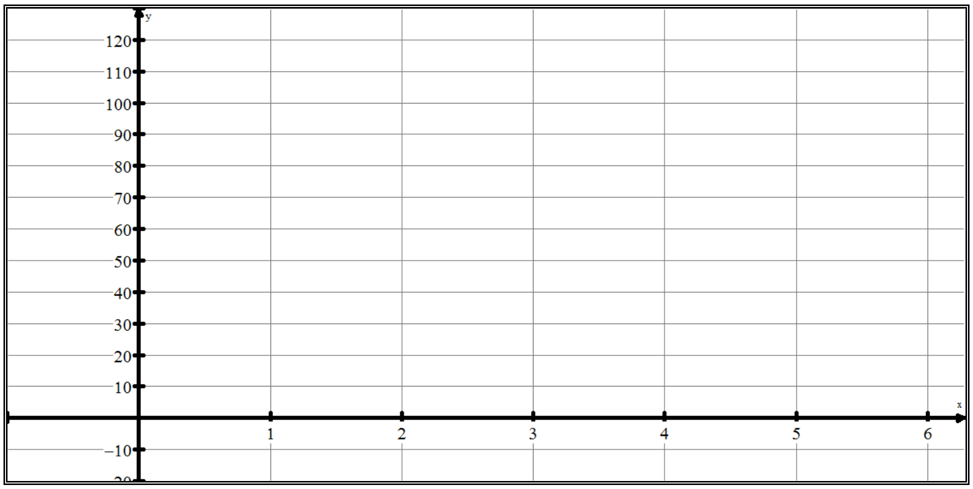
i. When will the temperature be predicted to be 12°?

j. Solve the equation $0 = -22\cos(30t) + 10$ and explain the solution in the context of the problem.

(J) In-Class Example

Each person's blood pressure is different. But there is a range of blood pressure values that is considered healthy. The function $P(t) = -20\cos(300t) + 100$ models the blood pressure, P, in millimetres of mercury, at time, t, in seconds of a person at rest.

a. Sketch the graph of $P(t) = -20\cos(300t) + 100$ for $0 \le t \le 6$.



b. What is the period of the function? What does the period represent for an individual?

c. What is the amplitude? Explain the amplitude in the context of the problem

d. How many times does this person's heart beat each minute?

e. What is the range of the function? Explain the meaning of the range in terms of a person's blood pressure.

f. What is the predicted blood pressure at 4 seconds of rest??

g. Evaluate $P(24)$ and explain the solution in the context of the problem.

h. When will the blood pressure be predicted to be 90 mm Hg?

i. Solve the equation $88 = -20\cos(300t) + 100$ and explain the solution in the context of the problem.

(K) ***In-Class Example***

The function $D(t)=4\sin\left[\frac{360}{365}(t-80)\right]^{\circ}+12$ is a model of the number of hours of daylight, *D*, on a specific day, *t*, on the 50° of north latitude.

a. Explain why a trigonometric function is a reasonable model for predicting the number of hours of daylight.

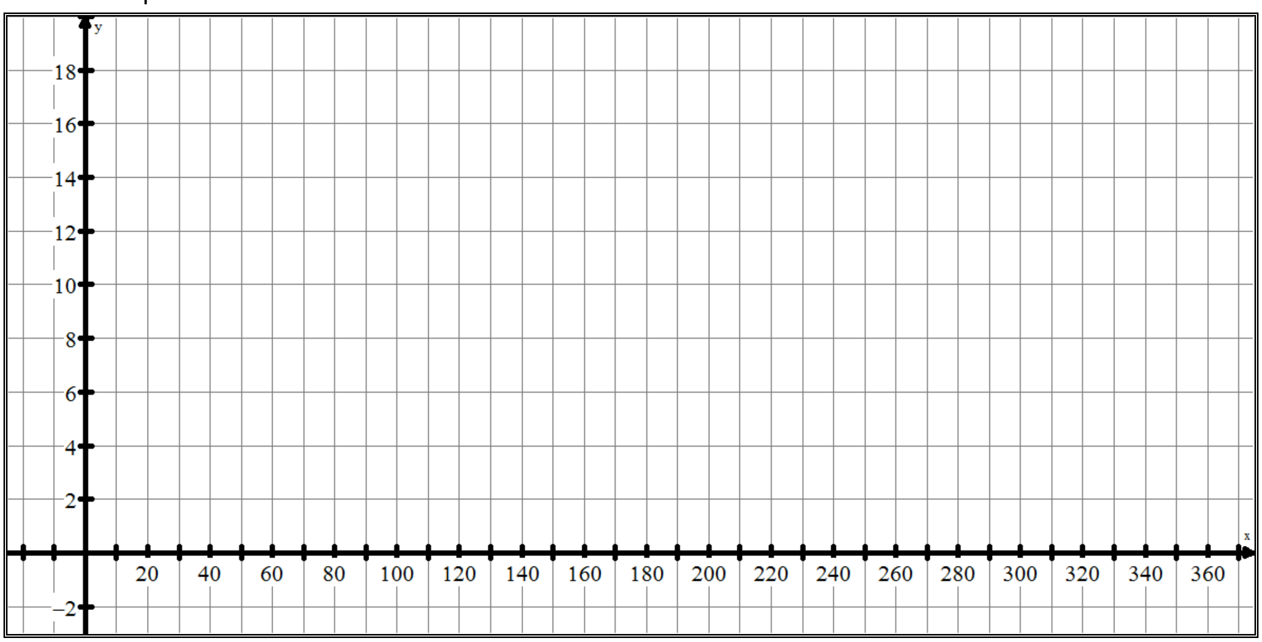
b. How many hours of daylight do March 21 and September 21 have? What is the significance of each of these days?

c. What is the significance of the number 80 in the model?

d. How many hours of daylight do June 21 and December 21 have? What is the significance of each of these days?

e. Explain what the number 12 represents in the model.

f. Graph the model.



g. What are the maximum hours of daylight? the minimum hours of daylight? On what days do these values occur?

h. Use the graph to determine *t* when $D(t)=15$. What dates correspond to *t* ?

i. Evaluate $D(246)$ and explain the solution in the context of the problem.

(L) *Example*

Thinking, Inquiry, Problem Solving: The population, *R*, of rabbits and the population, *F*, of foxes in a given region are modelled by the functions $R(t) = 10{,}000 + 5{,}000\cos(15t)$ and $F(t) = 1{,}000 + 500\sin(15t)$ where *t* is the time in months. Explain, referring to each graph, how the number of rabbits and the number of foxes are related by answering the following questions:

a. When does each population reach their maximum populations? Their minimum populations?
b. When does the rabbit population increase? How do you know? What is happening to the fox population in the same time interval?
c. Evaluate $R(5)$ as well as $F(5)$.
d. Unfortunately, a pathogenic bacteria gets introduced to the ecosystem. The rabbit population halves and now fluctuates by 2000 rabbits per period.
    i. Write a new equation to represent this situation.
    ii. What would happen to the fox population? Write a new equation to present a realistic model for the fox population. Explain your equation.

(M) *Example*

The average monthly temperature in a region of Australia is modelled by the function $T(t) = 9 + 23\cos(30t)$, where *T* is the temperature in degrees Celsius and *t* is the month of the year. For *t* = 0, the month is January.

a. Prepare a table for $0 \leq t \leq 13$.
b. Graph the data.
c. Explain how to use the axis of the curve and the amplitude to determine the maximum and minimum values of the function.
d. Determine the period of the function from the graph. Verify your answer algebraically.
e. Verify the graph in (b) by using a graphing calculator.
f. Explain how to sketch a similar graph using transformations of *y* = cos(t).

(N) Examples: No Equations

a. In Canada's wonderland there is a roller coaster that is a continuous series of identical hills that are 18m high from the ground. The platform to get on the ride is on top of the first hill. It takes 3 seconds for the coaster to reach the bottom of the hill 2m off the ground.
    i. Sketch a graph below which expresses the path of the roller coaster.
    ii. What is the sinusoidal equation (sine and cosine) that best reflects this roller coaster's motion?

b. Mr. Smith, disguised as Mathman, a costumed crime fighter, is swinging back and forth in front of the window to Ms. Aschenbrenner's math class. At t = 3s, he is at one end of his swing and 4m from the window. At t = 7s, he is at the other end of his swing and 20m from the window.
    i. Sketch the curve. Use the distance from the window on the vertical axis and the time in seconds along the horizontal axis.

ii. What is the equation (in terms of sine and cosine), which represents Mathman's motion?

c. John is floating on a tube in a wave tank. At t = 1second, John reaches a maximun height of 14m above the bottom of the pool. At t = 9seconds, John reaches a minimum height of 2m above the bottom of the pool.
   i. Sketch a graph below which expresses John's height from the bottom of the pool with respect to time.
   ii. What is the equation (in terms of sine and cosine), which represents John's motion? What is John's height from the bottom of the pool at 21 seconds?

d. A pendulum on a grandfather clock is swinging back and forth as it keeps time. A device is measuring the distance the pendulum is above the floor as it swings back and forth. At the beginning of the measurements the pendulum is at its highest point, 36cm high exactly one second later it was at its lowest point of 12cm. One second later it was back to its highest position.
   i. Use the information above to sketch a diagram of this sinusoidal movement.
   ii. Write the sinusoidal equation (sine and cosine) that describes this situation.

e. Sam is riding his bike home from school one day and picks up a nail in his tire. The nail hits the ground every 2 seconds and reaches a maximum height of 48 cm (assume the tire does not deflate).
   i. Use the information above to sketch a diagram of this sinusoidal movement.
   ii. Write the sinusoidal equation (sine and cosine) that describes the situation in part a.

f. Jackie, Nicolle and Maegan are playing skip rope. As the rope rotates it is observed that its maximum height is 2.75m after 1 second. The first minimum height of 0.25m occurs 2 seconds after the maximum height.
   i. Use the information above to sketch a diagram of this sinusoidal movement.
   ii. Write the sinusoidal equation (sine and cosine) that describes the situation in part a.

g. A skyscraper sways 55 cm back and forth from "the vertical" during high winds. At t = 5 s, the building is 55 cm to the right of vertical. The building sways back to the vertical and, at t = 35 s, the building sways 55 cm to the left of the vertical. Write an equation that models the motion of the building in terms of time.

h. The maximum height of a Ferris wheel is 35 m. The wheel takes 2 min to make one revolution. Passengers board the Ferris wheel 2 m above the ground at the bottom of its rotation.
   i. Write an equation to represent the position of a passenger at any time, t, in seconds.
   ii. How high is the passenger after 45 s?
   iii. The ride lasts for 4 min. When will the passenger be at the maximum height during this ride?