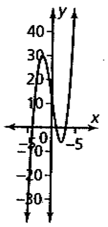
## Chapter 1 Review Test, page 80

1. (a) $f(x) = a_nx^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$; degree: $n$; leading coefficient: $a_n$
   (b) $n - 1$ (c) $n$ (d) degree $\geq 1$
   (e) degree: 0; leading coefficient < 0
2. (a) find the range, $y$-intercept, zeros, the degree, the leading coefficient from the equation; use this information to determine the end behaviour
   (b)



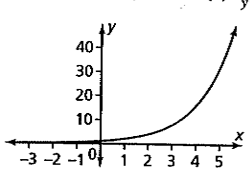

3. yes
4. (a) $(2x + 3)(x - 1)(x - 2)$ (b) $(2x - 3)(4x^2 + 6x + 9)$
   (c) $(x - 2)(4x^2 + 3)$
5. (a) $x = -1$ or $x = 1$ or $x = -4$ or $x = \frac{1}{2}$
   (b) $-2.5 < x < 1$ and $x > 1$
6. (a) $x^3 - 7x + 6$ (b) $x^4 - 4x^3 + 8x^2 - 16x + 16$
7. (a) males: $f(x) = 0.176x^3 - 4.052x^2 + 16.97x + 72.365$; females: $f(x) = 0.022x^3 - 0.64x^2 + 9.01x + 14.369$; in 1983, about 89.3 males per 100 000; in 1983, about 26.5 females per 100 000
   (b) when $x = 6.96$ or about 2010
8. the length of one side of the square must be between 0.113 cm and 13.560 cm
9. $f(x) = 3x^4 + 14x^3 - 11x^2 - 70x + 24$

# Chapter 2

## Getting Ready, page 84

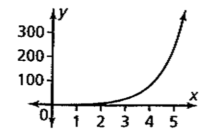
1. (a) $\left(\frac{1}{3}\right)^4$ (b) $\left(\frac{1}{7}\right)^2$ (c) $\left(\frac{1}{a}\right)^8$ (d) $\left(\frac{1}{x}\right)^5$
2. (a) 256 (b) 243 (c) $-512$ (d) $-16$
   (e) $-\frac{1}{36}$ (f) 371.4228 (g) $-\frac{1}{32}$ (h) 1
   (i) $\frac{4}{9}$ (j) $\frac{64}{27}$ (k) $\frac{161}{405}$ (l) $\frac{13}{16}$
3. (a) 2 (b) $-2$ (c) 0.25 (d) 0.17
   (e) 1.41 (f) 0.94 (g) 0.12 (h) 0.29
4. (a) $2^{11}$ (b) $2^9$ (c) $2^{4-m}$ (d) $2^{3n}$
5. (a) 9 (b) $\frac{257}{4}$ (c) $\frac{9}{10}$ (d) $\frac{1}{20}$
   (e) $\frac{9}{4}$ (f) $\frac{1}{625}$
6. (a) $-1$ (b) $\frac{4}{9}$
7. (a) $x^2y$ (b) $a^{18}b^{10}$ (c) $(cd)^6$ (d) $\frac{x^{18}}{y^2}$
8. (a)

| $x$ | $y$ |
|---|---|
| $-1$ | 0.5 |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |



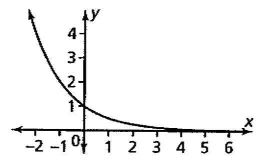

(b)

| $x$ | $y$ |
|---|---|
| $-1$ | $\frac{1}{3}$ |
| 0 | 1 |
| 1 | 3 |
| 2 | 9 |
| 3 | 27 |
| 4 | 81 |
| 5 | 243 |




(c)

| $x$ | $y$ |
|---|---|
| $-4$ | 16 |
| $-3$ | 8 |
| $-2$ | 4 |
| $-1$ | 2 |
| 0 | 1 |
| 1 | 0.5 |
| 2 | 0.25 |




9. (a) $y = 1.225\,043$ (b) $m = \pm 14$ (c) $p = 3$
   (d) $c = 512$ (e) $a = 32$ (f) $a = \pm\sqrt{2}$
10. (a) $x \doteq 2.9534$ (b) $x \doteq 1.2851$
    (c) $x \doteq 5.7824$ (d) $x \doteq 14.2067$
11. (a) 1st difference: 2; linear, $y = 2x$
    (b) 1st difference: 3; linear, $y = 3x - 1$
    (c) 1st differences: 4, 12, 36, 108, 323; 2nd differences: 8, 24, 72, 215; other, $y = 2(3^x)$
    (d) 1st differences: 1, 2, 4, 8, 16; 2nd differences: 1, 2, 4, 8; other, $y = 2^x$
12. (a) $x = -5, 3$ (b) $n = -10, 3$ (c) $y = -4, -0.5$
    (d) $m = \pm 2$ (e) $x = -3, -0.4$
    (f) $t \doteq -0.0237, 4.2237$ (g) $x = -0.5, 4$
13. (a) $x = 3$ (b) $x = -2$ (c) $x = -1$
    (d) $x = 3$ (e) $x = 1.44$ (f) $x = -5$
14. (a) $r = -2, t_n = 6(-2)^{n-1}, t_8 = -768$
    (b) $r = -\frac{2}{3}, t_n = \left(\frac{-2}{3}\right)^{n-1}, t_8 = -\frac{128}{2187}$
15. (a) \$4660.96 (b) \$5895.75
16. 195 min
17. 37 days

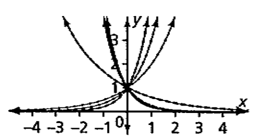
## 2.1 Exercises, page 92

1. (a) 162.0000 (b) 5766.5039 (b) 0.0977 (d) 0.0095
2. (a) 1.14 (b) 0.94 (c) 1.10 (d) 0.91
3. (a) 3.00 (b) 2.92 (c) 1.45 (d) 0.45
4. (a) 300 (b) 153 600 (c) 1200
   (d) $150(2)^t$ (e) 2 457 600 (f) $1.611 \times 10^{11}$
5. (a) \$10 400 (b) \$20 800 (c) \$41 600
   (d) \$29 416.64 (e) \$58 831.28 (f) \$93 388.84
6. (a) inflation rate: 6% (b) deflation rate: 2%
   (c) inflation rate: 1.2% (d) deflation rate: 5.8%
7. (a) Initial value: 1200; growth value: 200%
   (b) Initial value: 1; growth value: 300%
   (c) Initial value: 100; growth value: 104.8%
   (d) Initial value: 50; growth value: 500%
8. 0.1845 m
9. (a) 5 years: \$1924.68; 10 years: \$2894.06
   (b) \$1280 (c) 8.5 % (d) \$376.11
10. 10 737 418 240

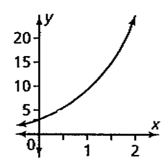
11. linear equation: $y$ is $y$-coordinate, $m$ is slope, $x$ is $x$-coordinate, $b$ is $y$-int; exponential equation: $y$ is $y$-coordinate, $c$ is initial value ($y$-int), $a$ is the factor, $x$ is $x$-coordinate; $b$ and $c$ are similar—they show initial value of function when $x = 0$, $m$ and $a$ are similar—show how the function increases or decreases as $x \to +\infty$ and $x \to -\infty$, difference of the functions is $y = mx + b$ is linear and $y = c(a)^x$ is a curve
12. \$10 197.40
13. $15^{180}$
14. 1648
15. 39 628
16. \$2980.23
17. $0 < a < 1$ when the rate of change is neg. Examples: deflation, decay, depreciation, decrease in population; $a > 1$ when the rate of change is pos. Examples: growth, inflation, increase in population, bank deposit, loan, investment
18. (a) 0.96. (b) $T = 55(0.96)^t$
    (c) no; after the coffee reaches the room temperature, it is going to stabilize
19. 72.9%, between 6 and 7 copies
20. 23.5 years

## 2.2 Exercises, page 103

1. graphs are all in the general form ($y = a^x$. When $a > 1$, graphs are similar in that as $x \to -\infty$, $y \to 0$, and as $x \to +\infty$, $y \to +\infty$. When $0 < a < 1$, graphs are similar in that as $x \to -\infty$, $y \to +\infty$, and as $x \to +\infty$, $y \to 0$

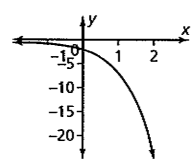



2. (a)



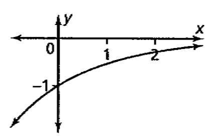

hor. asymp.: $y = 0$; $y$-int.: (0, 3)

(b)



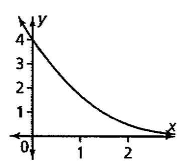

hor. asymp.: $y = 0$; $y$-int.: (0, −2)

(c)



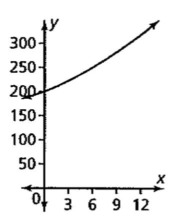

hor. asymp.: $y = 0$; $y$-int.: (0, −1)

(d)



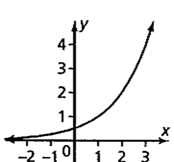

hor. asymp.: $y = 0$; $y$-int.: (0, 4)

(e)



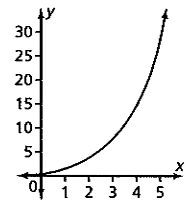

hor. asymp.: $y = 0$; $y$-int.: (0, 200)

(f)



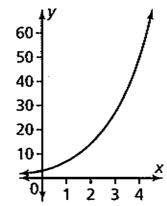

hor. asymp.: $y = 0$; $y$-int.: (0, 0.5)

3. (a)



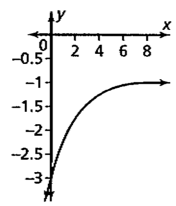

hor. asymp.: $y = -1$; $y$-int.: (0, 0)

(b)



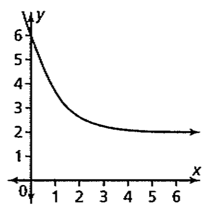

hor. asymp.: $y = 0$; $y$-int.: (0, 3)

(c)



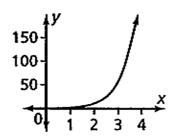

hor. asymp.: $y = -1$; $y$-int.: (0, −3)

(d)



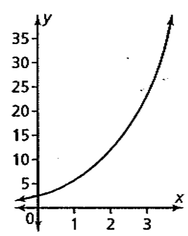

hor. asymp.: $y = 2$; $y$-int.: (0, 6)

4. (a)



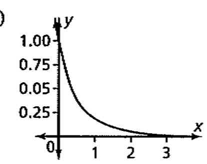

hor. asymp.: $y = 0$; $y$-int.: (0, 1); D: $x \in \mathbf{R}$; R: $y > 0$

(b)



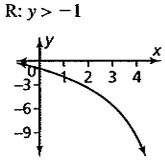

hor. asymp.: $y = -1$; $y$-int.: (0, 2); D: $x \in \mathbf{R}$; R: $y > -1$

(c)



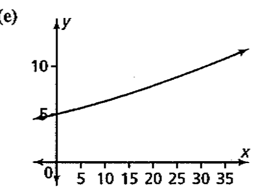

hor. asymp.: $y = 0$; $y$-int.: (0, 1); D: $x \in \mathbf{R}$; R: $y > 0$

(d)



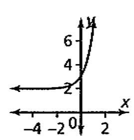

hor. asymp.: $y = 0$; $y$-int.: (0, −1); D: $x \in \mathbf{R}$; R: $y < 0$

(e)



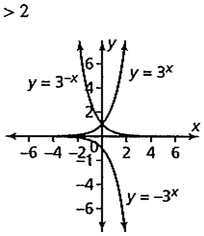

hor. asymp.: $y = 0$; $y$-int.: (0, 5); D: $x \in \mathbf{R}$; R: $y > 0$

(f)

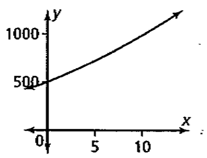



$y = 5^x + 2$, hor. asymp.: $y = 2$; $y$-int.: (0, 3); D: $x \in \mathbf{R}$; R: $y > 2$

5. $y = -3^x$ is the hor. reflection of $y = 3^x$ about $y = 0$; $y = 3^{-x}$ is the vert. reflection of $y = 3^x$ about $x = 0$

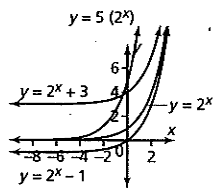


6. $y$-int.: 500; represents initial value of investment



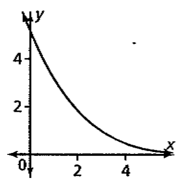

7. (a) 3.16 (b) 15.85 (c) 0.50
(d) 2.15 (e) 6.32 (f) 1.78

8. $y = 2^x + 3$ is a vert. translation (3 units up) of $y = 2^x$; $y = 2^x - 1$ is a vert. translation (1 unit down) of $y = 2^x$; $y = 5(2^x)$ is a vert. stretch by a factor of 5 of $y = 2^x$




9. (a) III; factor is larger than 1 and initial value is pos.
(b) I; factor is larger than 1 and initial value is neg.
(c) IV; factor is smaller than 1 and initial value is pos.
(d) II; factor is smaller than 1 and initial value is neg.

10. According to the graph, after 1 year the time required is 2.70 h and after 5 years the time required is 0.23 h; not a good model—not realistic that a 5-h job can be done in 14 min 5 years later.

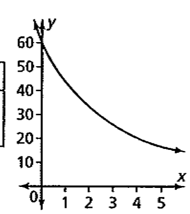


11.

| | Function | $x \to +\infty$ | $x \to -\infty$ | $x \to 0$ |
|---|---|---|---|---|
| (a) | $y = 2^x$ | $y \to +\infty$ | $y \to 0$ | $y \to 1$ |
| (b) | $y = (0.1)^x$ | $y \to 0$ | $y \to +\infty$ | $y \to 1$ |
| (c) | $y = (1.02)^x$ | $y \to +\infty$ | $y \to 0$ | $y \to 1$ |
| (d) | $y = 3^{x+1}$ | $y \to +\infty$ | $y \to 0$ | $y \to 3$ |

12. (a) i. 45 mg ii. 33.75 mg iii. 18.98 mg
(b)

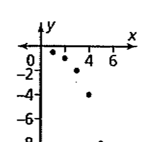
| Time (hour) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Amount of caffeine (mg) | 60 | 45 | 33.75 | 25.31 | 18.98 | 14.24 |



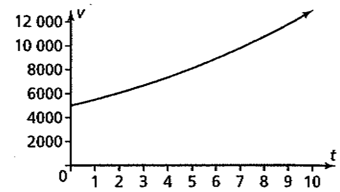
(c) $y = 60(0.75)^t$ where $y$ is amount of the caffeine left in mg and $t$ is time in hours
(d) 14.23 hours

13. Equations will vary. Both graphs have same $y$-int. but the factors are different. They are a vert. reflection about $y$-axis.

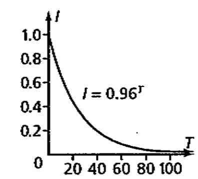
14. $y = \left(-\frac{1}{2}\right)(2)^{n-1}, n \geq 1$; it is an example of exponential equation with $-\frac{1}{2}$ as the initial value and 2 as the factor



15. (a) $V = 5000(1.10)^t$
(b) 7.3 years



16. (a) $I = 0.96^T$
(b) 5.47 units




17. For $y = a^x$, if $0 < a < 1$ the end behaviour is, as $x \to -\infty, y \to +\infty$ and $x \to +\infty, y \to 0$; if $a > 1$ the end behaviour is, as $x \to -\infty, y \to 0$ and $x \to +\infty, y \to +\infty$

18. Example:

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 1.1875 | 1.75 | 4 | 13 | 49 | 193 |
| First Difference | 0.5625 | 2.25 | 9 | 36 | 144 | |
| Second Difference | 1.6875 | 6.75 | 27 | 108 | | |

(a) common ratio is 4 (b) common ratio is 4

19. (a) $x = -\frac{1}{2}$ (b) $x = -3$ (c) $x = 1$

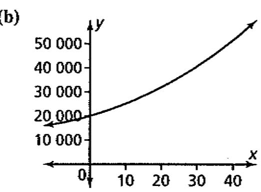
20. (a) X and Y1 correspond to B, X and Y2 correspond to A
(b) graph A (c) graph A: 8; graph B: 5

## 2.3 Exercises, page 110

1. A
2. (a) 2480 (b) 9920 (c) 62 988
(d) 40 632 320 (e) $1.745 \times 10^{17}$ (f) $3.219 \times 10^{36}$
3. (a) 1.25 kg (b) 1.77 kg
(c) 0.74 kg (d) 2.97 kg
4. (a)

| Time (years) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Population | 23 000 | 23 460 | 23 929 | 24 408 | 24 896 | 25 394 | 25 902 |

(b)



(c) $P = (23\,000)(1.02)^t$ where $t$ is time in years and $P$ is the population.
(d) 28 036 (e) 13.42 years (f) 30 910
5. (a) 1.02 (b) $P = (35\,000)(1.02)^t$ (c) $t = 44.14$
6. (a) 1.5 h
(b) $P = (3000)(2)^{\frac{t}{1.5}}$ where $t$ is time and $P$ is population.
(c) 120 952 bacteria (d) 4762 bacteria
7. 2.18 mg
8. 6.66 years
9. (a) $N = (1)\left(\frac{1}{2}\right)^{\frac{t}{2.5 \times 10^5}}$ where $N$ is mass in grams and $t$ is time in years
(b) 0.99 g

**10.** similarities: have the form $y = c(a)^x$; better to have several forms because these forms specialize on the particular field such as compound interest, growth, decay and geometric sequences

**11.** 72.42%

**12.** (a) \$183.54 (b) 7.86 years

**13.** (a) \$295 245 (b) 9.96 years

**14.** 0.966

**15.** 0.880

**16.** 6%/a

**17.** growth: $a > 1$; decay: $0 < a < 1$

**18.** 12.40%

**19.** (a) 1750 to 1800: 28.57%; 1800 to 1850: 31.11%; 1850 to 1900: 35.59%; 1900 to 1950: 56.25%

(b) $P = (7 \times 10^8)(1.319)^t$ where $t$ is the time and $P$ is the population

(c) no; adjust by using the population value of 1950 as a point to derive the formula

**20.** 2 036 579

## 2.4 Exercises, page 117

**1.**

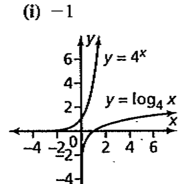
| Exponential Form | Logarithmic Form |
|---|---|
| $8 = 2^3$ | $3 = \log_2 8$ |
| $16 = 2^4$ | $4 = \log_2 16$ |
| $1000 = 10^3$ | $3 = \log_{10} 1000$ |
| $2^{\frac{1}{2}} = \sqrt{2}$ | $\frac{1}{2} = \log_2 \sqrt{2}$ |
| $2^{-3} = \frac{1}{8}$ | $-3 = \log_2 \frac{1}{8}$ |
| $25^{-\frac{1}{2}} = \frac{1}{5}$ | $-\frac{1}{2} = \log_{25} \frac{1}{5}$ |

**2.** (a) $5^2 = 25$ (b) $2^4 = 16$ (c) $3^3 = 27$
(d) $2^{-1} = 0.5$ (e) $4^{\frac{1}{2}} = 2$ (f) $7^0 = 1$

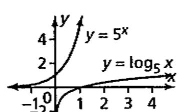
**3.** (a) $3 = \log_2 8$ (b) $\frac{1}{2} = \log_9 3$ (c) $4 = \log_{10} 10\,000$
(d) $-2 = \log_3 \frac{1}{9}$ (e) $\frac{1}{3} = \log_{125} 5$ (f) $\log_4 \frac{1}{2} = -\frac{1}{2}$

**4.** (a) 3 (b) $\frac{1}{2}$ (c) 1
(d) 0 (e) $\frac{1}{2}$ (f) 3
(g) 1 (h) 7 (i) −1

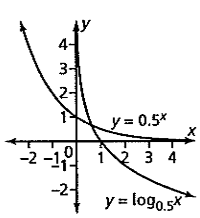
**5.** (a) Original: $y = 4^x$: D: $x \in$ R; R: $y > 0$; asymp.: $y = 0$; Inverse: $y = \log_4 x$: D: $x > 0$; R: $y \in$ **R**; asymp.: $x = 0$



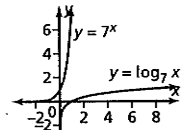

(b) Original: $y = \log_5 x$: D: $x > 0$; R: $y \in$ **R**; asymp.: $x = 0$; Inverse: $y = 5^x$: D: $x \in$ R; R: $y > 0$; asymp.: $y = 0$



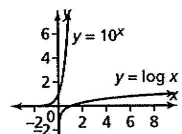

(c) Original: $y = (0.5)^x$: D: $x \in$ R; R: $y > 0$; asymp.: $y = 0$; Inverse: $y = \log_{0.5} x$: D: $x > 0$; R: $y \in$ **R**; asymp.: $x = 0$



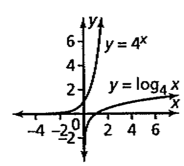

(d) Original: $y = \log_7 x$: D: $x > 0$; R: $y \in$ **R**; asymp.: $x = 0$; Inverse: $y = 7^x$: D: $x \in$ **R**; R: $y > 0$; asymp.: $y = 0$



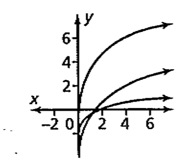

(e) Original: $y = 10^x$: D: $x \in$ **R**; R: $y > 0$; asymp.: $y = 0$; Inverse: $y = \log_{10} x$: D: $x > 0$; R: $y \in$ **R**; asymp.: $x = 0$




(f) Original: $y = \log_4 x$: D: $x > 0$; R: $y \in$ **R**; asymp.: $x = 0$; Inverse: $y = 4^x$: D: $x \in$ **R**; R: $y > 0$; asymp.: $y = 0$




**6.** $y = 3 \log_5 x$ is vertically stretched version of $y = \log_5 x$ and $y = 3 \log_5 x + 4$ is vertically stretched and vertically translated version of $y = \log_5 x$.

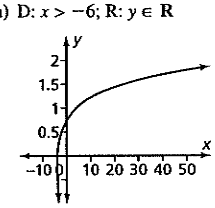


**7.** (a) $x = 2$ (b) $x = 100$ (c) $x = -2$
(d) $x = 6309.57$ (e) $x = -3$ (f) $x = 316.23$

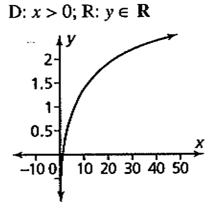
**8.** (a) $a > 1$: $f(x)$ passes through (1, 0). D: $x > 0$; R: $y \in$ **R**; asymp.: $x = 0$; $x \to 0$, $y \to -\infty$ and $x \to +\infty$, $y \to +\infty$.
$0 < a < 1$: $f(x)$ passes through (1, 0). D: $x > 0$; R: $y \in$ **R**; asymp.: $x = 0$; $x \to 0$, $y \to +\infty$ and $x \to +\infty$, $y \to -\infty$.

(b) logarithm must result in a unique exponent; if the base is 1, any exponent works

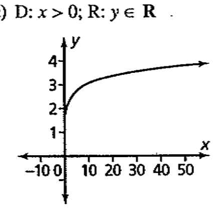
**9.** (a) D: $x > -6$; R: $y \in$ R



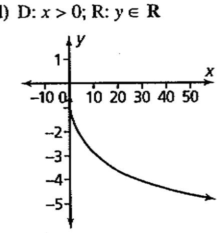
(b) D: $x > 0$; R: $y \in$ R



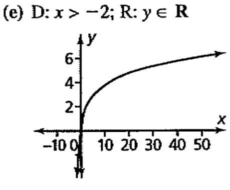
(c) D: $x > 0$; R: $y \in$ R



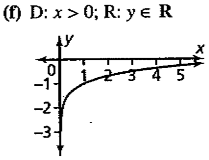
(d) D: $x > 0$; R: $y \in$ R



(e) D: $x > -2$; R: $y \in$ R



(f) D: $x > 0$; R: $y \in$ R



**10.** they are reflections of one another on the line $y = x$

**11.** $y = \log_3 x - 1$; 3 h 12 min