Chapter 1 Review

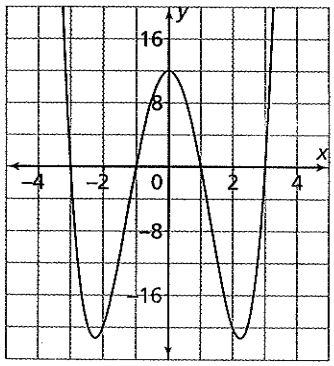
POLYNOMIAL FUNCTION MODELS

CHECK YOUR UNDERSTANDING

1. What features define a polynomial function?
2. What is the most number of zeros a degree-n polynomial function can have?
3. Sketch a degree-4 polynomial function with
   (a) four zeros (b) three zeros (c) two zeros
   (d) one zero (e) no zeros
4. Why do polynomial functions with an odd degree with no restrictions on the domain have at least one zero?
5. Explain the meaning of a turning point on the graph of a polynomial function.
6. What is the greatest number of turning points in the graph of a degree-n polynomial function? Sketch the graph of a degree-3 polynomial function with the maximum number of turning points and one with the minimum number of turning points.
7. Explain how the end behaviour of a polynomial function is determined by the leading coefficient and degree of the polynomial.
8. Explain how to use the factor theorem to find one factor of a polynomial function. How can you find the remaining factors?
9. A polynomial equation of degree 4 has exactly two real roots. How many other roots are possible? Describe the other roots.
10. Explain how to graphically determine the intervals of the domain where the value(s) of a polynomial function are positive. How are the intervals determined algebraically?

ADDITIONAL REVIEW QUESTIONS BY SECTION

1.1 Polynomial Functions



11. Expand, simplify, and write in standard form. State the degree.
    (a) $f(x) = (2x - 3)(3x + 1)(x + 4)$ (b) $f(x) = (x - 1)(2x - 3)^2(x + 2)$
    (c) $f(x) = (2x - 5)^3(3x + 1)$ (d) $f(x) = (x - 3)(x^3 + 4x^2 - x + 3)$
12. Determine the equation of the polynomial function of degree 3, with zeros -2, -1, and 4. The graph of this function passes through $(5, -84)$.
13. (a) Determine the equation of the function on the left.
    (b) Approximate the intervals in which the function increases and decreases.

1.2 Investigating the Characteristics of Polynomial Functions

14. Predict the value of the finite difference, then calculate that value.

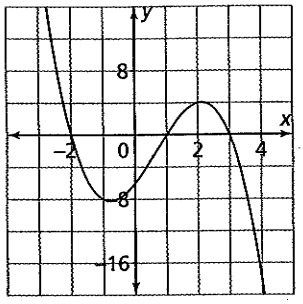
(a) $f(x) = 5x^2 - 2x + 3$, second finite difference

(b) $f(x) = -2x^3 + 3x^2 + 2x - 4$, third finite difference

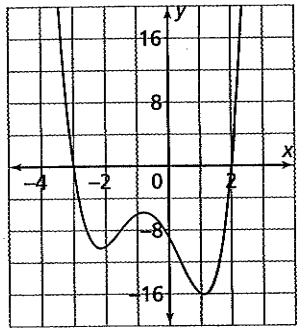
(c) $f(x) = 0.5x^4 - 3x^3 + 2x^2 - x + 4$, fourth finite difference

15. Describe the end behaviour and state the zeros of each function.

(a)



(b)



(c) $f(x) = 2x^3 + 3x^2 - x + 2$ **(d)** $f(x) = -5x^4 - 7x^3$

16. Graph each function. Visually approximate the coordinates of the turning points and the intervals in which $f(x)$ increases and decreases.

(a) $f(x) = x^3 - 3x^2 - 10x + 24$ **(b)** $f(x) = 0.5x^4 - x^3 - 6.5x^2 + 7x + 12$

1.3 Creating New Polynomial Functions

17. Given $f = \{(0, 6), (1, 3), (4, 7), (5, 8)\}$ and $g = \{(-1, 2), (1, 4), (2, 3), (4, 8), (8, 9)\}$, determine the following.

(a) $f(x) + g(x)$ **(b)** $f(x) - g(x)$ **(c)** $[f(x)][g(x)]$ **(d)** $f(g(x))$ **(e)** $g(f(x))$

18. Given $f(x) = 2x^2 - 2x$, $-2 \le x \le 3$ and $g(x) = -4x$, $-3 \le x \le 5$, graph the following.

(a) f **(b)** g **(c)** $f + g$ **(d)** $f - g$ **(e)** fg

19. Given $f(x) = x^3 - 3x^2 + 2x + 1$ and $g(x) = x - 2$, determine

(a) $f(g(2))$ **(b)** $g(f(2))$ **(c)** $f(g(x))$ **(d)** $g(f(x))$

20. A housing development begins in 2002 as a 200-m by 100-m parcel of land. Each year its length grows by 50 m and its width grows by 40 m.

(a) Express the length of the development as a function of time.

(b) Express the width as a function of time.

(c) What function represents the area of the development in terms of time?

(d) If the development continues to grow at the same rate what will be its area in 2010?

1.4 Dividing Polynomials

21. Divide each expression by $x + 2$. State the quotient and remainder.

(a) $2x^3 + 5x^2 - x - 5$ **(b)** $3x^3 + 13x^2 + 17x + 3$

(c) $2x^4 + 5x^3 - 16x^2 - 45x - 18$ **(d)** $2x^3 + 4x^2 - 5x - 4$

22. When $2x - 3$ is divided into the dividend, the quotient is $3x^3 + 10x^2 - 27x - 10$, with a remainder of 9. What is the dividend?

23. When $4x^4 - 15x^2 + px + 6$ is divided by $2x + 1$, the remainder is 2. Determine the value of p.

1.5 Factoring Polynomials

24. Use the remainder theorem to determine each remainder.

(a) $(2x^6 + 6x^5 - 3x^3 - 9x^2 + 3x + 1) \div (x + 3)$

(b) $(2x^3 - 9x^2 - 6x + 35) \div (2x - 5)$

25. Show whether $2x + 3$ is a factor of $6x^4 + 23x^3 + 7x^2 - 27x - 9$.

26. Factor fully.

(a) $x^3 - 4x^2 - 11x + 30$ **(b)** $x^4 - 15x^2 + 10x + 24$

(c) $x^4 - 2x^3 - 7x^2 + 8x + 12$ **(d)** $12x^3 + 28x^2 - 7x - 5$

(e) $6x^6 + 3x^5 + 2x^2 + x$ **(f)** $2x^4 - 10x^3 - x^2 + 5x$

1.6 Solving Polynomial Equations

27. Solve algebraically. Confirm the solutions using technology.

(a) $x^3 + 2x^2 = x + 2$ **(b)** $2x^4 + 11x^3 + 9x^2 - 14x - 8 = 0$

28. Solve algebraically, to two decimal places. Confirm using technology.

(a) $x^3 + x^2 - 11x + 10 = 0$ **(b)** $x^3 + 5x^2 = 2x + 6$

(c) $2x^4 - 9x^2 - x + 6 = 0$ **(d)** $x^4 - 13x^2 = -36$

29. What is the equation of a degree-4 polynomial function with zeros 1 and $2 - 3i$ and whose graph passes through $(-1, -72)$?

1.7 Solving Polynomial Inequalities

30. Solve graphically.

(a) $x^3 - 6x^2 + 5x + 12 > 0$ **(b)** $x^4 - 3x^3 - 3x^2 + 7x + 6 < 0$

(c) $-2x^3 + 7x^2 - 9x + 6 \leq 0$ **(d)** $-2x^4 + 7x^3 - 19x^2 + 21x - 15 > 0$

31. Solve algebraically and confirm graphically.

(a) $-x^3 - 2x^2 + 5x + 6 > 0$ **(b)** $x^4 + 2x^3 - 13x^2 - 14x + 24 < 0$

(c) $2x^3 - x^2 + x + 4 < 0$ **(d)** $2x^4 + 5x^3 + 7x^2 + 7x + 3 < 0$

32. Write using absolute value notation.

(a) $-2 < x < 2$ **(b)** $-5 \leq x \leq 5$ (c) $x < -3$ or $x > 3$

REVIEW QUESTIONS BY ACHIEVEMENT CHART CATEGORIES

Knowledge and Understanding

33. Which binomials are factors of $f(x) = x^4 + 2x^3 - 25x^2 - 26x + 120$?

(a) $x - 1$ **(b)** $x - 2$ **(c)** $x + 3$

(d) $x - 4$ **(e)** $x + 2$ **(f)** $x + 5$

34. Divide each polynomial using long division. Check using synthetic division.

(a) $(2x^3 - 3x^2 - 11x + 6) \div (x - 3)$

(b) $(2x^4 + 11x^3 + x^2 - 50x - 24) \div (2x + 1)$

(c) $(x^6 - 4x^3 + 3x^2 - 12) \div (x + 2)$

(d) $(x^4 + 2x^3 - 12x^2 - 8x + 32) \div (x^2 - 4)$

35. Solve for x, $x \in \mathbf{R}$.

(a) $x^3 - 7x - 6 = 0$ **(b)** $6x^3 - 5x^2 - 3x + 2 = 0$

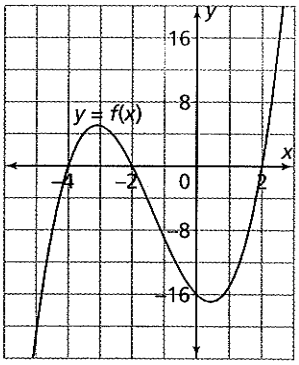
(c) $64x^3 - 27 = 0$ **(d)** $2x^3 + 6x^2 - x - 3 = 0$

(e) $2x^4 - x^3 = 8x^2 - x - 6$ **(f)** $x^5 - x^4 - 4x = -4$

Communication

36. **(a)** Describe how to determine a cubic polynomial function, given its x-intercepts, and a point that satisfies the function.

(b) Determine the cubic function with x-intercepts -1, 3, and 4. The graph of the function passes through $(5, -24)$.



37. The graph of a cubic polynomial function is shown on the left. Describe the graph in terms of end behaviour, x-intercepts, intervals of increase and decrease, turning points, local maxima and minima, intervals when the function is greater than 0, and intervals when it is less than 0. Approximate values are sufficient.

38. Sketch a quartic polynomial function, $f(x)$, in which as $x \to -\infty$, $f(x) \to -\infty$; as $x \to \infty$, $f(x) \to -\infty$; the x-intercepts are 2 and -3; and $f(x) \leq 0$ for all x.

Application

39. Prove that $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ using

(a) multiplication

(b) the factor theorem

40. During a normal five-second respiratory cycle in which a person inhales and then exhales, the volume of air in a person's lungs can be modelled by $V(t) = 0.027t^3 - 0.27t^2 + 0.675t$, where volume, V, is in litres and t is the time in seconds. In this cycle, when is the volume of air in the lungs

(a) at its maximum? **(b)** 0.4 L? (c) more than 0.3 L?

Year	Population (millions)
1950	2555
1955	2780
1960	3039
1965	3346
1970	3708
1975	4088
1980	4457
1985	4855
1990	5284
1995	5691
2000	6080

Source: U.S. Bureau of the Census

41. The population of the world from 1950 to 2000 is shown on the left. Prepare a scatter plot of the data. Model the data using a cubic function. State any restrictions on the domain. Estimate the population in 1963 and 1983. What will the world population be in 2040? Does this seem reasonable according to the growth pattern of the last 50 years? Explain.

Thinking, Inquiry, Problem Solving

42. One root of $3x^3 - 15x^2 + kx - 4 = 0$ is 2. Find k and the other roots.

43. The graph of a quartic function crosses the x-axis at -4 and -2, and touches at $x = 3$. State the equation of the family of curves. Sketch one possible function in this family. Sketch its reflected image in the x-axis. Compare the leading coefficients of the functions. Let the coordinates of the turning point of one of the functions be (a, b). Show that the distance between the turning point of this function and the corresponding turning point of the reflected function is $2|b|$.

44. What is the equation of the quartic function with roots $2 + 3i$ and $1 - 2i$ and whose graph passes through $(-2, 325)$?

Chapter 1 Performance Task

Katie is building a wooden rectangular toy storage box for her younger brother. The box will have an open top and volume of 9 m^3. For design purposes, Katie would like the length of its base to be triple its width. Thick wood for the base costs \$8/m^2 and thinner wood for the sides costs \$5/m^2.

1. Express the cost of wood as a function of the width of the base.
2. What is a reasonable domain and range for the function in this context?
3. Using graphing technology, such as a TI-83 Plus, graph this function with appropriate window settings. Explain why you chose the window settings that you did and comment on the shape of the graph.
4. Locate any maximum and minimum values. Explain what they mean in the context of the problem. What is the cost as the width approaches ∞ and as it approaches $-\infty$? Is this result realistic? Why or why not?
5. If Katie can afford to spend \$144 to make the toy storage box, what must the dimensions of the box be to keep the volume at 9 m^3?
6. What dimensions do you recommend that Katie use? Justify your answer.

Chapter 1 Review Test

POLYNOMIAL FUNCTION MODELS

1. **Knowledge and Understanding**
   (a) Write the standard form for a general polynomial function and state its degree and leading coefficient.
   (b) What is the most number of turning points this function can have?
   (c) What is the most number of zeros this function can have?
   (d) If the least number of zeros is 1, describe the degree of the polynomial.
   (e) If a polynomial function is less than 0 for all x, describe the degree and the leading coefficient of the polynomial.

2. **Communication:** Let $f(x) = (x - 3)(x + 4)(x - 1)$.
   (a) Describe how to graph $f(x)$ without technology.
   (b) Follow your directions and graph $f(x)$.

3. Divide $6x^3 + x^2 - 12x + 5$ by $2x - 1$. Is the divisor a factor of the dividend?

4. Factor.
   (a) $2x^3 - 3x^2 - 5x + 6$ (b) $8x^3 - 27$ (c) $4x^3 - 8x^2 + 3x - 6$

5. Solve for x algebraically.
   (a) $2x^4 + 7x^3 - 6x^2 = 7x - 4$ (b) $2x^3 + x^2 > 8x - 5$

6. The roots and degree of a polynomial function are given. Write the function in standard form.
   (a) 1, 2, -3, degree 3 (b) 2, $-2i$, degree 4

7. **Application:** The incidence of lung cancer in Canadians per 100 000 population is shown on the left.
   (a) Determine a cubic function to represent the curve of best fit for the male data and for the female data. Determine the number of males and females per 100 000 who had lung cancer in 1983.
   (b) According to your models, when will more females have lung cancer than males?

Year	Males	Females
1975	73.1	14.7
1980	83.2	21.7
1985	93.2	30.9
1990	92.7	36.5
1995	84.7	40.8
2000	78.6	46.4

Source: Cancer Bureau, Health Canada

8. **Thinking, Inquiry, Problem Solving:** Chris makes an open-topped box from a 30-cm by 30-cm piece of cardboard by cutting out equal squares from the corners and folding up the flaps to make the sides. What are the dimensions of each square, to the nearest hundredth of a centimetre, so that the volume of the resulting box is more than 100 cm^3?

9. The graph of $f(x) = 3x^4 + 14x^3 + px^2 + qx + 24$ has x-intercepts -4 and 2. Determine the function.