6.4 Exploring Transformations of Sinusoidal Functions

GOAL

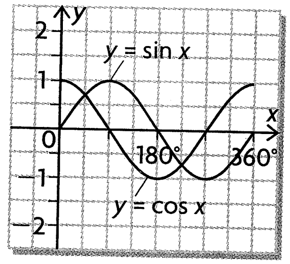
Determine how changing the values of a, c, d, and k affect the graphs of $f(x) = a\sin(k(x - d)) + c$ and $f(x) = a\cos(k(x - d)) + c$

YOU WILL NEED

- graphing calculator

EXPLORE the Math

Paula and Marcus know how various transformations affect several types of functions, such as $f(x) = x^2$, $f(x) = \frac{1}{x}$, $f(x) = \sqrt{x}$, and $f(x) = |x|$.



They want to know if these same transformations can be applied to $y = \sin x$ and $y = \cos x$, and if so, how the equations and graphs of these functions change.

? Can transformations be applied to sinusoidal functions in the same manner, and do they have the same effect on the graph and the equation?

Part 1 The graphs of $y = a \sin x$ and $y = a \cos x$

A. Predict what the graphs of $y = a \sin x$, $0° \leq x \leq 720°$, will look like for $a = 1, 2,$ and 3 and for $a = \frac{1}{2}$ and $a = \frac{1}{4}$. Sketch the graphs on the same axes. Verify your sketches using a graphing calculator.

B. On a new set of axes, repeat part A for the graphs of $y = a \sin x$, $0° \leq x \leq 720°$, for $a = -1, -2,$ and -3.

C. How do the graphs in part A compare with those in part B? Discuss how the zeros, amplitude, and maximum or minimum values change for each function.

D. Repeat parts A to C using $y = a \cos x$.

E. Explain how the value of a affects the graphs of $y = a \sin x$ and $y = a \cos x$.

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For Parts 1 and 2, verify your sketches by graphing the parent function ($y = \sin x$ or $y = \cos x$) in Y1 and each transformed function in Y2, Y3, and so on. Use an Xscl = 90°, and graph using ZoomFit by pressing ZOOM 0.

Part 2 The graphs of $y = \sin x + c$ and $y = \cos x + c$

F. Predict what the graphs of $y = \sin x + c$, $0° \leq x \leq 720°$, will look like for $c = -2, -1, 1,$ and 2. Sketch the graphs on the same axes, and then verify your predictions using a graphing calculator. Discuss which features of the graph have changed.

G. Predict what the graphs of $y = \cos x + c$, $0° \leq x \leq 720°$, will look like for $c = -2, -1, 1,$ and 2. Sketch the graphs on the same axes, and then verify your predictions using a graphing calculator. Discuss which features of the graph have changed.

H. Explain how the value of c affects the graphs of $y = a \sin x + c$ and $y = a \cos x + c$.

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> For Part 3, verify your sketches by graphing the parent function in Y1 and each transformed function in Y2. Use an Xscl = 90° and graph using ZoomFit.

Part 3 The graphs of $y = \sin kx$ and $y = \cos kx$

I. Predict what the graphs of $y = \sin kx$ will look like for $k = 2, 3,$ and 4, $0° \le x \le 720°$. Sketch each graph, and then verify your predictions using a graphing calculator. Discuss which features of the graph have changed. Clear the previous equation, but not the base equation, from the graphing calculator before entering another equation.

J. Repeat part I for $k = \frac{1}{2}$, $k = \frac{1}{4}$, and $k = -1$. Adjust the WINDOW on the graphing calculator so that you can see one complete cycle of each graph.

K. Repeat parts I and J using $y = \cos kx$.

L. How could you determine the period of $y = \sin kx$ and $y = \cos kx$ knowing that the period of both functions is 360°?

M. Explain how the value of k affects $y = \sin kx$ and $y = \cos kx$.

Part 4 The graphs of $y = \sin(x - d)$ and $y = \cos(x - d)$

N. **a)** Predict the effect of d on the graph of $y = \sin(x - d)$.

b) Copy and complete the tables of values at the left.

i) $y = \sin(x - 60°)$ **ii)** $y = \sin(x + 120°)$

i)

x	y
60°	
150°	
240°	
330°	
420°	

ii)

x	y
−120°	
−30°	
60°	
150°	
240°	

c) Use your tables to sketch the graphs of the two sinusoidal functions from part (b) on the same coordinate system. Include the graph of $y = \sin x$. Verify your sketches using a graphing calculator, and discuss which features of the graph have changed.

O. **a)** Predict the effect of d on the graph of $y = \cos(x - d)$.

b) Copy and complete the tables of values at the left.

i) $y = \cos(x + 45°)$ **ii)** $y = \cos(x - 120°)$

i)

x	y
−45°	
45°	
135°	
225°	
315°	

ii)

x	y
120°	
210°	
300°	
390°	
480°	

c) Use your tables to sketch the graphs of the two sinusoidal functions from part (b) on the same coordinate system. Include the graph of $y = \cos x$. Verify your sketches with a graphing calculator, and discuss which features of the graph have changed.

P. Explain how the value of d affects the graphs of $y = \sin(x - d)$ and $y = \cos(x - d)$.

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> For Part 4, verify your sketches using a domain of $0° \le x \le 360°$ and an Xscl = 30°. Graph using ZoomFit.

Reflecting

Q. What transformation affects the period of a sinusoidal function?

R. What transformation affects the equation of the axis of a sinusoidal function?

S. What transformation affects the amplitude of a sinusoidal function?

T. What transformations affect the location of the maximum and minimum values of the sinusoidal function?

U. Summarize how the graphs of $y = a\sin(k(x - d)) + c$ and $y = a\cos(k(x - d)) + c$ compare with the graphs of $y = \sin x$ and $y = \cos x$.

In Summary

Key Ideas

- The graphs of the functions $f(x) = a\sin(k(x - d)) + c$ and $f(x) = a\cos(k(x - d)) + c$ are periodic in the same way that the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ are. The differences are only in the placement of the graph and how stretched or compressed it is.
- The values a, k, c, and d in the functions $f(x) = a\sin(k(x - d)) + c$ and $f(x) = a\cos(k(x - d)) + c$ affect the graphs of $y = \sin x$ and $y = \cos x$ in the same way that they affect the graphs of $y = f(k(x - d)) + c$, where $f(x) = x^2$, $f(x) = \frac{1}{x}$, $f(x) = \sqrt{x}$, and $f(x) = |x|$.

Need to Know

- Changing the value of c results in a vertical translation and affects the equation of the axis, the maximum and minimum values, and the range of the function but has no effect on the period, amplitude, or domain.
- Changing the value of d results in a horizontal translation and slides the graph to the left or right but has no effect on the period, amplitude, equation of the axis, domain, or range unless the situation forces a change in the domain or range.
- Changing the value of a results in a vertical stretch or compression and affects the maximum and minimum values, amplitude, and range of the function but has no effect on the period or domain. If a is negative, a reflection in the x-axis also occurs.
- Changing the value of k results in a horizontal stretch or compression and affects the period, changing it to $\frac{360°}{|k|}$, but has no effect on the amplitude, equation of the axis, maximum and minimum values, domain, and range unless the situation forces a change in the domain or range. If k is negative, a reflection in the y-axis also occurs.

FURTHER Your Understanding

1. State the transformation to the graph of either $y = \sin x$ or $y = \cos x$ that has occurred to result in each sinusoidal function.

a) $y = 3\cos x$
b) $y = \sin(x - 50°)$
c) $y = -\cos x$
d) $y = \sin(5x)$
e) $y = \cos x - 6$
f) $y = \cos(x + 20°)$

2. Each sinusoidal function below has undergone one transformation that has affected either the period, amplitude, or equation of the axis. In each case, determine which characteristic has been changed and indicate its value.

a) $y = \sin x + 2$
b) $y = 4\sin x$
c) $y = \cos(8x)$
d) $y = \sin(2x + 30°)$
e) $y = 0.25\cos x$
f) $y = \sin(0.5x)$

3. Which two of these transformations do not affect the period, amplitude, or equation of the axis of a sinusoidal function?

a) reflection in the x-axis
b) vertical stretch/vertical compression
c) vertical translation
d) horizontal stretch/horizontal compression
e) horizontal translation

6.5 Using Transformations to Sketch the Graphs of Sinusoidal Functions

YOU WILL NEED

- graph paper

GOAL

Sketch the graphs of sinusoidal functions using transformations.

LEARN ABOUT the Math

Glen has been asked to graph the sinusoidal function $f(x) = 3\sin(2(x - 60°)) + 4$ without using technology.

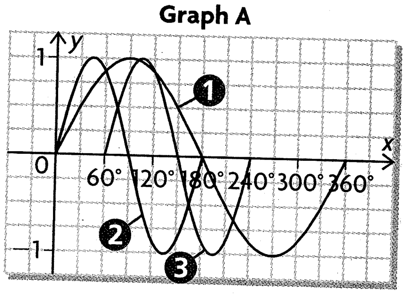
? How can you graph sinusoidal functions using transformations?

EXAMPLE 1 Using transformations to sketch the graph of a sinusoidal function

Sketch the graph of $f(x) = 3\sin(2(x - 60°)) + 4$.

Glen's Solution

Graph A

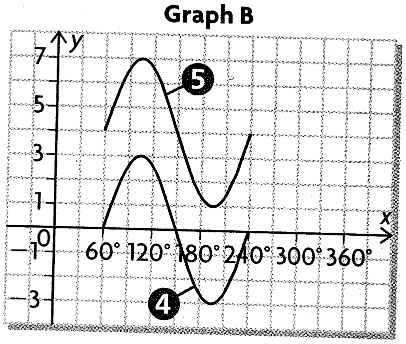


1 I started by graphing $y = \sin x$ (in green).

2 Then I graphed $y = \sin(2x)$ (in red). It has a horizontal compression of $\frac{1}{2}$, so the period is $\frac{360°}{2} = 180°$ instead of 360° because all the x-coordinates of the points on the graph of $y = \sin x$ have been divided by 2.

3 Then I graphed $y = \sin(2(x - 60°))$ (in blue) by applying a horizontal translation of $y = \sin(2x)$ 60° to the right because 60° has been added to all the x-coordinates of the points on the previous graph.

Graph B



4 Next, I graphed $y = 3\sin(2(x - 60°))$ (in purple) by applying a vertical stretch of 3 to $y = \sin(2(x - 60°))$. The amplitude is now 3 because all the y-coordinates of the points on the previous graph have been multiplied by 3.

5 Finally, I graphed $y = 3\sin(2(x - 60°)) + 4$ (in black) by applying a vertical translation of 4 to $y = 3\sin(2(x - 60°))$. This means that the whole graph has slid up 4 units and that the equation of the axis is now $y = 4$ because 4 has been added to all the y-coordinates of the points on the previous graph.

Reflecting

A. In what order were the transformations applied to the function $y = \sin x$?

B. If the equation of the function $y = 3\sin(2(x - 60°)) + 4$ were changed to $y = 3\sin(2(x - 60°)) - 5$, how would the graph of the function change? How would it stay the same?

C. If the equation of the function $y = 3\sin(2(x - 60°)) + 4$ were changed to $y = 3\sin(9(x - 60°)) + 4$, how would the graph of the function change?

D. Which transformations affect the range of the function? How?

E. Which transformations affect the period of the function? How?

F. Could Glen graph this function faster by combining transformations? If so, which ones?

APPLY the Math

EXAMPLE 2 Connecting transformations to the graph of a sinusoidal function

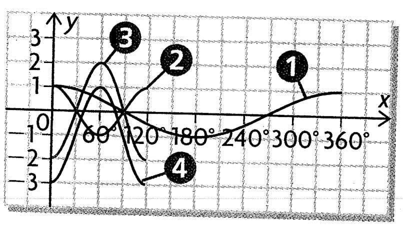
a) Graph $y = -2\cos(3x) - 1$ using transformations.

b) State the amplitude, period, equation of the axis, **phase shift**, and range of this sinusoidal function.

phase shift
the horizontal translation of a sinusoidal function

Steven's Solution

a)



1 I started by graphing $y = \cos x$ (in green).

2 I dealt with the horizontal compression first. I graphed $y = \cos(3x)$ (in red) using a period of $\frac{360°}{3} = 120°$ instead of 360°.

3 I dealt with the vertical stretch and the reflection in the x-axis. I graphed $y = -2\cos(3x)$ (in blue) starting at its lowest value due to the reflection, changing its amplitude to 2 due to the vertical stretch.

4 Finally, I did the vertical translation. I graphed $y = -2\cos(3x) - 1$ (in black) by sliding the previous graph down 1 unit, so the equation of the axis is $y = -1$.

b) The amplitude is 2.

The period is 120°.

The equation of the axis is $y = -1$.

The phase shift is 0.

The range is $\{y \in \mathbf{R} \mid -3 \leq y \leq 1\}$.

You can graph sinusoidal functions more efficiently if you combine and use several transformations at the same time.

EXAMPLE 3 Using a factoring strategy to determine the transformations

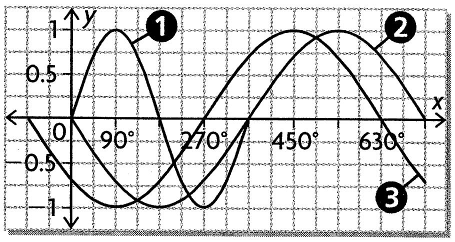
Graph $y = -\sin(0.5x + 45°)$ using transformations.

John's Solution

$y = -\sin(0.5x + 45°)$

$y = -\sin[0.5(x + 90°)]$

I factored the expression inside the brackets so that I could see all the transformations. I divided out the common factor 0.5 from 0.5x and 45.



1 I started by graphing $y = \sin x$ (in green).

2 Rather than graph this one transformation at a time, I dealt with all stretches/compressions and reflections at the same time. I graphed $y = -\sin(0.5x)$ (in red) by using a period of $\frac{360°}{0.5} = 720°$ and reflecting this across the x-axis.

3 I applied the phase shift and graphed $y = -\sin(0.5(x + 90°))$ (in black) by shifting all the points on the previous graph 90° to the left.

In Summary

Key Idea

- Functions of the form $g(x) = a\sin(k(x - d)) + c$ and $h(x) = a\cos(k(x - d)) + c$ can be graphed by applying the appropriate transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$, respectively, one at a time, following the order of operations (multiplication and division before addition and subtraction) for all vertical transformations and for all horizontal transformations. The horizontal and vertical transformations can be completed in either order.
- As with other functions, you can apply all stretches/compressions and reflections together followed by all translations to graph the transformed function more efficiently.

Need to Know

- To graph $g(x)$, you need to apply the transformations to the key points of $f(x) = \sin x$ or $f(x) = \cos x$ only, not to every point on $f(x)$.
  - Key points for $f(x) = \sin x$
    (0°, 0), (90°, 1), (180°, 0), (270°, −1), (360°, 0)
  - Key points for $f(x) = \cos x$
    (0°, 1), (90°, 0), (180°, −1), (270°, 0), (360°, 1)

(continued)

- By doing so, you end up with a function with
  - an amplitude of $|a|$
  - a period of $\frac{360°}{|k|}$
  - an equation of the axis $y = c$
- Horizontal and vertical translations of sine and cosine functions can be summarized as follows:

  Horizontal
  - Move the graph d units to the right when $d > 0$.
  - Move the graph $|d|$ units to the left when $d < 0$.

  Vertical
  - Move the graph $|c|$ units down when $c < 0$.
  - Move the graph c units up when $c > 0$.
- Horizontal and vertical stretches of sine and cosine functions can be summarized as follows:

  Horizontal
  - Compress the graph by a factor $\left|\frac{1}{k}\right|$ when $|k| > 1$.
  - Stretch the graph by a factor $\left|\frac{1}{k}\right|$ when $0 < |k| < 1$.
  - Reflect the graph in the y-axis if $k < 0$.

  Vertical
  - Stretch the graph by a factor $|a|$ when $|a| > 1$.
  - Compress the graph by a factor $|a|$ when $0 < |a| < 1$.
  - Reflect the graph in the x-axis if $a < 0$.

CHECK Your Understanding

1. State the transformations, in the order you would apply them, for each sinusoidal function.

a) $f(x) = \sin(4x) + 2$

b) $y = 0.25\cos(x - 20°)$

c) $g(x) = -\sin(0.5x)$

d) $y = 12\cos(18x) + 3$

e) $f(x) = -20\sin\left[\frac{1}{3}(x - 40°)\right]$

2. If the function $f(x) = 4\cos 3x + 6$ starts at $x = 0$ and completes two full cycles, determine the period, amplitude, equation of the axis, domain, and range.

3. Use transformations to predict what the graph of $g(x) = 5\sin(2(x - 30°)) + 4$ will look like. Verify with a graphing calculator.

PRACTISING

4. State the transformations in the order you would apply them for each sinusoidal function.

a) $y = -2\sin(x + 10°)$

b) $y = \cos(5x) + 7$

c) $y = 9\cos(2(x + 6°)) - 5$

d) $g(x) = \frac{1}{5}\sin(x - 15°) + 1$

e) $h(x) = -\sin\left[\frac{1}{4}(x + 37°)\right] - 2$

f) $d = -6\cos(3t) + 22$

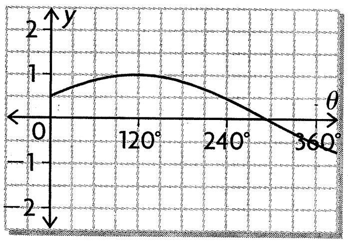
5. Match each function to its corresponding graph.

a) $y = \sin(2\theta - 90°), 0° \leq \theta \leq 360°$

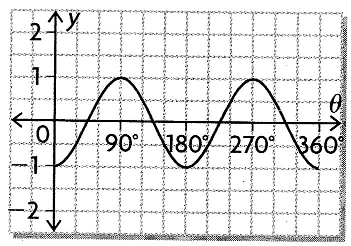
b) $y = \sin(3\theta - 90°), 0° \leq \theta \leq 360°$

c) $y = \sin\left(\frac{\theta}{2} + 30°\right), 0° \leq \theta \leq 360°$

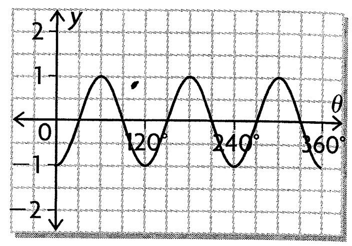
i)



ii)



iii)



6. If each function starts at $x = 0$ and finishes after three complete cycles, determine the period, amplitude, equation of the axis, domain, and range of each without graphing.

a) $y = 3\sin x + 2$

b) $g(x) = -4\cos(2x) + 7$

c) $h = -\frac{1}{2}\sin t - 5$

d) $h(x) = \cos(4(x - 12°)) - 9$

e) $d = 10\sin(180(t - 17°)) - 30$

f) $j(x) = 0.5\sin(2x - 30°)$

7. K Predict what the graph of each sinusoidal function will look like by describing the transformations of $y = \sin x$ or $y = \cos x$ that would result in the new graph. Sketch the graph, and then verify with a graphing calculator.

a) $y = 2\sin x + 3$

b) $y = -3\cos x + 5$

c) $y = -\sin(6x) + 4$

d) $y = 4\cos(2x) - 3$

e) $y = \frac{1}{2}\cos(3x - 120°)$

f) $y = -8\sin\left[\frac{1}{2}(x + 50°)\right] - 9$

8. Determine the appropriate WINDOW settings on your graphing calculator that enable you to see a complete cycle for each function. There is more than one acceptable answer.

a) $k(x) = -\sin(2x) + 6$

b) $j(x) = -5\sin\left(\frac{1}{2}x\right) + 20$

c) $y = 7\cos(90(x - 1°)) + 82$

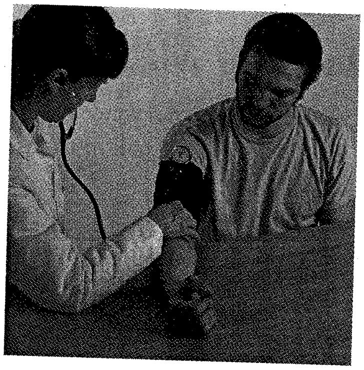
d) $f(x) = \frac{1}{2}\sin(360x + 72°) - 27$

9. Each person's blood pressure is different. But there is a range of blood pressure values that is considered healthy. For a person at rest, the function $P(t) = -20\cos(300t)° + 100$ models the blood pressure, $P(t)$, in millimetres of mercury at time t seconds.

A

a) What is the period of the function? What does the period represent for an individual?

b) What is the range of the function? Explain the meaning of the range in terms of a person's blood pressure.



10. a) Determine the equation of a sine function that would have the range $\{y \in \mathbf{R} \mid -1 \leq y \leq 7\}$ and a period of 720°.

T

b) Determine the equation of the cosine function that results in the same graph as your function in part (a).

11. Explain how you would graph the function $f(x) = -\frac{1}{2}\cos(120x) + 30$ using transformations.

C

Extending

12. If the functions $y = \sin x$ and $y = \cos x$ are subjected to a horizontal compression of 0.5, what transformation would map the resulting sine curve onto the resulting cosine curve?

13. The function $D(t) = 4\sin[\frac{360}{365}(t - 80)]° + 12$ is a model of the number of hours, $D(t)$, of daylight on a specific day, t, at latitude 50° north.

a) Explain why a trigonometric function is a reasonable model for predicting the number of hours of daylight.

b) How many hours of daylight do March 21 and September 21 have? What is the significance of each of these days?

c) How many hours of daylight do June 21 and December 21 have? What is the significance of each of these days?

d) Explain what the number 12 represents in the model.

Lesson 6.5, pp. 383–385

1. a) horizontal compression: $\frac{1}{4}$, vertical translation: 2, in any order

b) horizontal translation: 20, vertical compression: $\frac{1}{4}$, in any order

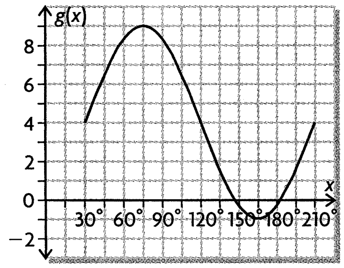
c) horizontal stretch: 2; reflection in x-axis, in any order

d) horizontal compression: $\frac{1}{18}$; vertical stretch: 12; vertical translation: 3, in any order

e) horizontal stretch: 3; horizontal translation: 40; vertical stretch: 20; reflection in x-axis, with the horizontal translation after the horizontal stretch

2. period: 120°; amplitude: 4; axis: $y = 6$;
domain: $\{x \in \mathbf{R} \mid 0° \le x \le 240°\}$;
range: $\{y \in \mathbf{R} \mid 2 \le y \le 10\}$

3.



4. Order may vary, as long as any horizontal translations are after any horizontal stretches or compressions, and any vertical translations are after any vertical stretches or compressions.

a) horizontal translation: −10; vertical stretch: 2; reflection in x-axis

b) horizontal compression: $\frac{1}{5}$; vertical translation: 7

c) horizontal compression: $\frac{1}{2}$; horizontal translation: −6; vertical stretch: 9; vertical translation: −5

d) horizontal translation: 15; vertical compression: $\frac{1}{5}$; vertical translation: 1

e) horizontal stretch: 4; horizontal translation: −37; reflection in x-axis; vertical translation: −2

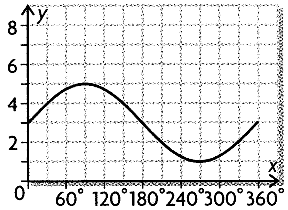
f) horizontal compression: $\frac{1}{3}$; vertical stretch: 6; reflection in x-axis; vertical translation: 22

5. a) (ii) b) (iii) c) (i)

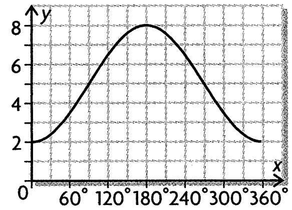
6.

	Period	Amplitude	Equation of the Axis	Domain	Range
a)	360°	3	$y = 2$	$\{x \in \mathrm{R} \mid 0° \le x \le 1080°\}$	$\{y \in \mathrm{R} \mid -1 \le y \le 5\}$
b)	180°	4	$g = 7$	$\{x \in \mathrm{R} \mid 0° \le x \le 540°\}$	$\{g \in \mathrm{R} \mid 3 \le g \le 11\}$
c)	360°	$\frac{1}{2}$	$h = -5$	$\{t \in \mathrm{R} \mid 0° \le t \le 1080°\}$	$\{h \in \mathrm{R} \mid -5.5 \le h \le -4.5\}$
d)	90°	1	$h = -9$	$\{x \in \mathrm{R} \mid 0° \le x \le 270°\}$	$\{h \in \mathrm{R} \mid -10 \le h \le -8\}$
e)	2°	10	$d = -30$	$\{t \in \mathrm{R} \mid 0° \le t \le 6°\}$	$\{d \in \mathrm{R} \mid -40 \le d \le -20\}$
f)	180°	$\frac{1}{2}$	$j = 0$	$\{x \in \mathrm{R} \mid 0° \le x \le 540°\}$	$\{j \in \mathrm{R} \mid -0.5 \le j \le 0.5\}$

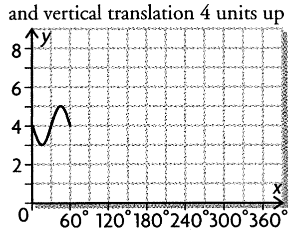
7. a) vertical stretch by a factor of 2 and vertical translation 3 units up



b) vertical stretch by a factor of 3, reflection in the x-axis, and vertical translation 5 units up

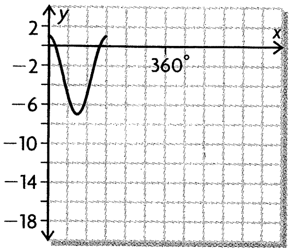


c) horizontal compression by a factor of $\frac{1}{6}$, reflection in the x-axis, and vertical translation 4 units up

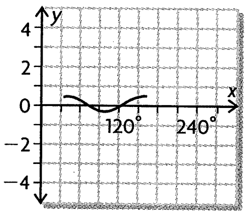


Answers

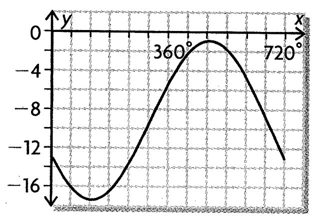
d) vertical stretch by a factor of 4, horizontal compression by a factor of $\frac{1}{2}$, and vertical translation 3 units down



e) vertical compression by a factor of $\frac{1}{2}$, horizontal compression by a factor of $\frac{1}{3}$, and horizontal translation 40° to the right



f) vertical stretch by a factor of 8, reflection in the x-axis, horizontal stretch by a factor of 2, horizontal translation 50° to the left, and vertical translation 9 units down



8.

	X min	X max	Y min	Y max
a)	0°	180°	5	7
b)	0°	720°	15	25
c)	0°	4°	75	89
d)	0°	1°	−27.5	−26.5

9. a) period: 1.2 s; one heart beat
   b) $\{P \in \mathbf{R} \mid 80 \leq P \leq 120\}$, maximum blood pressure of 120, minimum blood pressure of 80
10. Answers may vary. For example,
    a) $y = 4 \sin\left(\frac{1}{2}x\right) + 3$
    b) $y = 4 \cos\left(\frac{1}{2}x - 90\right) + 3$
11. Reflection in x-axis, vertical compression of $\frac{1}{2}$, vertical translation 30 up, horizontal compression of $\frac{1}{120}$.
12. horizontal translation: $-45°$
13. a) The number of hours of daylight increases to a maximum and decreases to a minimum in a regular cycle as Earth revolves around the Sun.
    b) Mar. 21: 12 h; Sept. 21: 12 h, spring and fall equinoxes
    c) June 21: 16 h; Dec. 21: 8 h, longest and shortest days of year; summer and winter solstices
    d) 12 is the axis of the curve representing half the distance between the maximum and minimum hours of daylight.

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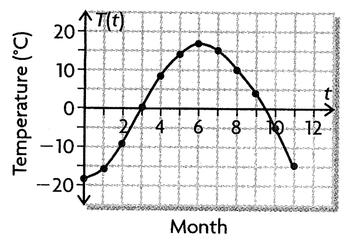
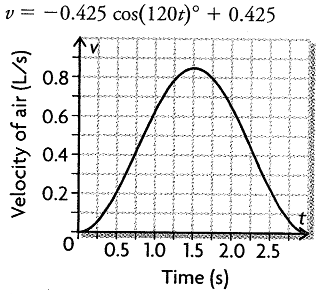
1. Answers may vary. For example,
   a) $y = 2\cos(4x) + 6$
   b) $y = \cos(2(x - 90°)) + 2$
   c) $y = 2\cos(3x) - 2$
2. $y = 2\cos(2x) + 7$
3. Answers may vary. For example, $y = 4\cos(3x) + 5$
4. Answers may vary. For example,
   a) i) $y = 3\cos(60(x - 4°)) + 5$;
   ii) $y = -0.5\cos(120x) + 1$;
   iii) $y = \cos(90(x - 3°)) - 2$
   b) i) $y = 5\cos(180(x - 1.5°)) + 25$;
   ii) $y = 5\cos(120(x - 2°)) + 10$;
   iii) $y = 10\cos(360x) - 5$
5. a) $y = \cos(3x) + 17$
   b) $y = -4\cos(0.5x) + 2$
   c) $y = 3\sin(1.5x) - 4$
   d) $y = 3\cos(3(x - 10°)) + 2$
6. a) $y = 3\cos x + 11$
   b) $y = 4\cos(2(x - 30°)) + 15$
   c) $y = 2\cos(9(x - 7°))$
   d) $y = 0.5\cos\left(\left(\frac{1}{2}\right)(x + 56°)\right) - 3$
7. Answers may vary. For example, $y = -6\cos(8x) + 7$
8. a)



   b) sinusoidal model because it changes with a cyclical pattern over time; the data is wave-shaped.
   c) $T(t) = -17.8\cos 30t - 0.8$
   d) Answers may vary. For example, 8.1°C or 10.3°C using the chart.
9. a) The respiratory cycle is an example of a periodic function because we inhale, rest, exhale, rest, inhale, and so on in a cyclical pattern.
   b) $v = -0.425\cos(120t)° + 0.425$



   c) The fit is somewhat close.
   d) 0 L/s; period is 3; troughs occur at 0, 3, and 6 s
   e) Answers may vary. For example, 0.8 s and 2.2 s from model or 0.6 s and 2.4 s from chart.