1 Mid-Chapter Review

FREQUENTLY ASKED Questions

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- See Lesson 1.1, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 1 and 2.

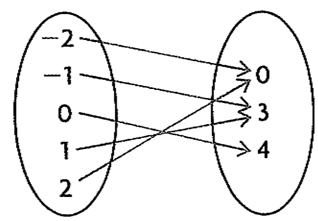
Q: How can you determine whether a relation is a function?

A1: For a relation to be a function, there must be only one value of the dependent variable for each value of the independent variable.

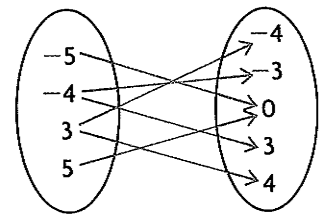
If the relation is described by a list of ordered pairs, you can see if any first elements appear more than once. If they do, the relation is not a function. For example, the relation $\{(-2, 0), (-1, 3), (0, 4), (1, 3), (2, 0)\}$ is a function; but the relation $\{(-5, 0), (-4, 3), (-4, -3), (3, -4), (3, 4), (5, 0)\}$ is not, because -4 and 3 each appear more than once as first elements.

A2: If the relation is shown in a mapping diagram, you can look at the arrows. If more than one arrow goes from an element of the domain (on the left) to an element of the range (on the right), then the relation is not a function. For example, diagram A shows a function but diagram B does not.

A

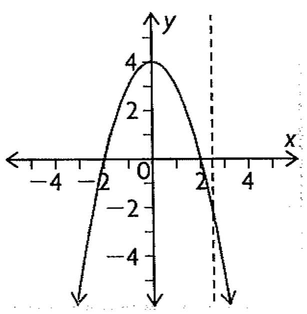


B

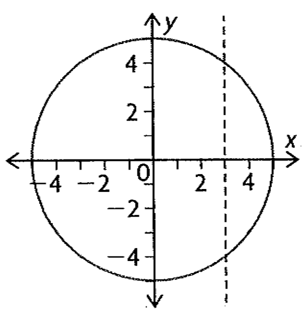


A3: If you have the graph of the relation, you can use the vertical-line test. If you can draw a vertical line that crosses the graph in more than one place, then an element in the domain corresponds to two elements in the range, so the relation is not a function. For example, graph A shows a function but graph B does not.

A



B



A4: If you have the equation of the relation, you can substitute numbers for x to see how many y-values correspond to each x-value. If a single x-value produces more than one corresponding y-value, the equation does not represent a function. For example, the equation $y = 4 - x^2$ is the equation of a function because you would get only one answer for y by putting a number in for x. The equation $x^2 + y^2 = 25$ does ***not*** represent a function because there are two values for y when x is any number between -5 and 5.

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- See Lesson 1.2, Examples 1, 3, and 4.
- Try Mid-Chapter Review Questions 3 and 4.

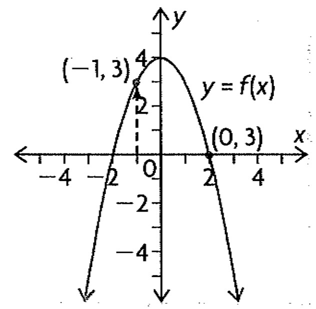
Q: What does function notation mean and why is it useful?

A: When a relation is a function, you can use function notation to write the equation. For example, you can write the equation $y = 4 - x^2$ in function notation as $f(x) = 4 - x^2$. f is a name for the function and $f(a)$ is the value of y or output when the input is $x = a$. The equation $f(-1) = 3$ means "When $x = -1, y = 3$;" in other words, the point $(-1, 3)$ belongs to the function.

To evaluate $f(-1)$, substitute -1 for x in the function equation:

$$f(-1) = 4 - (-1)^2$$
$$= 4 - 1$$
$$= 3$$

Or you can read the value from a graph.



Function notation is useful because writing $f(x) = 3$ gives more information about the function—you know that the independent variable is x—than writing $y = 3$. Also, you can work with more than one function at a time by giving each function a different name. You can choose meaningful names, such as $v(t)$ to describe velocity as a function of time, t, or $C(n)$ to describe the cost of producing n items.

Q: How can you determine the domain and range of a function?

A: The domain of a function is the set of input values for which the function is defined. The range is the set of output values that correspond to the input values. Set notation can be used to describe the domain and range of a function.

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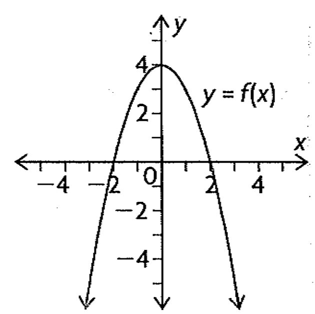
- See Lesson 1.4, Examples 2 and 3.
- Try Mid-Chapter Review Questions 6, 7, and 8.

If you have the graph of a function, you can see the domain and range, as in the following examples:

A



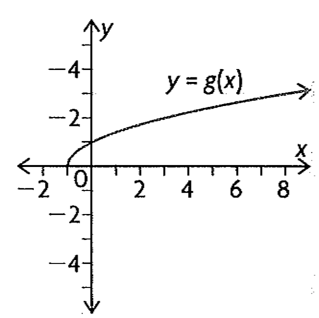
Because graph A goes on forever in both the positive and negative x direction, x can be any real number.

Because this function has a maximum value at the vertex, y cannot have a value greater than this maximum value.

You can express these facts in set notation:

$$\text{Domain} = \{x \in \mathbf{R}\};\ \text{Range} = \{y \in \mathbf{R} \mid y \leq 4\}$$

B



Graph B starts at the point $(-1, 0)$ and continues forever in the positive x direction and positive y direction. So x can be any real number greater than or equal to -1 and y can be any real number greater than or equal to 0.

$$\text{Domain} = \{x \in \mathbf{R} \mid x \geq -1\};\ \text{Range} = \{y \in \mathbf{R} \mid y \geq 0\}$$

You can also determine the domain and range from the equation of a function. For example, if $f(x) = 4 - x^2$, then any value of x will work in this equation, so $x \in \mathbf{R}$. Also, because x^2 is always positive or zero, $f(x)$ is always less than or equal to 4.

$$\text{Domain} = \{x \in \mathbf{R}\};\ \text{Range} = \{y \in \mathbf{R} \mid y \leq 4\}$$

If $g(x) = \sqrt{x + 1}$, then x cannot be less than -1, or the number inside the square root sign would be negative. Also, the square root sign refers to the positive square root, so $g(x)$ is always positive or zero.

$$\text{Domain} = \{x \in \mathbf{R} \mid x \geq -1\};\ \text{Range} = \{y \in \mathbf{R} \mid y \geq 0\}$$

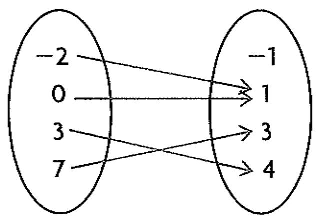
PRACTICE Questions

Lesson 1.1

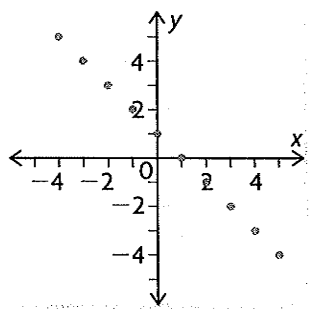
1. Determine which relations are functions. For those which are, explain why.

a) $\{(1, 2), (2, 3), (2, 4), (4, 5)\}$

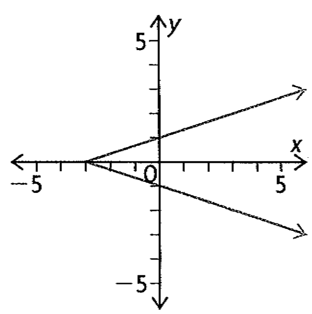
b)



c)



d)



e) $y = -(x - 3)^2 + 5$

f) $y = \sqrt{x - 4}$

2. Use numeric and graphical representations to show that $x^2 + y = 4$ is a function but $x^2 + y^2 = 4$ is not a function.

Lesson 1.2

3. a) Graph the function $f(x) = -2(x + 1)^2 + 3$.

b) Evaluate $f(-3)$.

c) What does $f(-3)$ represent on the graph of f?

d) Use the equation to determine **i)** $f(1) - f(0)$, **ii)** $3f(2) - 5$, and **iii)** $f(2 - x)$.

4. A teacher asked her students to think of a number, multiply it by 5, and subtract the product from 20. Then she asked them to multiply the resulting difference by the number they first thought of.

a) Use function notation to express the final answer in terms of the original number.

b) Determine the outputs for the input numbers 1, -1, and 7.

c) Determine the maximum result possible.

Lesson 1.3

5. Graph each function and state its domain and range.

a) $f(x) = x^2$

b) $f(x) = \frac{1}{x}$

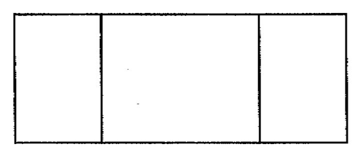
c) $f(x) = \sqrt{x}$

d) $f(x) = |x|$

Lesson 1.4

6. Determine the domain and range of each relation in question 1.

7. A farmer has 600 m of fencing to enclose a rectangular area and divide it into three sections as shown.



a) Write an equation to express the total area enclosed as a function of the width.

b) Determine the domain and range of this area function.

c) Determine the dimensions that give the maximum area.

8. Determine the domain and range for each.

a) A parabola has a vertex at $(-2, 5)$, and $y = 5$ is its maximum value.

b) A parabola has a vertex at $(3, 4)$, and $y = 4$ is its minimum value.

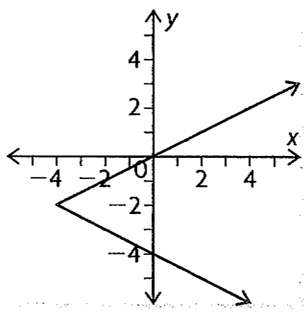
c) A circle has a centre at $(0, 0)$ and a radius of 7.

d) A circle has a centre at $(2, 5)$ and a radius of 4.

PRACTICE Questions

Lesson 1.1

1. For each relation, determine the domain and range and whether the relation is a function. Explain your reasoning.
   a) $\{(-3, 0), (-1, 1), (0, 1), (4, 5), (0, 6)\}$
   b) $y = 4 - x$
   c)



   d) $x^2 + y^2 = 16$

2. What rule can you use to determine, from the graph of a relation, whether the relation is a function? Graph each relation and determine which are functions.
   a) $\{(-2, 1), (1, 1), (0, 0), (1, -1), (1, -2), (2, -2)\}$
   b) $y = 4 - 3x$
   c) $y = (x - 2)^2 + 4$
   d) $x^2 + y^2 = 1$
   e) $y = \frac{1}{x}$
   f) $y = \sqrt{x}$

3. Sketch the graph of a function whose domain is the set of real numbers and whose range is the set of real numbers less than or equal to 3.

Lesson 1.2

4. If $f(x) = x^2 + 3x - 5$ and $g(x) = 2x - 3$, determine each.
   a) $f(-1)$
   b) $f(0)$
   c) $g\left(\frac{1}{2}\right)$
   d) $f(2b)$
   e) $g(1 - 4a)$
   f) x when $f(x) = g(x)$

5. a) Graph the function $f(x) = -2(x - 3)^2 + 4$, and state its domain and range.
   b) What does $f(1)$ represent on the graph? Indicate, on the graph, how you would find $f(1)$.
   c) Use the equation to determine each of the following.
      i) $f(3) - f(2)$
      ii) $2f(5) + 7$
      iii) $f(1 - x)$

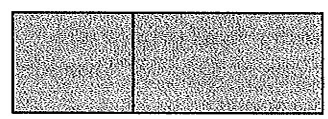
6. If $f(x) = x^2 - 4x + 3$, determine the input(s) for x whose output is $f(x) = 8$.

Lesson 1.4

7. A ball is thrown upward from the roof of a building 60 m tall. The ball reaches a height of 80 m above the ground after 2 s and hits the ground 6 s after being thrown.
   a) Sketch a graph that shows the height of the ball as a function of time.
   b) State the domain and range of the function.
   c) Determine an equation for the function.

8. State the domain and range of each function.
   a) $f(x) = 2(x - 1)^2 + 3$
   b) $f(x) = \sqrt{2x + 4}$

9. A farmer has 540 m of fencing to enclose a rectangular area and divide it into two sections as shown.



   a) Write an equation to express the total area enclosed as a function of the width.
   b) Determine the domain and range of this area function.
   c) Determine the dimensions that give the maximum area.

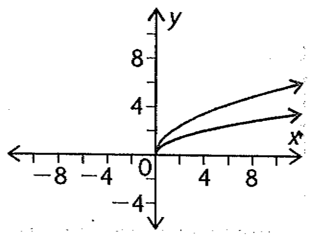
Lesson 1.5

10. Using the functions listed as examples, describe three methods for determining the inverse of a linear function. Use a different method for each function.
   a) $f(x) = 2x - 5$
   b) $f(x) = \frac{x + 3}{7}$
   c) $f(x) = 4 - \frac{1}{2}x$

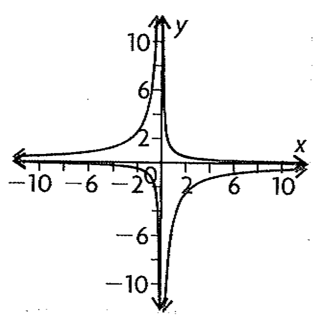
11. For a fundraising event, a local charity organization expects to receive \$15 000 from corporate sponsorship, plus \$30 from each person who attends the event.
  a) Use function notation to express the total income from the event as a function of the number of people who attend.
  b) Suggest a reasonable domain and range for the function in part (a). Explain your reasoning.
  c) The organizers want to know how many tickets they need to sell to reach their fundraising goal. Create a function to express the number of people as a function of expected income. State the domain of this new function.

Lesson 1.7

12. In each graph, a parent function has undergone a transformation of the form $f(kx)$. Determine the equations of the transformed functions graphed in red. Explain your reasoning.
  a)



  b)



13. For each set of functions, transform the graph of $f(x)$ to sketch $g(x)$ and $h(x)$, and state the domain and range of each function.
  a) $f(x) = x^2, g(x) = \left(\frac{1}{2}x\right)^2, h(x) = -(2x)^2$
  b) $f(x) = |x|, g(x) = |-4x|, h(x) = \left|\frac{1}{4}x\right|$

Lesson 1.8

14. Three transformations are applied to $y = x^2$: a vertical stretch by the factor 2, a translation 3 units right, and a translation 4 units down.
  a) Is the order of the transformations important?
  b) Is there any other sequence of these transformations that could produce the same result?

15. The point (1, 4) is on the graph of $y = f(x)$. Determine the coordinates of the image of this point on the graph of $y = 3f[-4(x + 1)] - 2$.

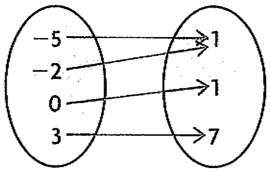
16. a) Explain what you would need to do to the graph of $y = f(x)$ to graph the function $y = -2f[(\frac{1}{3}x + 4)] - 1$.
  b) Graph the function in part (a) for $f(x) = x^2$.

17. In each case, write the equation for the transformed function, sketch its graph, and state its domain and range.
  a) The graph of $f(x) = \sqrt{x}$ is compressed horizontally by the factor $\frac{1}{2}$, reflected in the y-axis, and translated 3 units right and 2 units down.
  b) The graph of $y = \frac{1}{x}$ is stretched vertically by the factor 3, reflected in the x-axis, and translated 4 units left and 1 unit up.

18. If $f(x) = (x - 4)(x + 3)$, determine the x-intercepts of each function.
  a) $y = f(x)$
  b) $y = -2f(x)$
  c) $y = f\left(-\frac{1}{2}x\right)$
  d) $y = f(-(x + 1))$

19. A function $f(x)$ has domain $\{x \in \mathbf{R} \mid x \geq -4\}$ and range $\{y \in \mathbf{R} \mid y < -1\}$. Determine the domain and range of each function.
  a) $y = 2f(x)$
  b) $y = f(-x)$
  c) $y = 3f(x + 1) + 4$
  d) $y = -2f(-x + 5) + 1$

1 Chapter Self-Test



1. For each relation, determine the domain and range and whether the relation is a function. Explain your reasoning.
   a) The function shown at the left.
   b) $y = \sqrt{x + 2}$
2. An incandescent light bulb costs \$0.65 to buy and \$0.004/h for electricity to run. A fluorescent bulb costs \$3.50 to buy and \$0.001/h to run.
   a) Use function notation to write a cost equation for each type of bulb.
   b) State the domain and range of each function.
   c) After how long is the fluorescent bulb cheaper than the regular bulb?
   d) Determine the difference in costs after one year. Assume the light is on for an average of 6 h/day.
3. Determine the domain and range of each function. Show your steps.
   a) $f(x) = \dfrac{1}{x - 2}$
   b) $f(x) = \sqrt{3 - x} - 4$
   c) $f(x) = -|x + 1| + 3$
4. Explain what the term *inverse* means in relation to a linear function. How are the domain and range of a linear function related to the domain and range of its inverse?
5. For each function, determine the inverse, sketch the graphs of the function and its inverse, and state the domain and range of both the function and its inverse.
   a) $\{(-2, 3), (0, 5), (2, 6), (4, 8)\}$
   b) $f(x) = 3 - 4x$
6. At Phoenix Fashions, Rebecca is paid a monthly salary of \$1500, plus 4% commission on her sales over \$2500.
   a) Graph the relation between monthly earnings and sales.
   b) Use function notation to write an equation of the relation.
   c) Graph the inverse relation.
   d) Use function notation to write an equation of the inverse.
   e) Use the equation in part (d) to express Rebecca's sales if she earned \$1740 one month. Then evaluate.
7. The function $y = f(x)$ has been transformed to $y = f(kx)$. Determine the value of k for each transformation.
   a) a horizontal stretch by the factor 5
   b) a horizontal compression by the factor $\frac{1}{3}$ and a reflection in the y-axis
8. The function $y = f(x)$ has been transformed to $y = af[k(x - d)] + c$. Determine a, k, d, and c; write the equation; sketch the graph; and state the domain and range of each transformed function.
   a) vertical compression by the factor $\frac{1}{2}$, reflection in the y-axis, and translation 2 units right, applied to $y = \sqrt{x}$
   b) vertical stretch by the factor 4, reflection in the x-axis, translation 2 units left, and translation 3 units down, applied to $y = \dfrac{1}{x}$
   c) horizontal compression by the factor $\frac{1}{4}$, vertical stretch by the factor $\frac{3}{2}$, reflection in the x-axis, translation 3 units right, and translation 2 units down, applied to $y = |x|$