6 Chapter Review

FREQUENTLY ASKED Questions

Study | *Aid*

- See Lesson 6.4, Example 1.
- Try Chapter Review Questions 8 to 10.

Q: How can you solve a quadratic equation that is not factorable over the set of integers, without graphing?

A: If the quadratic equation is in the form $ax^2 + bx + c = 0$, you can use the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

EXAMPLE

Solve $3x^2 - 7x - 5 = 0$. Round to two decimal places.

Solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a = 3,\ b = -7, \text{ and } c = -5$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{7 \pm \sqrt{49 + 60}}{6}$$

$$x = \frac{7 \pm \sqrt{109}}{6}$$

$$x = \frac{7 + \sqrt{109}}{6} \quad \text{or} \quad x = \frac{7 - \sqrt{109}}{6}$$

$$x \doteq 2.91 \qquad\qquad x \doteq -0.57$$

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- See Lesson 6.5, Examples 1 and 2.
- Try Chapter Review Questions 11 and 12.

Q: How can you use part of the quadratic formula to determine the number of real solutions that a quadratic equation has?

A: You can use the discriminant, $D = b^2 - 4ac$. If $D < 0$, there are no real solutions. If $D = 0$, there is one real solution. If $D > 0$, there are two real solutions.

Study | *Aid*

- See Lesson 6.6, Examples 1 and 2.
- Try Chapter Review Questions 13 to 18.

Q: When using a quadratic model, how do you decide whether you should determine the vertex or solve the corresponding equation?

A: If you want to determine a maximum or minimum value, then you should locate the vertex of the relation. If you are given a specific value of y (any number, including 0), then you should solve the corresponding equation.

PRACTICE Questions

Lesson 6.1

1. Solve each equation.
   a) $(2x - 5)(3x + 8) = 0$
   b) $x^2 + 12x + 32 = 0$
   c) $3x^2 - 10x - 8 = 0$
   d) $3x^2 - 5x + 5 = 2x^2 + 4x - 3$
   e) $2x^2 + 5x - 1 = 0$
   f) $5x(x - 1) + 5 = 7 + x(1 - 2x)$

2. The safe stopping distance, in metres, for a boat that is travelling at v kilometres per hour in calm water can be modelled by the relation $d = 0.002(2v^2 + 10v + 3000)$.
   a) What is the safe stopping distance if the boat is travelling at 12 km/h?
   b) What is the initial speed of the boat if it takes 15 m to stop?

Lesson 6.2

3. Determine the value of c needed to create a perfect-square trinomial.
   a) $x^2 + 8x + c$
   b) $x^2 - 16x + c$
   c) $x^2 + 19x + c$
   d) $2x^2 + 12x + c$
   e) $-3x^2 + 15x + c$
   f) $0.1x^2 - 7x + c$

Lesson 6.3

4. Complete the square to write each quadratic relation in vertex form.
   a) $y = x^2 + 8x - 2$
   b) $y = x^2 - 20x + 95$
   c) $y = -3x^2 + 12x - 2$
   d) $y = 0.2x^2 - 0.4x + 1$
   e) $y = 2x^2 + 10x - 12$
   f) $y = -4.9x^2 - 19.6x + 12$

5. Consider the relation $y = -3x^2 - 12x - 2$.
   a) Write the relation in vertex form by completing the square.
   b) State the transformations that must be applied to $y = x^2$ to draw the graph of the relation.
   c) Graph the relation.

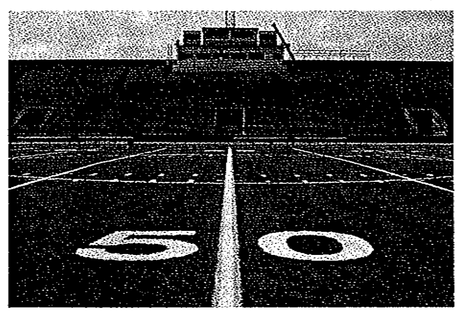
6. A basketball player makes a long pass to another player. The path of the ball can be modelled by $y = -0.2x^2 + 2.4x + 2$, where x is the horizontal distance from the player and y is the height of the ball above the court, both in metres. Determine the maximum height of the ball.

7. Cam has 46 m of fencing to enclose a meditation space on the grounds of his local hospital. He has decided that the meditation space should be rectangular, with fencing on only three sides. What dimensions will give the patients the maximum amount of meditation space?

Lesson 6.4

8. Solve each equation.
   a) $3x^2 - 4x - 10 = 0$
   b) $-4x^2 + 1 = -15$
   c) $x^2 = 6x + 10$
   d) $(x - 3)^2 - 4 = 0$
   e) $(2x + 5)(3x - 2) = (x + 1)$
   f) $1.5x^2 - 6.1x + 1.1 = 0$

9. The height, h, in metres, of a water balloon that is launched across a football stadium can be modelled by $h = -0.1x^2 + 2.4x + 8.1$, where x is the horizontal distance from the launching position, in metres. How far has the balloon travelled when it is 10 m above the ground?



10. A chain is hanging between two posts so that its height above the ground, h, in centimetres, can be determined by $h = 0.0025x^2 - 0.9x + 120$, where x is the horizontal distance from one post, in centimetres. How far from the post is the chain when it is 50 cm from the ground?

Lesson 6.5

11. Without solving, determine the number of solutions that each equation has.

a) $2x^2 - 5x + 1 = 0$
b) $-3.5x^2 - 2.1x - 1 = 0$
c) $x^2 + 5x + 8 = 0$
d) $4x^2 - 15 = 0$
e) $5(x^2 + 2x + 5) = -2(2x - 25)$

12. Without graphing, determine the number of x-intercepts that each relation has.

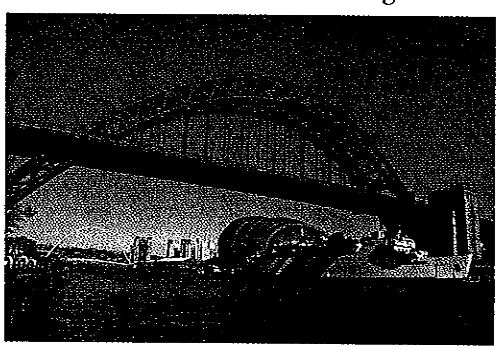
a) $y = (x - 4)(2x + 9)$
b) $y = -1.8(x - 3)^2 + 2$
c) $y = 2x^2 + 8x + 14$
d) $y = 2x(x - 5) + 7$
e) $y = -1.4x^2 - 4x - 5.4$

Lesson 6.6

13. Skydivers jump out of an airplane at an altitude of 3.5 km. The equation $H = 3500 - 5t^2$ models the altitude, H, in metres, of the skydivers at t seconds after jumping out of the airplane.

a) How far have the skydivers fallen after 10 s?
b) The skydivers open their parachutes at an altitude of 1000 m. How long did they free fall?

14. The arch of the Tyne bridge in England is modelled by $h = -0.008x^2 - 1.296x + 107.5$, where h is the height of the arch above the riverbank and x is the horizontal distance from the riverbank, both in metres. Determine the height of the arch.



15. Tickets to a school dance cost \$5, and the projected attendance is 300 people. For every \$0.50 increase in the ticket price, the dance committee projects that attendance will decrease by 20. What ticket price will generate \$1562.50 in revenue?

16. A room has dimensions of 5 m by 8 m. A rug covers $\frac{3}{4}$ of the floor and leaves a uniform strip of the floor exposed. How wide is the strip?

17. Two integers differ by 12 and the sum of their squares is 1040. Determine the integers.

18. The student council at City High School is thinking about selling T-shirts. To help them decide what to do, they conducted a school-wide survey. Students were asked, "Would you buy a school T-shirt at this price?" The results of the survey are shown.

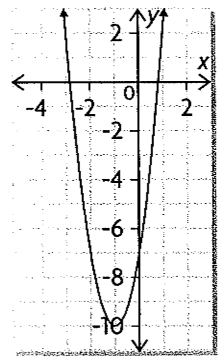
T-Shirt Price, t (\$)	Students Who Would Buy, N	Revenue, R (\$)
4.00	923	
6.00	752	
8.00	608	
10.00	455	
12.00	287	

a) Use the table to determine the revenue for each possible price.
b) Draw a scatter plot relating the revenue, R, to the T-shirt price, t. Sketch a curve of good fit.
c) Verify that the number of students, N, who would buy a T-shirt for t dollars can be approximated by the relation $N = 1230 - 78t$.
d) Use the equation in part c) to create an algebraic expression for the revenue.
e) The student council needs to bring in revenue of at least \$4750. What price range can they consider?

6 Chapter Self-Test

Round all answers to two decimal places where necessary.

1. Use the graph of $y = 3x^2 + 6x - 7$ at the right to estimate the solutions to each equation.
   a) $3x^2 + 6x - 7 = 0$
   b) $3x^2 + 6x - 7 = -7$
   c) $3x^2 + 6x - 9 = 0$



$y = 3x^2 + 6x - 7$

2. Determine the roots of each equation.
   a) $x^2 + 5x - 14 = 0$
   b) $5x^2 - 9x + 1 = 0$
   c) $2x^2 - 8 = 24$
   d) $2(x - 1)^2 - 5 = 0$

3. Complete the square to determine the vertex of each parabola.
   a) $y = 2x^2 + 12x - 14$
   b) $y = 3x^2 - 15x - 24$

4. Can all quadratic relations be written in vertex form by completing the square? Justify your answer.

5. Without solving, determine the number of real roots that each relation has. Justify your answers.
   a) $y = 2x^2 - 4x + 7$
   b) $y = 3(x - 4)(x - 4)$
   c) $y = (x - 3)^2$

6. April sells specialty teddy bears at various summer festivals. Her profit for a week, P, in dollars, can be modelled by $P = -0.1n^2 + 30n - 1200$, where n is the number of teddy bears she sells during the week.
   a) According to this model, could April ever earn a profit of \$2000 in one week? Explain.
   b) How many teddy bears would she have to sell to break even?
   c) How many teddy bears would she have to sell to earn \$500?
   d) How many teddy bears would she have to sell to maximize her profit?

7. Serge and Francine have 24 m of fencing to enclose a vegetable garden at the back of their house. Determine the dimensions of the largest rectangular garden they could enclose, using the back of their house as one of the sides of the rectangle.

8. Give two reasons why $3x^2 + 6x + 6$ cannot be a perfect square.

9. A rapid-transit company has 5000 passengers daily, each currently paying a \$2.25 fare. For each \$0.50 increase, the company estimates that it will lose 150 passengers daily. If the company must be paid at least \$15 275 each day to stay in business, what minimum fare must they charge to produce this amount of revenue?

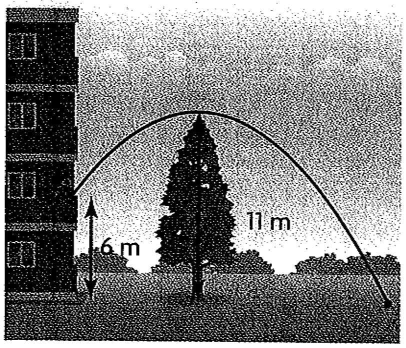
Process | *Checklist*

✔ Question 1: Did you compare the algebraic and graphical representations to help you estimate?

✔ Questions 4 and 5: Did you communicate using correct mathematical vocabulary as you justified your answers?

✔ Question 6: Did you make connections between the quadratic equation and the situation that it is modelling?

Up and Over



On Earth, the quadratic relation $h = -5t^2 + ut + h_0$ can be used to determine the height of an object that has been thrown as it travels through the air, measured from a reference point. In this relation, h is the height of the object in metres, t is the time in seconds since the object was thrown, u is the initial velocity, and h_0 is the initial height.

Myrtle throws a ball upward from a second-floor balcony, 6 m above the ground, with an initial velocity of 2 m/s. In this situation, $u = 2$ and $h_0 = 6$, so the relation that models the height of the ball is $h = -5t^2 + 2t + 6$. Myrtle knows that changing the velocity with which she throws the ball will change the maximum height of the ball. Myrtle wants to know with what velocity she must throw the ball to make it pass over a tree that is 11 m tall.

? What initial velocity will result in a maximum height of 11 m?

A. Suppose that Myrtle just dropped the ball from the balcony, with an initial velocity of 0 m/s. Write a quadratic relation to model this situation.

B. What is the maximum height of the ball in part A?

C. Complete the square of $h = -5t^2 + 2t + 6$ to determine the maximum height of the ball when Myrtle throws the ball with an initial velocity of 2 m/s.

D. Will Myrtle have to increase or decrease the initial velocity with which she throws the ball for it to clear the tree? Explain how you know.

E. Create relations to model the height of the ball when it is thrown from a second-floor balcony with initial velocities of 4 m/s and 6 m/s. Then determine the maximum height of the ball for each relation.

F. Create a scatter plot to show the maximum heights for initial velocities of 0 m/s, 2 m/s, 4 m/s, and 6 m/s. Is this relation quadratic? Explain how you know.

G. Use quadratic regression to determine an algebraic model for your graph for part F.

H. Use the model you created for part G to determine the initial velocity necessary for the ball to clear the tree.

Task | ***Checklist***

- ✔ Did you show all your calculations?
- ✔ Did you draw and label your graph accurately?
- ✔ Did you answer all the questions reflectively, using complete sentences?
- ✔ Did you explain your thinking clearly?