Guided Practice 2A.1.2

Example 1

Simplify $(2x^2 + x + 10) + (7x^2 + 14)$.

1. Rewrite the sum so that any like terms are together.

   The first polynomial, $2x^2 + x + 10$, has a term with a power of 2, a term with a power of 1, and a constant term. The second polynomial, $7x^2 + 14$, has a term with a power of 2 and a constant term. The terms with the same powers and variables are like terms, as are the constants.

   $(2x^2 + x + 10) + (7x^2 + 14)$

   $= 2x^2 + 7x^2 + x + 10 + 14$

2. Find the sum of any constants.

   The previous expression contains two constants: 10 and 14.

   $2x^2 + 7x^2 + x + 10 + 14$

   $= 2x^2 + 7x^2 + x + 24$

3. Find the sum of any terms with the same variable raised to the same power by adding the coefficients of the terms.

   $2x^2 + 7x^2 + x + 24$

   $= (2 + 7)x^2 + x + 24$

   $= 9x^2 + x + 24$

   $(2x^2 + x + 10) + (7x^2 + 14)$ is equivalent to $9x^2 + x + 24$.

Example 2

Simplify $(6x^4 - x^3 - 3x^2 + 20) + (10x^3 - 4x^2 + 9)$.

1. Rewrite any subtraction using addition.

   Subtraction can be rewritten as adding a negative.

   $(6x^4 - x^3 - 3x^2 + 20) + (10x^3 - 4x^2 + 9)$

   $= [6x^4 + (-x^3) + (-3x^2) + 20] + [10x^3 + (-4x^2) + 9]$

2. Rewrite the sum so that any like terms are together.

   Be sure to keep any negatives with the terms.

   $[6x^4 + (-x^3) + (-3x^2) + 20] + [10x^3 + (-4x^2) + 9]$

   $= 6x^4 + (-x^3) + 10x^3 + (-3x^2) + (-4x^2) + 20 + 9$

3. Find the sum of any constants.

   The previous expression contains two constants: 20 and 9.

   $6x^4 + (-x^3) + 10x^3 + (-3x^2) + (-4x^2) + 20 + 9$

   $= 6x^4 + (-x^3) + 10x^3 + (-3x^2) + (-4x^2) + 29$

4. Find the sum of any terms with the same variable raised to the same power.

   The previous expression contains the following like terms: $(-x^3)$ and $10x^3$; $(-3x^2)$ and $(-4x^2)$.

   Add the coefficients of any like terms, being sure to keep any negatives with the coefficients.

   $6x^4 + (-x^3) + 10x^3 + (-3x^2) + (-4x^2) + 29$

   $= 6x^4 + (-1 + 10)x^3 + [-3 + (-4)]x^2 + 29$

   $= 6x^4 + 9x^3 - 7x^2 + 29$

   $(6x^4 - x^3 - 3x^2 + 20) + (10x^3 - 4x^2 + 9)$ is equivalent to $6x^4 + 9x^3 - 7x^2 + 29$.

Example 3

Simplify $(-x^6 + 7x^2 + 11) - (12x^6 + 4x^5 - 2x + 1)$.

1. Rewrite the difference as a sum.

   A difference can be written as the sum of a negative quantity.

   Distribute the negative in the second polynomial.

   $(-x^6 + 7x^2 + 11) - (12x^6 + 4x^5 - 2x + 1)$

   $= (-x^6 + 7x^2 + 11) + [-(12x^6 + 4x^5 - 2x + 1)]$

   $= (-x^6 + 7x^2 + 11) + (-12x^6) + (-4x^5) + 2x + (-1)$

2. Rewrite the sum so that any like terms are together.

   Be sure to keep any negatives with the coefficients.

   $(-x^6 + 7x^2 + 11) + (-12x^6) + (-4x^5) + 2x + (-1)$

   $= -x^6 + (-12x^6) + (-4x^5) + 7x^2 + 2x + 11 + (-1)$

3. Find the sum of any constants.

   The previous expression contains two constants: 11 and (–1).

   $-x^6 + (-12x^6) + (-4x^5) + 7x^2 + 2x + 11 + (-1)$

   $= -x^6 + (-12x^6) + (-4x^5) + 7x^2 + 2x + 10$

4. Find the sum of any terms with the same variable raised to the same power.

   The previous expression contains the following like terms: $-x^6$ and $(-12x^6)$.

   $-x^6 + (-12x^6) + (-4x^5) + 7x^2 + 2x + 10$

   $= [(-1) + (-12)]x^6 + (-4x^5) + 7x^2 + 2x + 10$

   $= -13x^6 + (-4x^5) + 7x^2 + 2x + 10$

   $= -13x^6 - 4x^5 + 7x^2 + 2x + 10$

   $(-x^6 + 7x^2 + 11) - (12x^6 + 4x^5 - 2x + 1)$ is equivalent to $-13x^6 - 4x^5 + 7x^2 + 2x + 10$.

Practice 2A.1.2: Adding and Subtracting Polynomials

Simplify each expression.

1. $(-x^6 + 10x^3 + 5x + 6) + (12x^3 - 8x + 7)$

2. $(14y^3 + 8y^2 - 8y - 19) + (18y^2 + 5y - 14)$

3. $(20x^4 + x^3 + 18) - (3x^4 + 14x^3 + 11x^2 + 2)$

4. $(11x^5 - 4x^4 + 19x) - (15x^4 + 13x^3 - 6x + 10)$

5. $(-10z^4 + 2z^3 + 14z^2 + 15) + (5z^3 - 17z^2 - 13z + 8)$

6. $(-4x^6 + 3x^4 + 20) - (-17x^6 + 12x^5 - 6x^4)$

7. $(12h^2 - 9h - 15) - (3h^3 + 7h^2 + 8h + 10)$

The perimeter of a rectangle is the sum of its sides. Find the perimeter of a rectangle with each given length and width. All measurements are given in centimeters.

8. length: $x - 5$; width: $x + 10$

9. length: $x^2 + 1$; width: $4x$

10. length: $-8x + 24$; width: $2x + 3$

Guided Practice 2A.1.3

Example 1

Simplify the expression $(x^2 + 3)(x + 6)$.

1. Rewrite the product using the Distributive Property.

   Multiply each term in the first polynomial by each term in the second polynomial.

   $(x^2 + 3)(x + 6) = (x^2 \bullet x) + (3 \bullet x) + (x^2 \bullet 6) + (3 \bullet 6)$

2. Use properties of exponents to simplify the expression.

   x is x raised to the first power, or x^1.

$(x^2 \bullet x) + (3 \bullet x) + (x^2 \bullet 6) + (3 \bullet 6)$	Distributed expression
$= (x^2 \bullet x^1) + (3 \bullet x) + (x^2 \bullet 6) + (3 \bullet 6)$	Substitute x^1 for x when multiplying terms that both have variables.
$= (x^{2+1}) + (3 \bullet x) + (x^2 \bullet 6) + (3 \bullet 6)$	Since $x^n \bullet x^m = x^{n+m}$, add the exponents.
$= x^3 + (3 \bullet x) + (x^2 \bullet 6) + (3 \bullet 6)$	Simplify.

3. Simplify any remaining products.

   The product of a number and a variable is written with the number first, as the coefficient of the variable.

$x^3 + (3 \bullet x) + (x^2 \bullet 6) + (3 \bullet 6)$	Expression from the previous step
$= x^3 + 3x + 6x^2 + 18$	Simplify.
$= x^3 + 6x^2 + 3x + 18$	Rewrite in descending order.

The expression $(x^2 + 3)(x + 6)$ is equivalent to $x^3 + 6x^2 + 3x + 18$.

Example 2

Simplify the expression $(-5x + 2)(3x^2 - x + 4)$.

1. Rewrite the product using the Distributive Property.

   Multiply each term in the first polynomial by each term in the second polynomial.

   $(-5x + 2)(3x^2 - x + 4)$

   $= (-5x \bullet 3x^2) + (-5x \bullet -x) + (-5x \bullet 4) + (2 \bullet 3x^2) + (2 \bullet -x) + (2 \bullet 4)$

2. Use properties of exponents and multiplication to simplify the expression.

   Start with the expression from the previous step.

   $(-5x \bullet 3x^2) + (-5x \bullet -x) + (-5x \bullet 4) + (2 \bullet 3x^2) + (2 \bullet -x) + (2 \bullet 4)$

   Substitute x^1 for x when multiplying terms that both have variables.

   $= (-5x^1 \bullet 3x^2) + (-5x^1 \bullet -x^1) + (-5x \bullet 4) + (2 \bullet 3x^2) + (2 \bullet -x) + (2 \bullet 4)$

   Since $x^n \bullet x^m = x^{n+m}$, add the exponents. Remember that the product of a number and a variable is written with the number first, as the coefficient of the variable.

   $= [(-5)(3)x^{1+2}] + [(-5)(-1)x^{1+1}] + (-5x \bullet 4) + (2 \bullet 3x^2) + (2 \bullet -x) + (2 \bullet 4)$

   Simplify.

   $= [(-5)(3)x^3] + [(-5)(-1)x^2] + (-5x \bullet 4) + (2 \bullet 3x^2) + (2 \bullet -x) + (2 \bullet 4)$

   $= -15x^3 + 5x^2 + (-20x) + 6x^2 + (-2x) + 8$

3. Combine like terms.

   Only terms with the same variable raised to the same power can be combined. Write the polynomial in descending order according to the power of each term, from highest power to lowest.

$-15x^3 + 5x^2 + (-20x) + 6x^2 + (-2x) + 8$	Expression from the previous step
$= -15x^3 + 5x^2 + 6x^2 + (-20x) + (-2x) + 8$	Reorder like terms in descending order.
$= -15x^3 + 11x^2 + (-22x) + 8$	Combine like terms.
$= -15x^3 + 11x^2 - 22x + 8$	Simplify.

   The expression $(-5x + 2)(3x^2 - x + 4)$ is equivalent to $-15x^3 + 11x^2 - 22x + 8$.

Example 3

Simplify the expression $(3x^4 + 10x^2 - 4x)(x^3 - 8x^2 + x)$.

1. Rewrite the product using the Distributive Property.

   Multiply each term in the first polynomial by each term in the second polynomial.

   $(3x^4 + 10x^2 - 4x)(x^3 - 8x^2 + x)$

   $= (3x^4 \bullet x^3) + (3x^4 \bullet -8x^2) + (3x^4 \bullet x) + (10x^2 \bullet x^3) + (10x^2 \bullet -8x^2) +$
   $(10x^2 \bullet x) + (-4x \bullet x^3) + (-4x \bullet -8x^2) + (-4x \bullet x)$

Practice 2A.1.3: Multiplying Polynomials

Simplify each expression.

1. $(11x + 3)(-x^2 + 7)$

2. $(6x^3 + 5x^2 - 1)(2x^3 + 4)$

3. $(-y^2 + 10)(8y^3 + 2y)$

4. $(10x^5 + 4x^2)(2x^2 - 6x + 3)$

5. $(-3x^4 + 5x^3 - x^2)(x^3 - 6)$

6. $(7y^6 - 9y^4 + 2)(4y^3 + y^2 - 1)$

7. $(5x^2 + 4x + 3)(7x^3 - 5x - 2)$

The area of a rectangle is found using the formula *length* • *width*. Find the area of a rectangle with the given length and width. All measurements are given in meters.

8. length: $x + 8$; width: $3x - 2$

9. length: $x^2 + 1$; width: $4x + 10$

10. length: $6x + 5$; width: $2x^2 - 3$