Key Concepts

- A function is a set of ordered pairs in which, for every x, there is only one y.
- If any vertical line passes through more than one point on the graph of a relation, then the relation is not a function.
- The set of the first elements in a relation is called the domain. The set of the second elements in a relation is called the range.
- An equation that is a function can be named using function notation.
- In function notation, the symbol $f(x)$ is another name for y and represents the value of the function f at x.

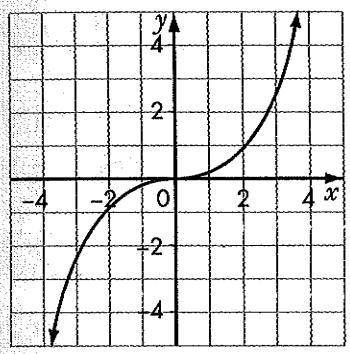
Communicate Your Understanding

1. a) Is every function a relation? Explain using examples.
b) Is every relation a function? Explain using examples.

2. Describe how you would determine if each of the following relations is a function.
a) $\{(0, 2), (1, 3), (2, 4), (1, 5), (0, 6)\}$
b) $\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

3. Describe how you would determine if each graph models a function.

a)



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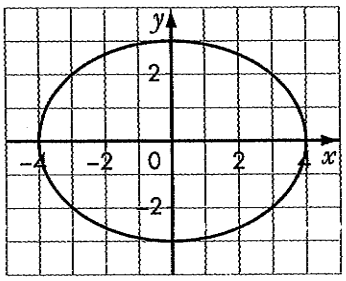


4. Describe how you would determine the domain and range of each of the following relations.
a) $\{(2, -3), (3, -1), (4, 1), (5, 3)\}$ b) $y = 3x + 2$

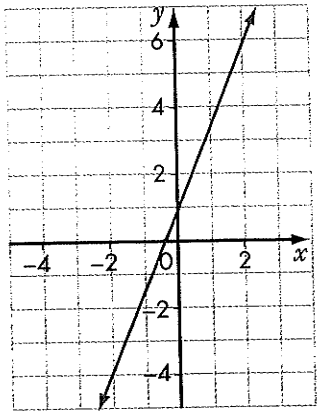
5. Describe how you would evaluate $f(4)$ for the function $f(x) = 2x - 3$.

Practise

A

1. Determine if each relation is a function.

a)



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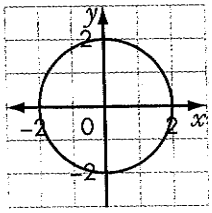

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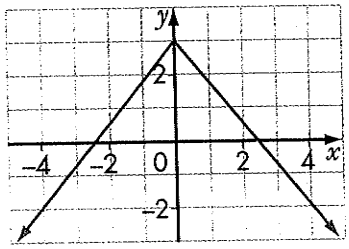

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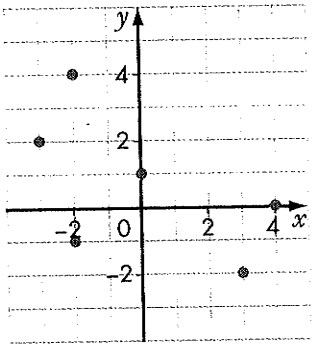


2. State the domain and range of each relation.

a) {(0, 5), (1, 6), (2, 7), (3, 8)}

b) {(1, 4), (2, 3), (2, 5)}

c) {(−2, −1), (−1, 0), (0, −1), (1, 2), (2, −1)}

d) {(−2, 1), (0, 1), (3, 1), (4, 1), (7, 1)}

e)

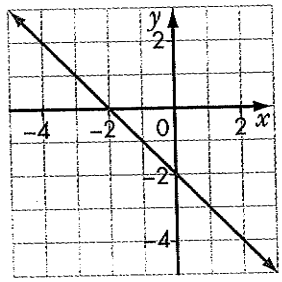
x	y
−2	5
−1	3
0	−1
1	−4
2	−5

f)

x	y
0	−1
0	0
0	2
0	4

3. State the domain and range of each relation.

a)



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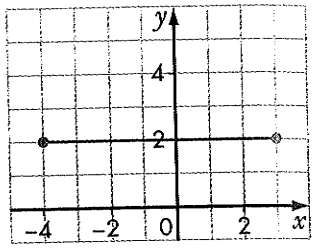

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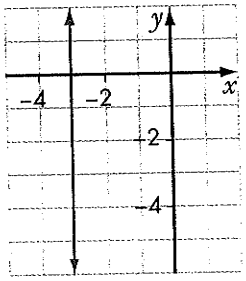

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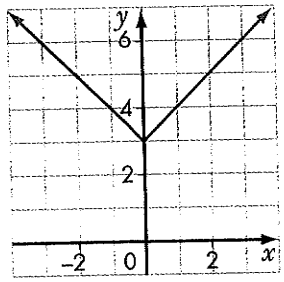

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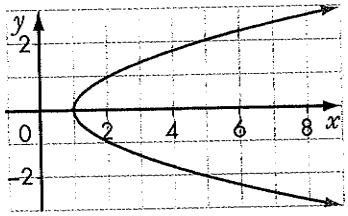

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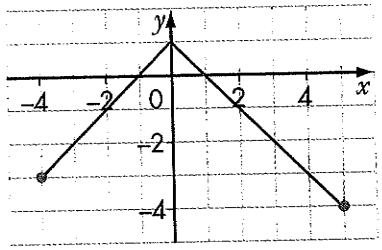
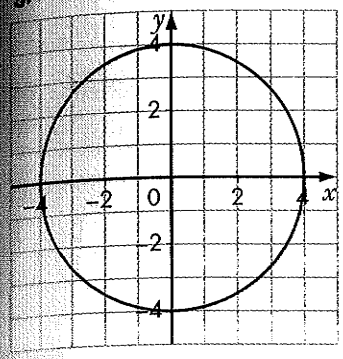
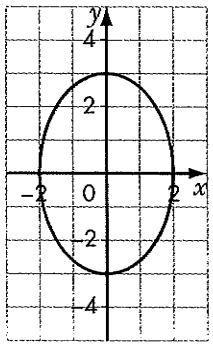


g)



h)



4. a) Graph the equation $y = x^2 - 3$.
b) Is the relation a function?

5. Determine if each relation is a function.
a) $y = 2 - 4x$
b) $y = 2x^2 + 3x - 5$
c) $x^2 + y^2 = 25$

6. If $f(x) = x - 5$, find
a) $f(8)$ b) $f(5)$ c) $f(1)$
d) $f(0)$ e) $f(-2)$

7. If $g(x) = 3x + 4$, find
a) $g(2)$ b) $g(0)$ c) $g(-1)$
d) $g(-3)$ e) $g(0.5)$

8. If $f(x) = x^2 + 2x - 1$, find
a) $f(0)$ b) $f(5)$ c) $f(-2)$
d) $f(1.5)$ e) $f(-0.5)$

9. If $h(x) = 2x^2 - 3x + 6$, find
a) $h(1)$ b) $h(10)$ c) $h(-3)$
d) $h(0.5)$ e) $h(5)$

10. If $f(n) = -4n^2 + 5$, find
a) $f(0)$ b) $f(1)$ c) $f(4)$
d) $f(-3)$ e) $f(-1.5)$

Apply, Solve, Communicate

11. List the ordered pairs of the function $f(x) = 7x - 1$ when the domain is $\{-1, 0, 2, 5\}$.

12. If $f(x) = 4x + 1$, find the value of x when the value of $f(x)$ is
a) 21 b) -7 c) 53 d) -19 e) 11

13. If the domain and range of a relation each contain exactly one real number, describe the graph of the relation.

14. Cost The cost, C dollars, of purchasing one type of ballpoint pen is related to the number of pens purchased, p, by the equation $C = 1.25p$.
a) Identify the dependent variable and the independent variable.
b) Is the cost a function of the number of pens purchased? Explain.

B

15. Determine if each of the following relations is a function. If so, state the dependent variable and the independent variable.
a) the time it takes to drive 100 km and the speed of the car
b) the ages of students and the numbers of CDs they own
c) the number of tickets sold for a school play at \$8 per ticket and the revenue from ticket sales

16. If a point is on the $f(x)$-axis of a coordinate grid, what is the x-coordinate of the point? Explain.

17. If the graph of a relation is a vertical line, is the relation a function? Explain.

18. Find the range of each function when the domain is $\{-2, 0, 0.5, 3\}$.

a) $f(x) = 2x - 1$ **b)** $f(x) = 8x^2 - 4x + 7$

19. Application Mario sells home theatre systems. He is paid a weekly salary, plus commission on his sales. His weekly earnings, $E(s)$ dollars, can be determined from his weekly sales, s dollars, using the following functions.
When his weekly sales are \$3000 or less: $E(s) = 0.05s + 400$
When his weekly sales are over \$3000: $E(s) = 0.06s + 400$

a) Interpret each function in words.

b) Identify the dependent variable and the independent variable in each equation.

c) State the domain and range of each function.

d) What are Mario's weekly earnings when his weekly sales are \$2000? \$4500?

20. Measurement The area of a circle, $A(r)$, is a function of the radius, r, where $A(r) = \pi r^2$. State the domain and range of the function.

21. a) Determine the domain and range of the relation $x^2 + y^2 = 1$.

b) Is the relation a function?

22. Discount prices Neeru is holding a year-end clearance sale in her clothing store. All prices are discounted by 25%.

a) Write an equation that expresses the sale price of an item as a function of its original price.

b) If the sale price of a shirt is \$42, what was its original price?

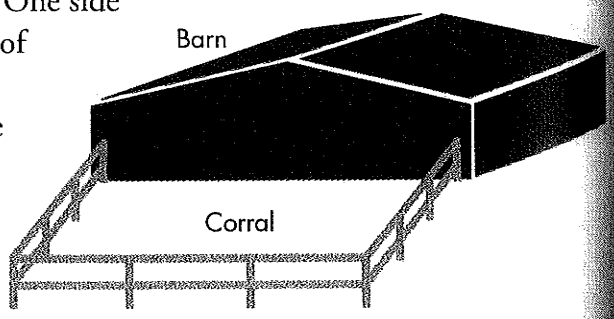
23. Measurement a) Write an equation that expresses the surface area of a cube as a function of its edge length.

b) Determine the surface area of a cube with an edge length of 2.5 cm.

24. Inquiry/Problem Solving Natalia wants to build a rectangular corral with the largest possible area for her horses. One side of the corral will be the barn wall. She has 100 m of fencing to build the other three sides.

a) Write an equation that expresses the area of the corral as a function of the width of the corral.

b) Find the maximum area of the corral and the dimensions that give this area.



25. Communication Does the graph of $2x - 3y = 6$ model a function? Explain.

26. Algebra If $f(x) = x^2 + 4x$, find the value(s) of x if the value of $f(x)$ is
a) 5 **b)** -4 **c)** 0 **d)** -3

C

27. a) Sketch a graph of a function that has all real numbers in its domain and all real numbers less than or equal to -3 in its range.
b) Sketch a graph of a relation that is not a function and that has all real numbers in its domain and all real numbers less than or equal to -3 in its range.

28. Jogging Chelsea jogs several laps around a running track at a steady speed.
a) Is the distance she covers a function of the time for which she jogs? Explain.
b) Is her distance from her starting point a function of the time for which she jogs? Explain.

29. Describe how the value of $\frac{f(4) - f(1)}{4 - 1}$ is related to the graph of $f(x) = 6x - 5$.

30. Algebra Solve the following system algebraically.
$f(x) = 2x - 3$
$f(x) = 3x + 2$

31. Measurement Express the area of a circle as a function of its circumference.

32. Algebra a) If $f(x) = 4x + 3$, write and simplify $f(2a)$.
b) If $f(x) = 2 - 3x$, write and simplify $f(n + 1)$.
c) If $g(x) = x^2 + 1$, write and simplify $g(m - 1)$.
d) If $f(x) = 2x^2 - 3$, write and simplify $f(2k + 1)$.
e) If $g(x) = x^2 + 4x - 1$, write and simplify $g(3t - 1)$.
f) If $f(x) = 3x^2 - 2x + 4$, write and simplify $f(3 - 2w)$.

Chapter 3

Getting Started: Human Physiology, p. 168

1. b) the first quadrant, since $t \geq 0$ **2. b)** The y-coordinates of $y = 30x$ are 6 times those of $y = 5x$. **3. b)** time, in terms of the number of litres of blood pumped **4. b)** equal **5. a)** $y = 12x$ **6.** $y = \frac{x}{12}$; the number of breaths divided by 12 produces the number of minutes.

Review of Prerequisite Skills, p. 169

1. a) (−3, 2) **b)** (−3, −8) **c)** (−6, −2) **d)** (2, −2) **e)** (1, −4) **f)** (−12, 6) **g)** (−13, −9) **h)** (8, 10) **2. a)** 12 units to the left **b)** 8 units upward **c)** 8 units to the left and 1 unit downward **d)** 6 units to the left and 5 units downward **e)** 11 units to the right and 21 units downward **f)** 7 units to the right and 19 units downward **3. a)** A′(−3, 0), B′(−6, −4), C′(0, −6) **b)** A′(4, 5), B′(1, 1), C′(7, −1) **c)** A′(6, −1), B′(3, −5), C′(9, −7) **4. a)** A′(−2, −4), B′(−4, −1), C′(5, 2) **b)** A′(2, 4), B′(4, 1), C′(−5, −2) **c)** A′(2, −4), B′(4, −1), C′(−5, 2) **5. a)** vertical stretch by a factor of 3 **b)** vertical compression by a factor of $\frac{1}{3}$ **6.** reflection in the x-axis **7. a)** 3 units upward **b)** 7 units downward **c)** 4 units to the right **d)** 6 units to the left **e)** 3 units to the left and 8 units downward **f)** 7 units to the right and 2 units upward **8. a)** opens up; vertex: (0, −4); axis of symmetry: $x = 0$; domain: all real numbers; range: $y \geq -4$; minimum value: −4 **b)** opens down; vertex: (0, 5); axis of symmetry: $x = 0$; domain: all real numbers; range: $y \leq 5$; maximum value: 5 **c)** opens up; vertex: (2, 3); axis of symmetry: $x = 2$; domain: all real numbers; range $y \geq 3$; minimum value: 3 **d)** opens down; vertex: (−3, −5); axis of symmetry: $x = -3$; domain: all real numbers; range: $y \leq -5$; maximum value: −5 **9. c)** equal **f)** congruent parabolas at differing locations **g)** For the function $y = x$, the graph is the collection of all ordered pairs of the form (x, x); the translation of the graph upward by 1 unit produces the collection $(x, x + 1)$, which is equivalent to the collection $(x - 1, x)$, produced by the translation of 1 unit to the left.

Section 3.1, pp. 178–181

1. a) a function **b)** not a function **c)** a function **d)** not a function **2. a)** domain: {0, 1, 2, 3}, range: {5, 6, 7, 8} **b)** domain: {1, 2}, range: {3, 4, 5} **c)** domain: {−2, −1, 0, 1, 2}, range: {−1, 0, 2} **d)** domain: {−2, 0, 3, 4, 7}, range: {1} **e)** domain: {−2, −1, 0, 1, 2}, range: {−5, −4, −1, 3, 5} **f)** domain: {0}, range: {−1, 0, 2, 4} **3. a)** domain: the set of real numbers, range: the set of real numbers **b)** domain: $-4 \leq x \leq 3$, range: {2} **c)** domain: {−3}, range: the set of real numbers **d)** domain: the set of real numbers, range: $y \geq 3$ **e)** domain: $x \geq 1$, range: the set of real numbers **f)** domain: $-4 \leq x \leq 5$, range: $-4 \leq y \leq 1$ **g)** domain: $-4 \leq x \leq 4$, range: $-4 \leq y \leq 4$ **h)** domain: $-2 \leq x \leq 2$, range: $-3 \leq y \leq 3$ **4. b)** yes **5. a)** a function **b)** a function **c)** not a function **6. a)** 3 **b)** 0 **c)** −4 **d)** −5 **e)** −7 **7. a)** 10 **b)** 4 **c)** 1 **d)** −5 **e)** 5.5 **8. a)** −1 **b)** 34 **c)** −1 **d)** 4.25 **e)** −1.75 **9. a)** 5 **b)** 176 **c)** 33 **d)** 5 **e)** 41 **10. a)** 5 **b)** 1 **c)** −59 **d)** −31 **e)** −4 **11.** (−1, −8), (0, −1), (2, 13), (5, 34) **12. a)** 5 **b)** −2 **c)** 13 **d)** −5 **e)** 2.5 **13.** a point **14. a)** dependent variable: C, independent variable: p **b)** Yes; there is only one cost for each number of pens purchased **15. a)** a function; dependent variable: time, independent variable: speed **b)** not a function **c)** a function; dependent variable: revenue, independent variable: number of tickets sold **16.** 0 **17.** No; the vertical line test fails. **18. a)** {−5, −1, 0, 5} **b)** {7, 47, 67} **19. a)** salary of \$400/week plus 5% commission; salary of \$400/week plus 6% commission **b)** dependent variable: E, independent variable: s **c)** domain: $s \geq 0$, range: $E \geq 400$ **d)** \$500, \$670 **20.** domain: $r \geq 0$, range: $A \geq 0$ **21. a)** domain: $-1 \leq x \leq 1$, range: $-1 \leq y \leq 1$ **b)** no **22. a)** $S(p) = 0.75p$ **b)** \$56 **23. a)** $S(x) = 6x^2$ **b)** 37.5 cm^2 **24. a)** $A(x) = x(100 - 2x)$ **b)** 1250 m^2, 25 m by 50 m **25.** Yes; for every value of x there is only one value of y. **26. a)** 1, −5 **b)** −2 **c)** 0, −4 **d)** −1, −3 **28. a)** Yes; jogging for a given period of time results in exactly one value for the distance covered. **b)** Yes; at any given time, she can be only one distance from the starting point. **29.** It is the slope of the line segment that joins the two points (1, 1) and (4, 19) on the graph. **30.** −5 **31.** $A = \frac{C^2}{4\pi}$ **32. a)** $8a + 3$ **b)** $-1 - 3n$ **c)** $m^2 - 2m + 2$ **d)** $8k^2 + 8k - 1$ **e)** $9t^2 + 6t - 4$ **f)** $12w^2 - 32w + 25$

Section 3.2, pp. 182–183

1. a) 0, 1, 2, 3, 4 **b)** The square root is defined only for positive values of x, and is defined as the positive square root. **c)** $x \geq 0$ **d)** No; the function is always increasing. **e)** $y \geq 0$ **2. a)** 9, 4, 1, 0, 1, 4, 9 **b)** the set of all real numbers **c)** $y \geq 0$ **3.** The graph of $f(x) = \sqrt{x}$ is the reflection of the right half of the graph of $f(x) = x^2$ in the line $y = x$. **4. b)** It is the reflection of $y = \sqrt{x}$ in the x-axis, and $y = \sqrt{x}$ appears only in the first quadrant. **c)** $x \geq 0$ **d)** $y \leq 0$ **e)** The graphs of $f(x) = \sqrt{x}$ and $f(x) = -\sqrt{x}$, taken together, are the reflection of the graph of $f(x) = x^2$ in the line $y = x$. **5. a)** 4, 3, 2, 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; −4, −3, −2, −1, $-\frac{1}{2}$, $-\frac{1}{3}$, $-\frac{1}{4}$ **b)** For $x > 0$, $y = \frac{1}{x}$ is positive and so has its graph in the first quadrant. For $x < 0$, $y = \frac{1}{x}$ is negative and so has its graph in the third quadrant.

6. Table 1: 1, 0.1, 0.01, 0.001; 1, 10, 100, 1000; Table 2: −1, −0.1, −0.01, −0.001; −1, −10, −100, −1000