# 2 Mid-Chapter Review

## *FREQUENTLY ASKED* Questions

**Study Aid**
- See Lesson 2.1, Example 1.
- Try Mid-Chapter Review Questions 1 and 2.

**Q: How do you determine the coordinates of the midpoint of a line segment if you know the coordinates of the endpoints?**

**A:** You can use the midpoint formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. This formula shows that the coordinates of the midpoint are the means of the coordinates of the endpoints.

**Study Aid**
- See Lesson 2.2, Examples 1 to 3.
- Try Mid-Chapter Review Questions 6 to 9.

**Q: How do you determine the length of a line segment if you know the coordinates of the endpoints?**

**A:** If the endpoints have the same $x$-coordinate, then the line segment is vertical. The length of the line segment is the difference in the $y$-coordinates of the endpoints. Similarly, if the endpoints have the same $y$-coordinate, then the line segment is horizontal. The length of the line segment is the difference in the $x$-coordinates of the endpoints.

For all types of line segments, including those which are neither vertical nor horizontal, you can use the distance formula to calculate its length.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Study Aid**
- See Lesson 2.3, Examples 1 and 3.
- Try Mid-Chapter Review Question 11.

**Q: How do you determine the equation of a circle that has its centre at the origin?**

**A1:** The equation of a circle with centre (0, 0) is $x^2 + y^2 = r^2$, where $r$ is the radius. For example, the equation of a circle with centre (0, 0) and a radius of 4 units is $x^2 + y^2 = 4^2$, or $x^2 + y^2 = 16$.

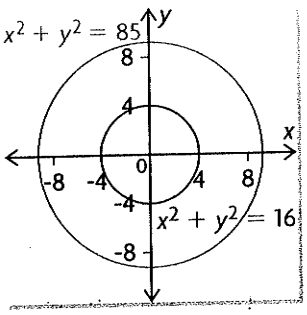
**A2:** If you only know the coordinates of a point on the circle, you can substitute these values for $x$ and $y$ and then solve for $r$. For example, suppose that you want to determine the equation of a circle that has its centre at the origin and passes through point $(2, -9)$. You substitute 2 for $x$ and $-9$ for $y$.

$2^2 + (-9)^2 = r^2$

$4 + 81 = r^2$

$85 = r^2$

The circle has equation $x^2 + y^2 = 85$.



# *PRACTICE* Questions

**Lesson 2.1**

**1.** Determine the coordinates of the midpoint of the line segment with each pair of endpoints.
- **a)** $(-1, -2)$ and $(-7, 10)$
- **b)** $(5, -1)$ and $(-2, 9)$
- **c)** $(0, -4)$ and $(0, 12)$
- **d)** $(6, 4)$ and $(0, 0)$

**2.** A diameter of a circle has endpoints $A(9, -4)$ and $B(3, -2)$. Determine the centre of the circle.

**3.** Describe all the points that are the same distance from points $A(-3, -1)$ and $B(5, 3)$.

**4.** A hockey arena is going to be built to serve two rural towns. On a plan of the area, the towns are located at $(1, 7)$ and $(8, 5)$. If the arena needs to be the same distance from both towns, determine an equation to describe the possible locations for the arena.

**5.** $\triangle PQR$ has vertices at $P(12, 4)$, $Q(-6, 2)$, and $R(-4, -2)$.
- **a)** Determine the coordinates of the midpoints of its sides.
- **b)** Determine the equation of the median from vertex $Q$.
- **c)** What is the equation of the perpendicular bisector of side $PQ$?

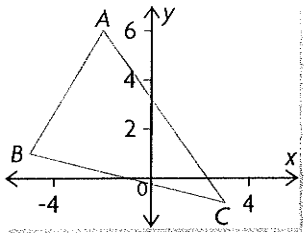
**Lesson 2.2**

**6.** Calculate the distance between each pair of points.
- **a)** $(2, 2)$ and $(7, 4)$
- **b)** $(-3, 0)$ and $(8, -5)$
- **c)** $(2, 9)$ and $(-5, 9)$
- **d)** $(9, -3)$ and $(12, -4)$

**7.** A power line is going to be laid from $A(-22, 15)$ to $B(7, 33)$ to $C(10, 18)$ to $D(-1, 4)$. If the units are metres, what length will the power line be?

**8.** Determine the distance between point $(-4, 4)$ and the line $y = 3x - 4$.

**9.** Show that $\triangle ABC$ has three unequal sides.




**Lesson 2.3**

**10.** **i)** State the coordinates of the centre of the circle described by each equation below.
**ii)** State the radius and the $x$- and $y$-intercepts of the circle.
**iii)** Sketch a graph of the circle.
- **a)** $x^2 + y^2 = 169$
- **b)** $x^2 + y^2 = 2.89$
- **c)** $x^2 + y^2 = 98$

**11.** Determine the equation of a circle that has its centre at $(0, 0)$ and passes through each point.
- **a)** $(-5, 0)$
- **b)** $(0, 7)$
- **c)** $(-3, -8)$
- **d)** $(4, -9)$

**12.** A raindrop falls into a puddle, creating a circular ripple. The radius of the ripple grows at a steady rate of 5 cm/s. If the origin is used as the location where the raindrop hits the puddle, determine the equation that models the ripple exactly 6 s after the raindrop hits the puddle.

**13.** Determine whether each point is on, inside, or outside the circle $x^2 + y^2 = 45$. Explain your reasoning.
- **a)** $(6, -3)$
- **b)** $(-1, 7)$
- **c)** $(-3, 5)$
- **d)** $(-7, -2)$

**14.** A line segment has endpoints $A(6, -7)$ and $B(2, 9)$.
- **a)** Verify that the endpoints of $AB$ are on the circle with equation $x^2 + y^2 = 85$.
- **b)** Determine the equation of the perpendicular bisector of $AB$.
- **c)** Explain how you can tell, from its equation, that the perpendicular bisector goes through the centre of the circle.

## PRACTICE Questions

**Lesson 2.1**

1. On the design plan for a garden, a straight path runs from $(-25, 20)$ to $(40, 36)$. A lamp is going to be placed at the midpoint of the path. Determine the coordinates for the lamp.

2. $\triangle ABC$ has vertices at $A(-4, 4)$, $B(-4, -2)$, and $C(2, -2)$.
   a) Determine the equation of the median from $B$ to $AC$.
   b) Is the median for part a) also an altitude? Explain how you know.

3. $\triangle LMN$ has vertices at $L(0, 4)$, $M(-5, 2)$, and $N(2, -2)$. Determine the equation of the perpendicular bisector that passes through $MN$.

**Lesson 2.2**

4. Which point is closer to the origin: $P(-24, 56)$ or $Q(35, -43)$?

5. A builder needs to connect a partially built house to a temporary power supply. On the plan, the coordinates of the house are $(20, 110)$ and the coordinates of the power supply are $(105, 82)$. What is the least amount of cable needed?

6. $\triangle QRS$ has vertices at $Q(2, 6)$, $R(-3, 1)$, and $S(6, 2)$. Determine the perimeter of the triangle.

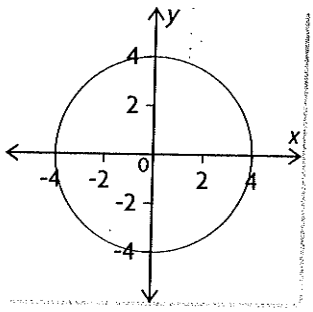
7. $\triangle XYZ$ has vertices at $X(1, 6)$, $Y(-3, 2)$, and $Z(9, 4)$. Determine the length of the longest median in the triangle.

**Lesson 2.3**

8. a) Determine the equation of the circle that is centred at $(0, 0)$ and passes through point $(-8, 15)$.
   b) Identify the coordinates of the intercepts and three other points on the circle.

9. A circle has a diameter with endpoints $C(20, -21)$ and $D(-20, 21)$. Determine the equation of the circle.

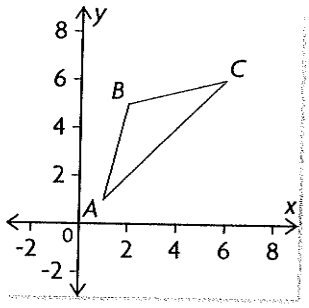
10. Determine the equation of this circle.




11. The point $(-2, k)$ lies on the circle $x^2 + y^2 = 20$. Determine the values of $k$. Show all the steps in your solution.

**Lesson 2.4**

12. $\triangle ABC$ has vertices as shown. Use analytic geometry to show that $\triangle ABC$ is isosceles.




13. A triangle has vertices at $A(1, 1)$, $B(-2, -1)$, and $C(3, -2)$. Calculate the side lengths to determine whether the triangle is isosceles, equilateral, or scalene.

14. Show that the quadrilateral with vertices at $J(-1, 1)$, $K(3, 4)$, $L(8, 4)$, and $M(4, 1)$ is a rhombus.

15. Determine the type of quadrilateral described by the vertices $R(-3, 2)$, $S(-1, 6)$, $T(3, 5)$, and $U(1, 1)$. Show all the steps in your solution.

**Lesson 2.5**

16. A quadrilateral has vertices at $A(-3, 1)$, $B(-5, -9)$, $C(7, -1)$, and $D(3, 3)$. Show that the midsegments of the quadrilateral form a parallelogram.

**17.** Show that points (10, 10), (−7, 3), and (0, −14) lie on a circle with centre (5, −2).

**18.** A triangle has vertices at $P(-2, 7)$, $Q(-4, 2)$, and $R(6, -2)$.

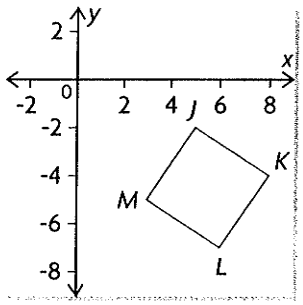
**a)** Show that $\triangle PQR$ is a right triangle.

**b)** Show that the midpoint of the hypotenuse is the same distance from each vertex.

**19. a)** Show that points (6, 7) and (−9, 2) are the endpoints of a chord in a circle with centre (0, 0).

**b)** A line is drawn through the centre of the circle so that it is perpendicular to the chord. Verify that this line passes through the midpoint of the chord.

**20. a)** Quadrilateral $JKLM$ has vertices as shown. Show that the diagonals of the quadrilateral bisect each other.

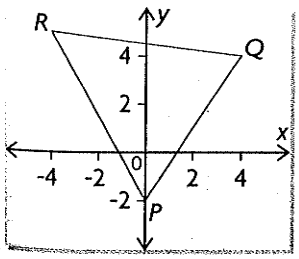



**b)** Make a conjecture about the type of quadrilateral $JKLM$ could be.

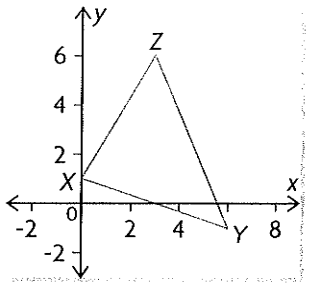
**c)** Use analytic geometry to verify your conjecture.

**Lesson 2.7**

**21.** $\triangle PQR$ has vertices at $P(0, -2)$, $Q(4, 4)$, and $R(-4, 5)$. Use analytic geometry to determine the coordinates of the orthocentre (the point where the altitudes intersect).




**22.** $\triangle XYZ$ has vertices at $X(0, 1)$, $Y(6, -1)$, and $Z(3, 6)$. Use analytic geometry to determine the coordinates of the centroid (the point where the medians intersect).




**23.** A new lookout tower is going to be built so that it is the same distance from three ranger stations. If the stations are at $A(-90, 28)$, $B(0, -35)$, and $C(125, 20)$ on a grid, determine the coordinates of the point where the new tower should be built.

**24.** Predict the type of quadrilateral that is formed by the points of intersection of the lines $3x + y - 4 = 0$, $4x - 5y + 30 = 0$, $y = -3x - 1$, and $-4x + 5y + 10 = 0$. Give reasons for your prediction. Verify that your prediction is correct by solving this problem.

**25.** A builder wants to run a temporary line from the main power line to a point near his site office. On the site plan, the site office is at $S(25, 18)$ and the main power line goes through points $T(1, 5)$ and $U(29, 12)$. Each unit represents 1 m.

**a)** At what point should the builder connect to the main power line?

**b)** What length of cable will the builder need?

# 2 Chapter Self-Test

**Process | *Checklist***

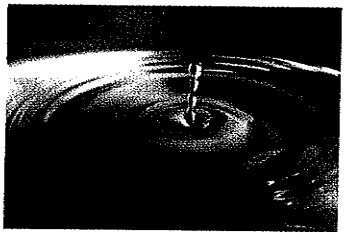
- ✔ Questions 1, 2, 6, and 8: Did you make connections between analytical geometry and the situation?
- ✔ Questions 3 and 4: Did you apply reasoning skills to construct a mathematical argument to confirm each figure type?
- ✔ Question 7: Did you use appropriate mathematical vocabulary to communicate your thinking?

**1.** An underground cable is going to be laid between points $A(-6, 23)$ and $B(14, -12)$.

**a)** If each unit represents 1 m, what length of cable will be needed? Give your answer to the nearest metre.

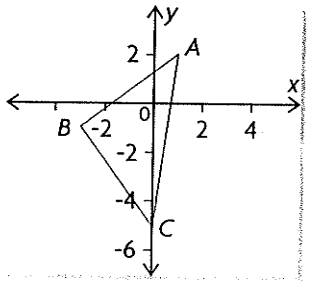
**b)** An access point will be located halfway between the endpoints of the cable. At what coordinates should the access point be built?

**2.** A stone is tossed into a pond, creating a circular ripple. The radius of the ripple increases by 12 cm/s.



**a)** Write an equation that describes the ripple exactly 3 s after the stone lands in the water. Use the origin as the point where the stone lands in the water.

**b)** A bulrush is located at point $(-36, 48)$. When will the ripple reach the bulrush?




**3.** The triangle at the left has vertices at $A(1, 2)$, $B(-3, -1)$, and $C(0, -5)$. Use analytic geometry to show that the triangle is an isosceles right triangle.

**4.** The corners of a building lot are marked at $P(-39, 39)$, $Q(-78, -13)$, $R(26, -91)$, and $S(65, -39)$ on a grid.

**a)** Verify that $PQRS$ is a rectangle.

**b)** What is the perimeter of the building?

**5.** Quadrilateral $JKLM$ has vertices at $J(2, 4)$, $K(6, 1)$, $L(2, -2)$, and $M(-2, 1)$. What type of quadrilateral is $JKLM$?

**6.** Three straight paths in a park form a triangle with vertices at $A(-24, 16)$, $B(56, -16)$, and $C(-72, -32)$. A new fountain is the same distance from the intersections of the three paths. Determine the location of the new fountain.

**7.** Explain how you can use analytic geometry to calculate the distance from a known point to a line that passes through two other known points.

**8.** The sides of a triangle are defined by the equations $x + 2y - 2 = 0$, $2x - y - 4 = 0$, and $3x + y + 9 = 0$. Determine the type of triangle that is formed by these three sides.