

A. Lesson Context

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> What is meant by the term FUNCTIONS and how do we work with them? mastery with working with basics & applications of linear functions mastery with working with basics & applications of linear systems understanding basics of function concepts and apply them to lines & linear systems 		
CONTEXT of this LESSON:	Where we've been In Grade 8, you studied linear RELATIONS & you've heard the terms domain and range	Where we are What do we mean by the term "functions" and how do linear relations from Grade 8 fit into this bigger understanding?	Where we are heading Mastery of working with multiple representations of $f(x) = mx + b$

B. Lesson Objectives

- Work with number patterns to revisit the "rule of four" or multiple representations
- Introduce fundamental concepts related to Functions → relations, functions, domain, range, notations
- Review basic algebra about linear relations

C. Fast Five (Skills Review Focus)

a. Solve $3x + 5 = 15 - 2x$

b. Solve $3(2 - x) = -(x - 2)$

c. Evaluate $4^{-2} - 2^{-3}$

d. Evaluate $(x - 4)(3 - 2x)$ when $x = 2$

e. Find the slope of the line through $A(2, 5)$ and $B(6, -3)$.

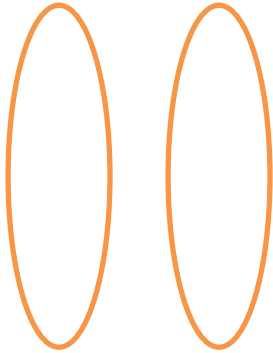
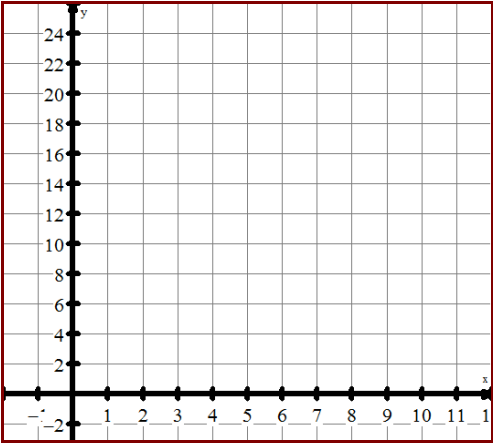
f. Find the equation of the line through $A(2, 5)$ and $B(6, -3)$.

g. Graph the line through $A(2, 5)$ and $B(6, -3)$.

h. Determine the slope of a line perpendicular to line AB.

D. Working with Number Patterns

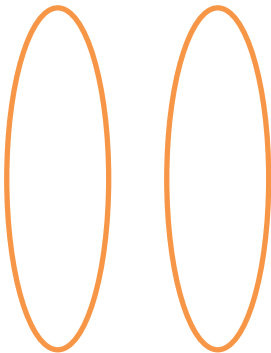
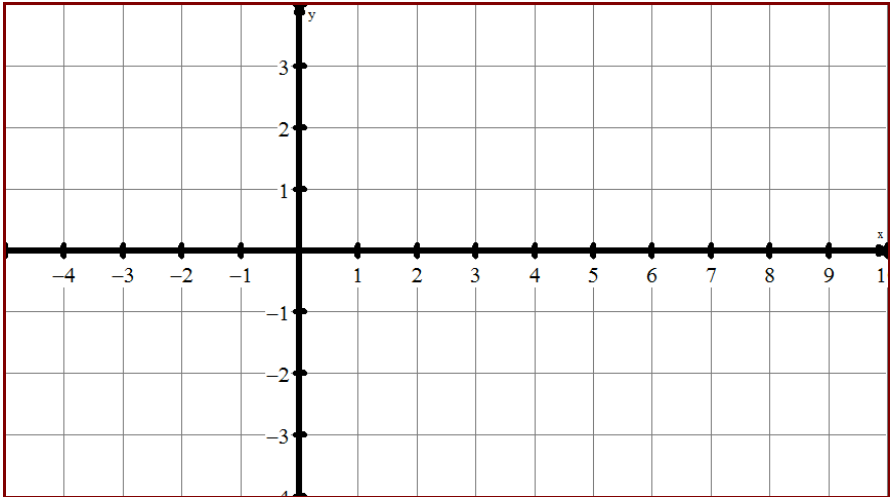
- a. We will work with something “familiar” (linear relations) in order to create some new understandings (multiple representations) and explore/introduce some new concepts (functions).

<u>VERBAL (V)</u>	<u>NUMERIC (N)</u>																						
<p>Our relation will be defined by the following verbal description:</p> <p><i>To “create/generate” a new value, a number is doubled and then increased by four.</i></p> <p>The CONDITION on the number is that it must be a real number between and including 0 and 10.</p>	<p>Table of Values</p> <table border="1" data-bbox="646 527 894 909"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table>	x	y																			<p>Mapping Diagram</p> 	<p>Ordered Pairs</p>
x	y																						
<p><u>ALGEBRAIC (A)</u></p>	<p><u>GRAPHIC (G)</u></p> 																						

b. REVIEW from Grade 8 Math

- What is the SLOPE of this line?
- What does slope MEAN?
- What are the x- and y-intercepts of this line?

E. Working with Number Patterns

<u>VERBAL (V)</u>	<u>NUMERIC (N)</u>																						
<p>Our relation will be defined by the following verbal description:</p> <p><i>To “create/generate” a new value, a “starting” number is reduced by 2 and then this result is halved.</i></p> <p>The CONDITION on the number is that it must be a real number between but excluding -4 and 8.</p>	<p>Table of Values</p> <table border="1" data-bbox="641 443 886 827"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table>	x	y																			<p>Mapping Diagram</p> 	<p>Ordered Pairs</p>
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F. REVIEW from Grade 8 Math

- What does the term “relation” mean?
- What is the domain of this relation?
- What is the range of this relation?

G. NEW CONCEPT: Function Basics**b. Relations:**

- i. A "relation" is just a relationship between sets of information;
- ii. A relation refers to a set of input and output values, usually represented in ordered pairs
- iii. A relation is simply a set of ordered pairs.

c. Functions:

- i. A function is a "well-behaved" relation → When we say that a function is "a well-behaved relation", we mean that, given a starting point, we know exactly where to go; given an x , we get only and exactly one y .
- ii. Function is a relation in which each element of the domain is paired with exactly one element of the range.
- iii. A function is a set of ordered pairs in which each x -element has only ONE y -element associated with it.
- iv. A function is a rule that takes an input, does something to it, and gives a unique corresponding output.

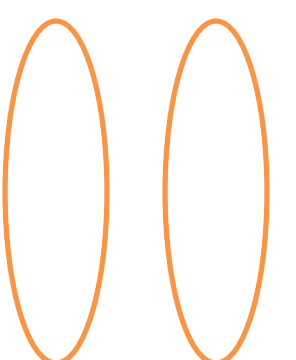
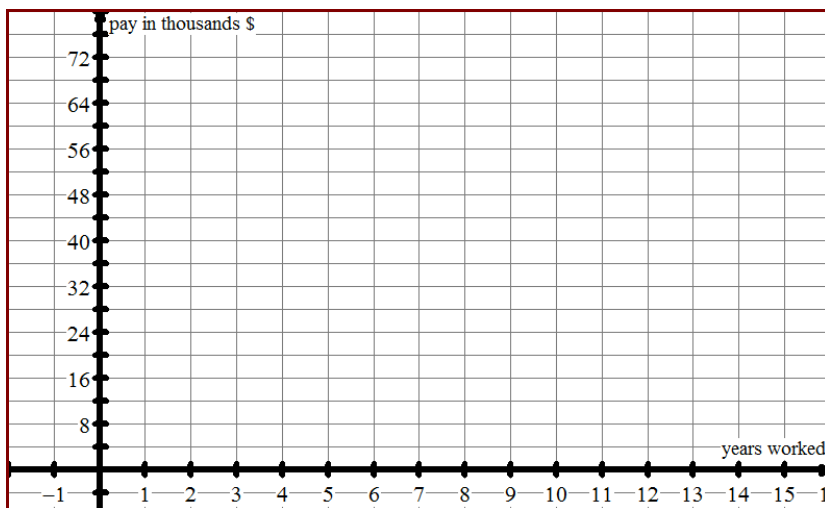
d. Notations:

- i. Rather than writing linear equations in the typical $y = mx + b$ format, we will now write them in function notation → as $f(x) = mx + b$ where " f " simply refers to the function name and the x refers to the input
- ii. Evaluate → if $f(x) = 2x + 4$, then we can evaluate $f(3)$ as
- iii. Solve → if $f(x) = 2x + 4$, then we can solve $12 = f(x)$ as

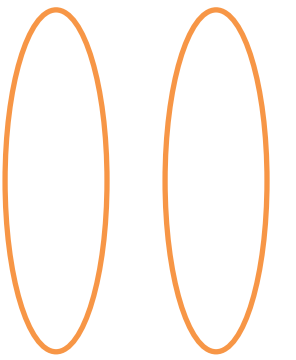
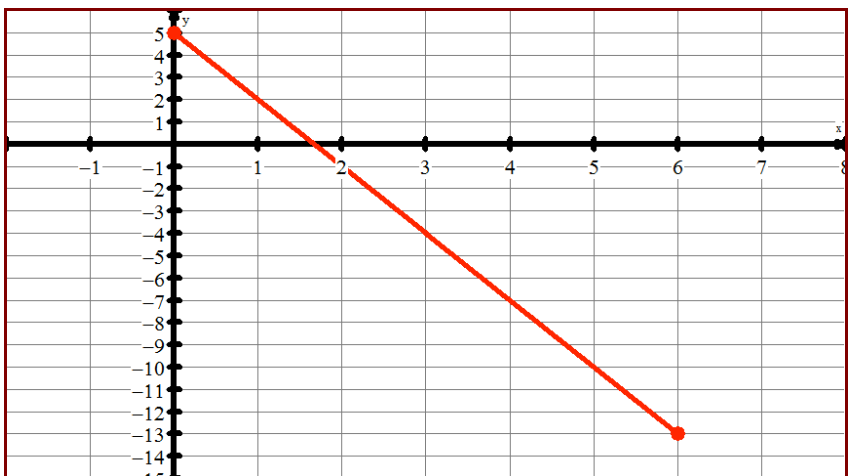
e. Understanding Domain and Range:

- i. The set of all the starting points is called "the domain" and the set of all the ending points is called "the range."
- ii. The domain is what you start with; the range is what you end up with.
- iii. The domain is the x 's; the range is the y 's

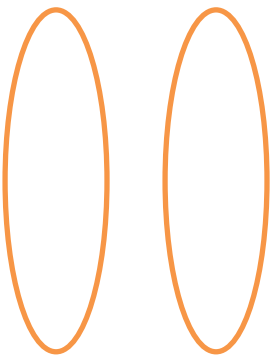
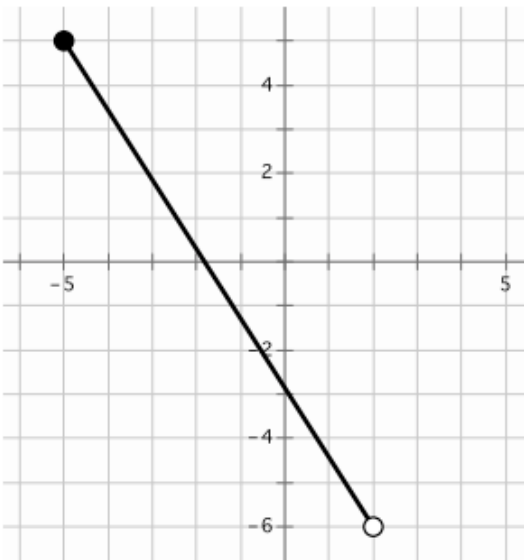
H. Further Examples: Working with Functions in Contextual Applications

<u>VERBAL (V)</u>	<u>NUMERIC (N)</u>																						
<p>Our function will be defined by the following verbal description:</p> <p><i>Mr. S works at CAC and earned a base salary of \$40,000 in his first year and then he receives an annual bonus of \$2,000 for each additional year he works here.</i></p>	<p>Table of Values</p> <table border="1"> <thead> <tr> <th>Years worked</th> <th>Salary</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table>	Years worked	Salary																			<p>Mapping Diagram</p> 	<p>Ordered Pairs</p>
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<u>NEW CONCEPTS: Functions</u>																							
<p>Domain of $S(y)$: Represents →</p> <p>Range of $S(y)$: Represents →</p> <p>$S(6)$ → Evaluate & interpret</p> <p>$\\$56,000 = S(y)$ → Solve and interpret</p>																							

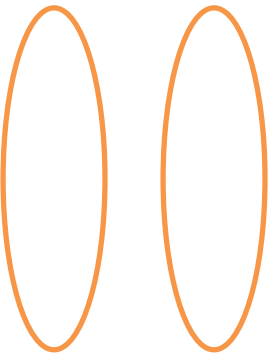
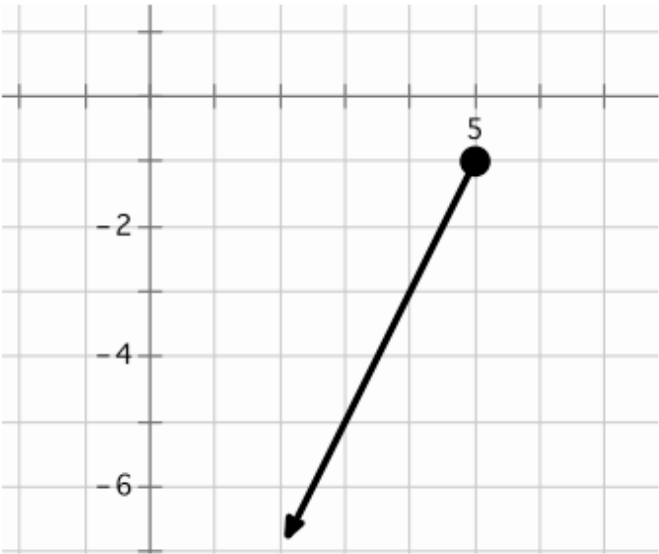
I. Further Examples: Working with Functions Graphically

<u>VERBAL (V)</u>	<u>NUMERIC (N)</u>																				
<p>Our function will be defined by the following GRAPHIC description:</p> <p>So, to write a verbal description: To “create/generate” an output value, the input value is ??????</p>	<p>Table of Values</p> <table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table>	x	f(x)																	<p>Mapping Diagram</p> 	<p>Ordered Pairs</p>
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<p>Domain of $f(x)$ →</p> <p>Range of $f(x)$ →</p> <p>Evaluate $f(2.75)$ →</p> <p>Solve $-6 = f(x)$ →</p>																					

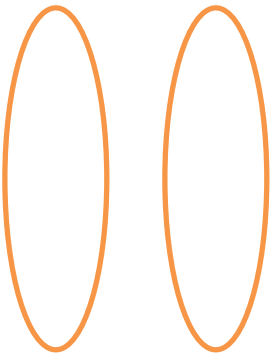
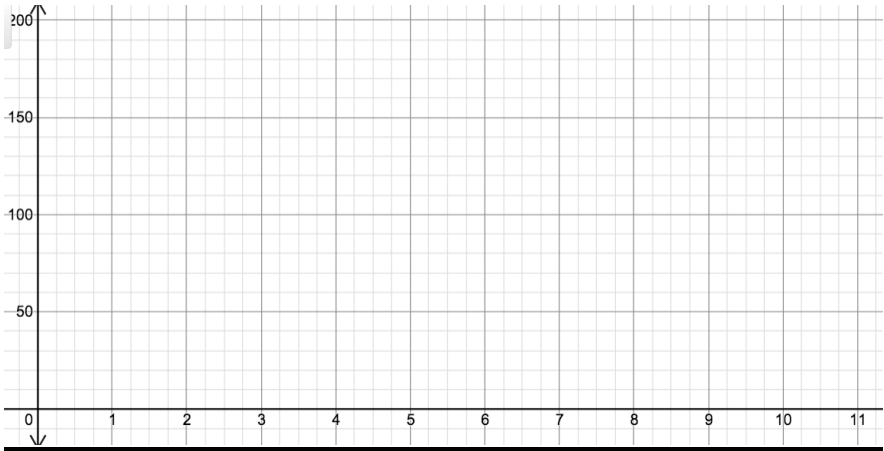
J. Working with Functions using Graphic Representations

<u>VERBAL (V)</u>	<u>NUMERIC (N)</u>																				
<p>Our function will be defined by the following GRAPHIC description:</p> <p>So, to write a verbal description: To “create/generate” a new value, ??????</p>	<p>Table of Values</p> <table border="1"> <tbody> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </tbody> </table>																			<p>Mapping Diagram</p> 	<p>Ordered Pairs</p>
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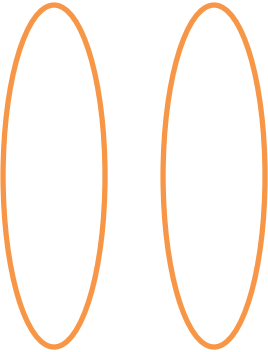
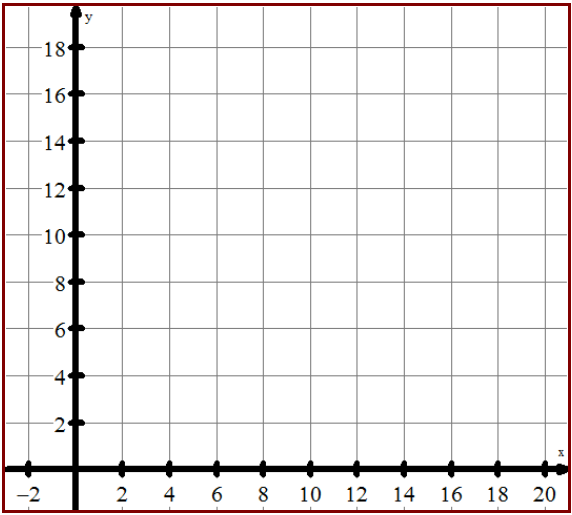
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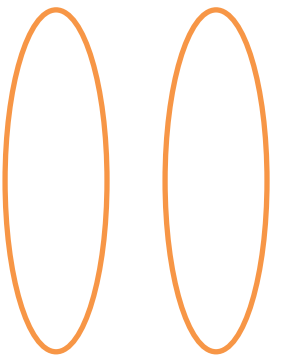
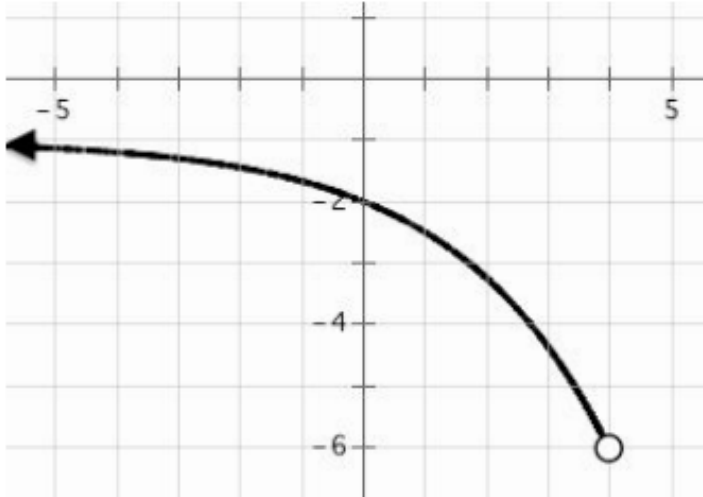
L. ADVANCED LEVEL: Working with NON-LINEAR Functions in Contextual Applications

<u>VERBAL (V)</u>	<u>NUMERIC (N)</u>																				
<p>Our function will be defined by the following verbal description:</p> <p><i>Mr. S started with 196 M&M's in a bag and he invited students to come by, one at a time, to remove HALF of the remaining M&Ms from the bag.</i></p>	<p>Table of Values</p> <table border="1" data-bbox="592 443 841 827"> <thead> <tr> <th>Student</th> <th>M&Ms</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>196</td> </tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table>	Student	M&Ms	0	196															<p>Mapping Diagram</p> 	<p>Ordered Pairs</p>
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0	196																				
<p><u>ALGEBRAIC (A)</u></p> <p>$N(s)$ will be a notation used to represent the idea of → Number of M&Ms in the bag as a function of the number of students who have come by to take M&Ms</p>	<p><u>GRAPHIC (G)</u></p> 																				
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<p>Domain of $N(s)$: Represents →</p> <p>Range of $N(s)$: Represents →</p> <p>$N(3)$ → Evaluate & interpret</p> <p>$12 = N(s)$ → Solve and interpret</p>																					

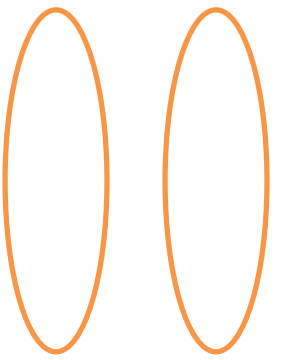
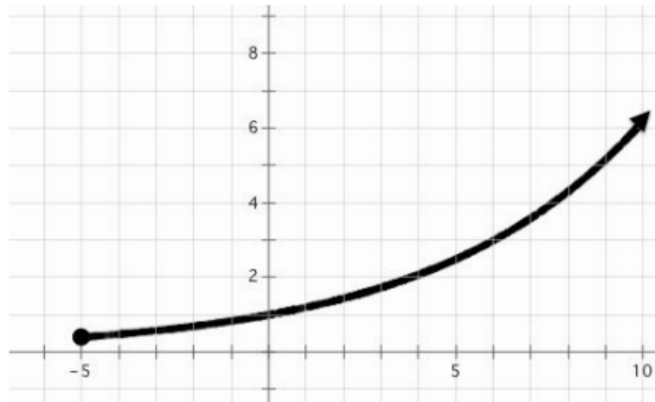
M. ADVANCED LEVEL: Working with NON-LINEAR Functions in Contextual Applications

<u>VERBAL (V)</u>	<u>NUMERIC (N)</u>																						
<p>Our function will be defined by the following verbal description/context:</p> <p><i>Calculate the AREA of a rectangle if the perimeter of a rectangle is 36 m</i></p> <div data-bbox="224 667 584 741" style="border: 1px solid black; height: 35px; width: 222px; margin: 10px auto;"></div>	<p>Table of Values</p> <table border="1" data-bbox="646 457 898 842"> <thead> <tr> <th>w</th> <th>A(w)</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table>	w	A(w)																			<p>Mapping Diagram</p> <div data-bbox="943 499 1208 848" style="text-align: center;">  </div>	<p>Ordered Pairs</p>
w	A(w)																						
<p><u>ALGEBRAIC (A)</u></p> <p>A(w) will be a notation used to represent the idea of → Area as a function of the width of the field</p>	<p><u>GRAPHIC (G)</u></p> <div data-bbox="781 982 1349 1493" style="text-align: center;">  </div>																						
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<p>Domain of A(w): Represents →</p> <p>Range of A(w): Represents →</p> <p>A(4) → Evaluate & interpret</p> <p>45 = A(w) → Solve and interpret</p>																							

N. ADVANCED LEVEL: NON-LINEAR Functions With Graphic Representations

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O. **ADVANCED LEVEL: NON-LINEAR Functions With Graphic Representations**

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