3.1 Quadratic Relations

- Sometimes a **curve of best fit** is a more appropriate model for data than a line of best fit. This is true when the data points seem to fit a recognizable pattern that is not a straight line. In such a case, try to draw a smooth curve that passes through as many of the data points as possible. Visualize where the curve should lie between the actual plotted data points. A piece of string may help you to decide the shape and location of the curve.
- The values of the **first differences** in a table of values determine if the relation is linear.
    - For constant increments of the independent variable, a relation is linear if the first differences of the dependent variable are constant.

      For example, the first differences in this table of values are constant, so the relation is linear.

x	0	1	2	3	4
y	2	3	4	5	6
First Difference	1	1	1	1	

    - For constant increments of the independent variable, a relation is **quadratic** if the second differences of the dependent variable are constant.

      For example, the second differences in this table of values are constant, so the relation is quadratic.

x	0	1	2	3	4
y	2	3	6	11	18
First Difference	1	3	5	7	
Second Difference	2	2	2		

- A linear relation models a phenomenon where the rate of change is constant. A nonlinear relation models a phenomenon with a variable rate of change.
- The **degree** of a one-variable polynomial is the highest exponent that appears in any term of the expanded form of the polynomial.
- A polynomial of degree 2 models a quadratic relation.

Example

A model rocket is shot straight up into the air. The table shows its height, h, in metres, at t seconds.

t (s)	0	1	2	3	4	5	6
h (m)	0.0	25.1	41.5	40.4	45.9	27.5	3.6

(a) Draw the curve of best fit.

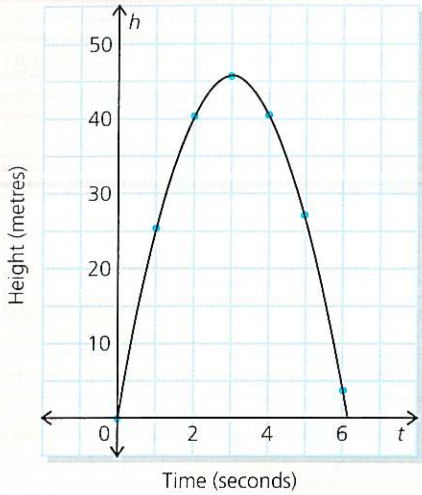
(b) Identify the type of relationship between time and height. Explain.

(c) Estimate the height after 4.5 s.

(d) When does the rocket reach 20 m?

Solution

(a)



(b) To identify the relationship between time and height, examine the first and second differences.

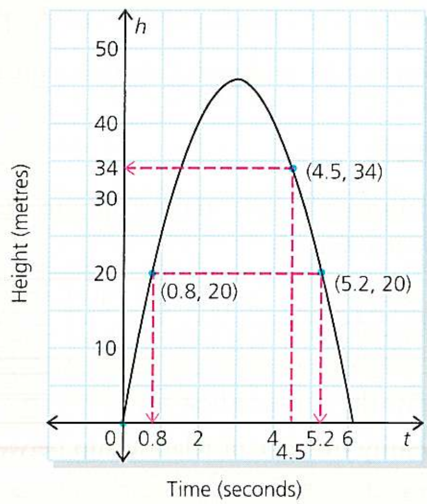
t (s)	0	1	2	3	4	5	6
h (m)	0.0	25.1	40.4	45.9	41.6	27.5	3.6
First Difference	25.1	15.3	5.5	4.3	−14.1	−23.9	
Second Difference	−9.8	−9.8	−9.8	−9.8	−9.8		

The first differences are not constant, so the relationship is nonlinear.

The second differences are constant, so the relationship is quadratic.

(c) Reading the graph tells you that at 4.5 s, the rocket is about 34 m high.

(d) The rocket reaches 20 m at two different times. Once on the way up, at about 0.8 s, and again on the way down, at 5.2 s.



Extra Practice

1. In each case,

i. use finite differences to determine whether the relationship is best represented by a linear expression, a quadratic expression, or another type of expression

ii. draw a curve of best fit to verify your answer in (i)

(a)

x	−3	−2	−1	0	1
y	−7	−4	−1	2	5

(b)

x	−1	0	1	2	3
y	−1	0	1	8	27

(c)

x	−2	−1	0	1	2
y	0.25	0.5	1	2	4

(d)

x	−2	−1	0	1	2
y	−7.5	−4.5	−1.5	1.5	4.5

(e)

x	0	1	2	3	4
y	8	10	12	14	16

2. Examine the pattern of numbers, $N_1 = 6$, $N_2 = 20$, $N_3 = 42$, $N_4 = 72$, and $N_5 = 110$.

(a) Find the next three terms in the sequence.

(b) Use finite differences to show that the relationship between n and the value of N_n is quadratic.

3. Over the ocean, a sandbag is dropped from a hot air balloon, so the balloon will rise. The table shows the height of the sandbag at different times as it falls.

t (s)	0	1	2	3	4	5	6	7	8	9	10
h (m)	1200	1195	1180	1155	1120	1075	1020	955	880	795	700

(a) Draw the scatter plot of the data.

(b) What type of model represents the relationship between the height of the sandbag and time?

(c) Follow the pattern and extend the table of values until the sandbag hits the water.

(d) Draw the curve of best fit.

(e) About how long does it take for the sandbag to reach the water?

3.2 Properties of Quadratic Equations

- The graph of a quadratic relation is called a **parabola.**
- The **vertex** of a parabola is the point on the graph with the greatest y-coordinate if the graph opens down or the least y-coordinate if the graph opens up.
- When a quadratic relation is used to model a situation, the y-coordinate of the vertex corresponds to an **optimal value.** Depending on the direction of opening of the parabola, this represents either a **maximum** or a **minimum** value of the quantity being modelled by the dependent variable. The maximum or minimum value is always associated with the vertex.
- The direction of opening of the parabola can be determined from the sign of the second differences in the table of values of the quadratic relation.
    - ♦ If the constant value of the second differences is positive, then the parabola opens up (figure 1).
    - ♦ If the constant value of the second differences is negative, then the parabola opens down (figure 2).
- A parabola is symmetrical with respect to a vertical line through its vertex. The line is called the **axis of symmetry** of the parabola and the vertex lies on the axis of symmetry. If the coordinates of the vertex are (h, k), then the equation of the axis of symmetry is $x = h$ (figure 3).
- The axis of symmetry is the perpendicular bisector of the segment joining any two points on the parabola that have the same y-coordinates. If the parabola crosses the x-axis, the x-coordinates of these points are called the **zeros**, or ***x*-intercepts** of the relation, and the vertex is directly above or below the midpoint of the segment joining the zeros (figure 4).

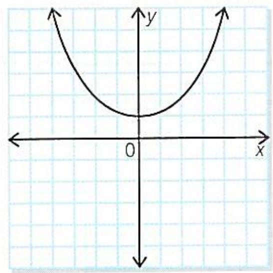


figure 1

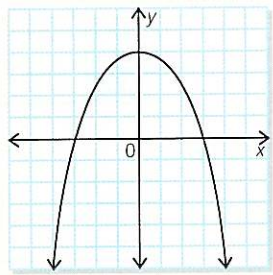


figure 2

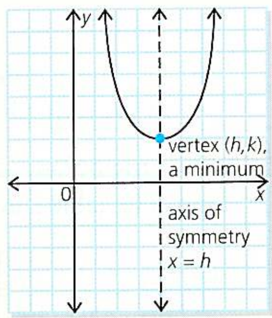


figure 3

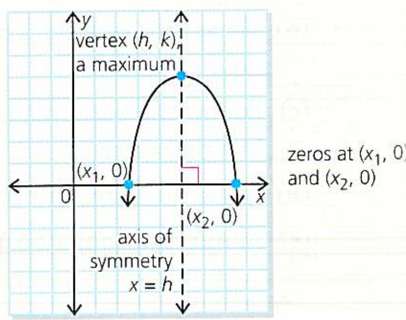


figure 4

Example

A golf ball is hit and its height is given by $h = 29.4t - 4.9t^2$, where h is its height in metres and t is the time in seconds.

(a) When is the golf ball on the ground?

(b) At what time does the golf ball reach its maximum height?

(c) What is the ball's maximum height?

(d) What will be the shape of the graph of this relation?

(e) Sketch the graph.

Solution

(a) The zeros of the relation occur when the height is zero, that is, when the ball is on the ground. Then,

$$h = 29.4t - 4.9t^2 \text{ or}$$
$$0 = 29.4t - 4.9t^2$$
$$0 = t(29.4 - 4.9t) \qquad \text{The common factor is } t.$$

So either $t = 0$ or $29.4 - 4.9t = 0$.

$$29.4 = 4.9t$$
$$\frac{29.4}{4.9} = \frac{4.9t}{4.9}$$
$$6 = t$$

The zeros are 0 and 6. The ball is on the ground at 0 s and 6 s.

(b) The golf ball reaches its maximum height at a value of the independent variable that is midway between 0 and 6. This value is $\frac{0 + 6}{2} = 3$. The golf ball reaches its maximum height at 3 s.

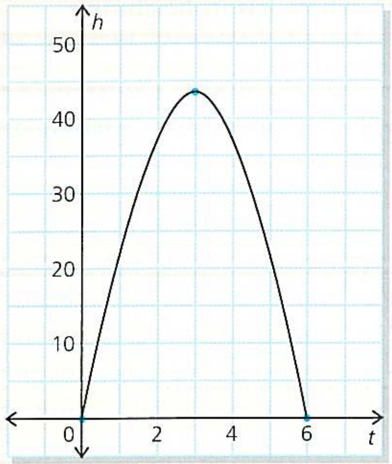
(c) To find the maximum height, substitute $t = 3$ into the equation.

$$h = 29.4t - 4.9t^2$$
$$h = 29.4(3) - 4.9(3)^2$$
$$h = 88.2 - 44.1$$
$$h = 44.1$$

The maximum height is 44.1 m.

(d) The relation $h = 29.4t - 4.9t^2$ is a polynomial of degree 2, which means it is a quadratic relation. The graph of a quadratic relation is a parabola. Since the coefficient of the t^2 term is negative, the parabola opens downward.

(e) The parabola has zeros, or x-intercepts, at 0 and 6. The vertex is at (3, 44.1)



Extra Practice

4. Each pair of points lies on a different parabola. Determine the axis of symmetry of each parabola.

(a) $(-5, 4)$ and $(-9, 4)$

(b) $(2.5, 3.5)$ and $(3.7, 3.5)$

(c) $(16, -2)$ and $(-18, -2)$

(d) $(\frac{3}{4}, -5)$ and $(-\frac{1}{2}, -5)$

(e) $(5\frac{5}{8}, -5)$ and $(4\frac{1}{4}, -5)$

(f) $(-3\frac{2}{3}, 7)$ and $(5\frac{1}{6}, 7)$

5. The zeros of a quadratic relation are -2 and 5, and the second differences are all negative.

(a) Explain whether the optimal value will be a maximum or a minimum.

(b) What value of the independent variable will produce the optimal value?

(c) Will the optimal value be positive or negative? Explain.

6. For each parabola,

i. find where it crosses the x-axis

ii. state the equation of the axis of symmetry

iii. without graphing, determine whether the vertex represents a maximum or a minimum value

iv. find the coordinates of the vertex

(a) $A = 18w - w^2$

(b) $A = -L^2 + 10L$

(c) $y = 4x - 16x^2$

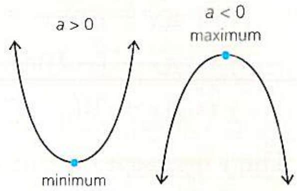
(d) $h = 25t - 5t^2$

(e) $y = 15x + 6x^2$

(f) $A = 42w - 6w^2$

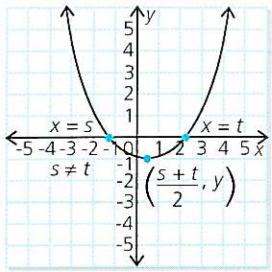
3.3–3.4 The Role of the Zeros of a Quadratic Relation

- Relations in the form $y = a(x - s)(x - t)$ are quadratic, provided that $a \neq 0$.
- A quadratic relation is said to be in **factored form** if its algebraic expression appears in the form $y = a(x - s)(x - t)$.
- If $a > 0$, the parabola opens up and has a minimum. If $a < 0$, the parabola opens down and has a maximum.

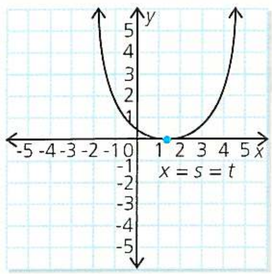


- When a quadratic relation is in factored form, the values of x that are the solutions to $0 = a(x - s)(x - t)$ are called the **zeros** of the quadratic relation. These values correspond to the x-intercepts of the graph of the relation. The zeros can be determined by setting each factor equal to 0 and solving the resulting equation for x. In $y = a(x - s)(x - t)$, the zeros are $x = s$ and $x = t$.
- If $s \neq t$, then the relation has two distinct zeros. If $s = t$, the relation has only one zero at $x = s = t$.
- If the zeros of a quadratic relation are s and t, then the x-coordinate of the vertex is $\frac{s+t}{2}$.
- When the zeros of a quadratic relation are known, the value of a can be determined if some other point (x_1, y_1) on the graph is known. Substitute the coordinates of the point in the factored form, giving $y_1 = a(x_1 - s)(x_1 - t)$, then solve for a.

This quadratic relation has two zeros.



This quadratic relation has one zero.



Example

The zeros of a parabola are −2 and 7, and it crosses the y-axis at −28.

(a) What is the equation of the quadratic relation?

(b) What are the coordinates of the vertex?

Solution

(a) The factored equation of any parabola is in the form $y = a(x - s)(x - t)$, where s and t are the zeros. In this case, $s = -2$ and $t = 7$. Substitute these values.

$$y = a(x - s)(x - t)$$
$$y = a(x - (-2))(x - 7)$$
$$y = a(x + 2)(x - 7)$$

The y-intercept of -28 occurs when $x = 0$. Then,

$$-28 = a(0 + 2)(0 - 7)$$
$$-28 = a(2)(-7)$$
$$-28 = -14a$$
$$a = \frac{-28}{-14}$$
$$a = 2$$

The equation of the quadratic relation is $y = 2(x + 2)(x - 7)$.

(b) The vertex occurs midway between -2 and 7, so

$$x = \frac{-2 + 7}{2}$$
$$x = \frac{5}{2}$$
$$x = 2\tfrac{1}{2}$$

Substitute this value into the equation.

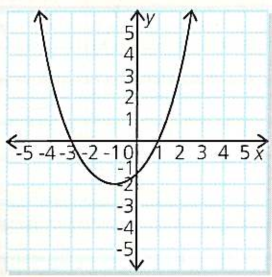
$$y = 2(2\tfrac{1}{2} + 2)(2\tfrac{1}{2} - 7)$$
$$y = 2\left(4\tfrac{1}{2}\right)\left(-4\tfrac{1}{2}\right)$$
$$y = 2\left(\tfrac{9}{2}\right)\left(-\tfrac{9}{2}\right)$$
$$y = -\frac{81}{2}$$
$$y = -40\tfrac{1}{2}$$

The coordinates of the vertex are $(2\tfrac{1}{2}, -40\tfrac{1}{2})$.

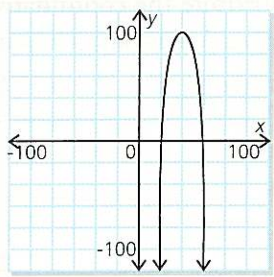
Extra Practice

7. Determine the equation of each parabola.

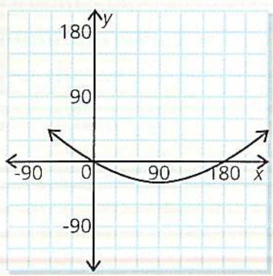
(a)



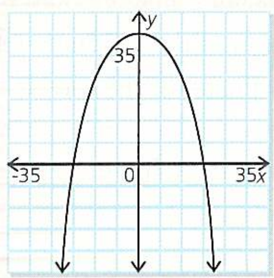
(b)



(c)



(d)



8. **(a)** Determine the zeros for each parabola in question 7.

(b) What is the value of the independent variable at the optimal point on each curve?

9. Determine the quadratic equation for a parabola with

(a) zeros at 5 and 9, and an optimal value of -2

(b) zeros at -3 and 7, and an optimal value of 4

(c) zeros at -6 and 2, and a y-intercept of -9

(d) zeros at -9 and -5, and a y-intercept of 8

10. The city bus company usually transports 12 000 riders per day at a ticket price of $1. The company wants to raise the ticket price and knows that for every 10¢ increase the number of riders decreases by 400.

(a) What price for a ticket will maximize revenue?

(b) What other factors can influence the number of people who use public transportation?

(c) How might the company use the answers from (a) and (b) in its decision to set ticket prices?

3.5–3.7 Standard Form of a Quadratic Relation

- The **factored form** of a quadratic relation, $y = (x - s)(x - t)$, $y = a(x - s)(x - t)$, or $y = (ax - s)(bx - t)$, can be expanded using the distributive property and simplified to give a **standard form** trinomial, $y = ax^2 + bx + c$. Standard form is often called **expanded form**.
- Equivalent factored and standard forms

Factored Form	Standard (or Expanded Form)
$y = (x - s)(x - t)$	$y = x^2 - (s + t)x + st$
$y = a(x - s)(x - t)$	$y = ax^2 - a(s + t)x + ast$
$y = (ax - s)(bx - t)$	$y = abx^2 - (at + bs)x + st$

- The **quadratic regression** feature of a graphing calculator provides the algebraic expression for a curve of best fit in **standard** (or **expanded**) **form**, $y = ax^2 + bx + c$.

Example

A pizza company's research shows that a 25¢ increase in the price of a pizza results in 50 fewer pizzas being sold. The usual price of \$15 for a three-item pizza results in sales of 1000 pizzas.

(a) Write the algebraic expression that models the revenue for this situation.

(b) Expand and simplify the expression in standard form.

(c) Graph the expressions in (a) and (b). Compare the graphs.

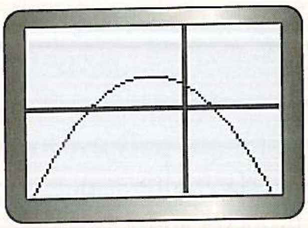
Solution

(a) Revenue = price of one pizza × number of pizzas sold

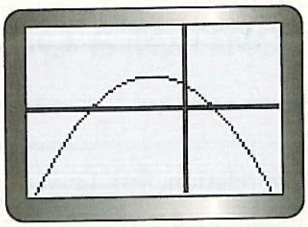
Let x be the number of 25¢ price increases and R be the revenue in dollars. Then, the price of one pizza is $15 + 0.25x$ and the number of pizzas sold is $1000 - 50x$. So, the expression for revenue is $R = (15 + 0.25x)(1000 - 50x)$.

(b)
$$R = (15 + 0.25x)(1000 - 50x)$$
$$R = (15)(1000) + (0.25x)(1000) + (15)(-50x) + (0.25x)(-50x)$$
$$R = 15\,000 + 250x - 750x - 12.5x^2$$
$$R = 15\,000 - 500x - 12.5x^2$$

(c)



$R = (15 + 0.25x)(1000 - 50x)$



$R = 15\,000 - 500x - 12.5x^2$

Although the algebraic expressions for both graphs look different, the graphs are the same and represent the same relation.

Extra Practice

11. Expand and simplify these expressions.

(a) $(x + 5)(x + 4)$

(b) $(x - 2)(x - 5)$

(c) $(x + 6)(x - 7)$

(d) $(2x - 3)(2x + 3)$

(e) $(4x + 5)(3x - 2)$

(f) $(6x - 2)(5x + 7)$

(g) $(5 - 3x)(6 + 2x)$

(h) $(4a - 2b)(5a + 3b)$

(i) $(2m + 3n)(2m + 3n)$

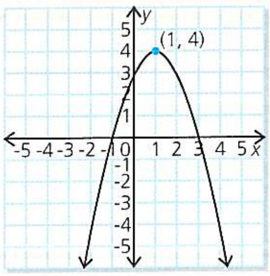
(j) $-3(2x - 1)(x + 4)$

(k) $5(3u + 2v)(2u - 5v)$

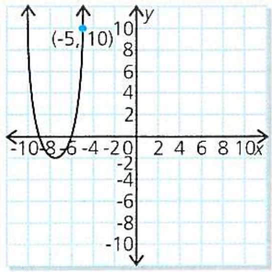
(l) $-2(5 - 2x)(3x + 4)$

12. Determine the equation of each parabola in standard form. Verify your answer using the quadratic regression function of a graphing calculator.

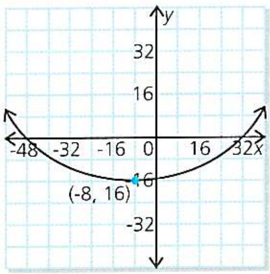
(a)



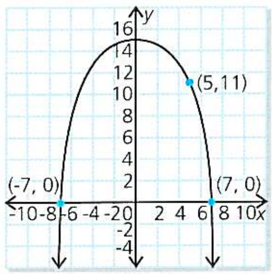
(b)



(c)



(d)



13. Express, in standard form, the equation of the parabola

(a) that has zeros at 4 and -2, and whose optimal value for y is 12

(b) that has zeros at -4 and 8, and goes through (2, 18)

3.8 Extending Algebra Skills: Factoring Quadratic Expressions

- Many quadratic relations in standard form $y = ax^2 + bx + c$ can be expressed as the product of two binomial factors. Finding this product is called **factoring**.
- Factoring is the opposite operation of expanding.
- If the quadratic expression $x^2 + bx + c$ can be factored, the factors are of the form $(x - s)(x - t)$, where $b = -(s + t)$ and $c = st$.
- If the quadratic expression $ax^2 + bx + c$ (where $a \neq 1$) can be factored, the factors can be found using a guess-and-check strategy.
  - ♦ Choose factors that produce the correct first and last term of the quadratic (the x^2-term and the constant term) when multiplied.

- Sometimes quadratics in the form $ax^2 + bx + c$ have a common factor. If the trinomial remaining after a common factor is removed is in the form $x^2 + bx + c$, you may be able to factor it as above. For example, $5x^2 - 5x - 150 = 5(x - 6)(x + 5)$, since $5x^2 - 5x - 150 = 5(x^2 - x - 30)$ and $(x^2 - x - 30) = (x - 6)(x + 5)$.
- The middle (x) term of the quadratic comes from multiplying the outside terms of the binomial factors together, then the inside terms, then adding the results. For example, $6x^2 - 23x + 20 = (3x - 4)(2x - 5)$, since $(3x)(2x) = 6x^2$, $(-4)(-5) = 20$, and $(3x)(-5) + (-4)(2x) = (-15x) + (-8x)$ or $-23x$. Check that the factors you choose give the correct x-term in the standard form.

- If a trinomial is of the form $a^2x^2 - b^2$, then it is a **difference of squares** and can be factored as $(ax + b)(ax - b)$.
- If a trinomial is of the form $a^2x^2 + 2abx + b^2$, then it is a **perfect square** and can be factored as $(ax + b)(ax + b) = (ax + b)^2$.

Example

Factor completely.

(a) $x^2 - 13x + 42$

(b) $6x^2 + x - 15$

(c) $4x^2 - 25$

(d) $9x^2 - 12x + 4$

Solution

(a) $x^2 - 13x + 42 = (x - 6)(x - 7)$

(b) $6x^2 + x - 15 = (3x + 5)(2x - 3)$

(c) $4x^2 - 25 = (2x + 5)(2x - 5)$

(d) $9x^2 - 12x + 4 = (3x - 2)(3x - 2) = (3x - 2)^2$

Extra Practice

14. Factor each expression.

(a) $x^2 + 2x - 15$ **(b)** $m^2 + 3m - 4$ **(c)** $r^2 - 4r + 4$

(d) $q^2 - 3q - 10$ **(e)** $6x^2 + 5x - 1$ **(f)** $6d^2 - d - 2$

(g) $x^2 - 9$ **(h)** $4x^2 - 25$ **(i)** $9x^2 + 12x + 4$

(j) $3m^2 + 3m - 18$ **(k)** $8p^2 + 8p - 6$ **(l)** $27x^2 - 48$

15. The Wheely Fast Co. makes custom skateboards for professional riders. They model their profit with the relation $P = -2b^2 + 14b - 20$, where b is number of skateboards they produce (in thousands), and P is the company's profit in hundred thousands of dollars.

(a) When does Wheely Fast break even? That is, when is their profit 0?

(b) How many skateboards does Wheely Fast need to produce to maximize profit?

3.9 Solving Problems with Quadratic Equations

- If the value of y is known for the relation $y = ax^2 + bx + c$, then the corresponding values of x can be found either graphically or algebraically.
- If the quadratic relation $y = ax^2 + bx + c$ is graphed, then for any value of the dependent variable, y, the values of the independent variable, x, can be read from the graph.
- In a quadratic relation, when y is replaced with a number, the result is a **quadratic equation**.
- In some cases, quadratic equations can be solved by factoring. Follow this procedure:
  - Replace y with the given value.
  - Rearrange the quadratic equation to the form $ax^2 + bx + c = 0$.
  - Factor the quadratic expression, if possible.
  - Set each factor equal to zero and solve the resulting equations.

For example, to solve $x^2 - x = 6$,

$$x^2 - x = 6$$
$$x^2 - x - 6 = 0$$
$$(x + 2)(x - 3) = 0$$
$$x + 2 = 0 \text{ or } x - 3 = 0$$
$$x = -2 \text{ or } x = 3$$

- The solutions to a quadratic equation are often called the **roots** of the equation.

Example

The population, P, of an Ontario city is modelled by $P = 14t^2 + 820t + 42\ 000$, where t is the time in years. When $t = 0$, the year is 2000.

(a) What will the population be in 2008?

(b) What was the population in 1991?

(c) In which year(s) was the population 30 000?

(d) Determine the year when the fewest people lived in the town.

Solution

(a) In 2008, $t = 8$. Substitute $t = 8$ into the model.

$P = 14t^2 + 820t + 42\ 000$

$P = 14(8)^2 + 820(8) + 42\ 000$

$P = 49\ 456$

In 2008, the population will be 49 456.

(b) In 1991, $t = -9$. Substitute $t = -9$ into the model.

$P = 14t^2 + 820t + 42\ 000$

$P = 14(-9)^2 + 820(-9) + 42\ 000$

$P = 35\ 754$

In 1991, the population was 35 754.

(c) To find the year in which the population is 30 000, substitute $P = 30\ 000$ into the model.

$$P = 14t^2 + 820t + 42\ 000$$
$$30\ 000 = 14t^2 + 820t + 42\ 000$$
$$0 = 14t^2 + 820t + 42\ 000 - 30\ 000$$
$$0 = 14t^2 + 820t + 12\ 000$$
$$0 = (2t + 60)(7t + 200)$$

Then $2t + 60 = 0$ or $7t + 200 = 0$.

Solve to get

$2t = -60$ or $7t + 200 = 0$

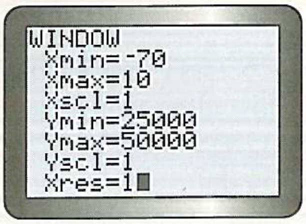
$t = -30$ $\qquad t = -\frac{200}{7}$

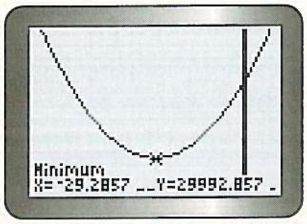
$t \doteq -28.5$

The population was 30 000 twice, once in 1970 (2000 − 30) and once again around 1971 (2000 − 28.5).

(d) The population of the city is at its lowest point on the vertex of the parabola. In this example, we will use a graphing calculator to find this point.

This graph is drawn using the TI-83 Plus and the vertex is determined by tracing. The vertex is at (−29.3, 29 993). So, the population was at its lowest level around 28.8 years before 2000, or around 1971.





Extra Practice

16. Solve each equation.

(a) $x^2 - x - 30 = 0$
(b) $x^2 - 4x - 32 = 0$
(c) $x^2 + 12x + 35 = 0$
(d) $x^2 + 4x = 21$
(e) $-5x = x^2 - 36$
(f) $x^2 = 10x - 25$
(g) $6x^2 + 2 = 7x$
(h) $9x - 4 = -9x^2$
(i) $3x^2 - 39x = -126$

17. Determine all the values of x for each value of y.

(a) $y = x^2 + x - 3$ for $y = 27$
(b) $y = x^2 + 7x - 9$ for $y = -21$
(c) $y = 2x^2 - x + 20$ for $y = 23$
(d) $y = 6x^2 - 13x + 1$ for $y = -5$

18. Graph each relation in question 17 and confirm the values of x that you found for the given values of y.

19. Boris throws a ball vertically upward from the top of a cliff. The height of the ball above the base of the cliff is approximated by the model $h = 65 + 10t - 5t^2$, where h is the height in metres and t is the time in seconds.

(a) How high is the cliff?
(b) How long does it take for the ball to reach a height of 50 m above the base of the cliff?
(c) After how many seconds does the ball hit the ground?

20. Helga owns a campground, and she has installed a rectangular swimming pool measuring 10 m by 20 m. She wants to put a wooden deck of uniform width around the pool.

(a) Helga has budgeted \$1920 to spend on the deck and knows that construction costs are \$30/m^2. What is the widest that the deck can be?
(b) Helga decides that the deck she can build with \$1920 is not big enough, so she budgets \$6000 for the deck. How wide can the deck be now?

Chapter 3 Summary

In this chapter you studied quadratic relationships and saw that they are better described by a curve of best fit than a line of best fit. You can use finite difference tables to determine whether a relationship is best represented by a linear expression, a quadratic expression, or some other type of algebraic expression.

The graph of a quadratic relationship is called a **parabola**. The axis of symmetry of a parabola passes through its vertex, and is the perpendicular bisector of the line segment that joins the zeros. If the relationship is quadratic and the zeros and optimal value are known, the algebraic relationship can be determined. The relationship's equation can be expressed in factored form $y = a(x - s)(x - t)$, where s and t are the zeros, or x-intercepts of the parabola.

To find a quadratic curve of best fit for a set of data, you can use the quadratic regression feature of a graphing calculator. The calculator will give the regression equation in standard form, $y = ax^2 + bx + c$. An equation in standard form can be converted into factored form if it can be factored.

Chapter 3 Review Test

Analyzing and Applying Quadratic Models

1. Determine, without graphing, which type of relationship best models each table of values: linear, quadratic, or neither.

(a)

x	−1	0	1	2	3
y	1	2	−3	−14	−31

(b)

x	−2	−1	0	1	2
y	−10	−2	−4	4	22

(c)

x	0	2	4	6	8
y	3	4	5	6	7

(d)

x	0	1	2	3	4
y	10	7	3	5	−1

2. The zeros of a quadratic relation are -3 and 9. The second differences are positive.

(a) Explain whether the optimal value will be a maximum or a minimum.
(b) What value of the independent variable will produce the optimal value?
(c) Explain whether the optimal value is a negative or positive value.

3. **(a)** Points $(-9, 0)$ and $(19, 0)$ lie on the curve of a parabola. What is the axis of symmetry for the parabola?
(b) What are the zeros of the parabola?
(c) The optimal value of the parabola is -28. Write the algebraic expression of the parabola in standard form.

4. Sketch each graph, using the x-intercepts and the optimal value as reference points. Clearly identify the x-intercepts and vertex.

(a) $y = (x - 6)(x + 2)$ **(b)** $y = (4 + x)(6 - x)$

5. Expand and simplify each expression.

(a) $(2x - 3)(5x + 2)$ **(b)** $(5a + 2b)(3a - 4b)$ **(c)** $-5(x - 6)(2x + 7)$

6. Factor each expression.

(a) $x^2 - 9x + 14$ **(b)** $16x^2 - 25$
(c) $6x^2 + 5x - 4$ **(d)** $2x^2 + 10x + 12$

7. Solve each equation.

(a) $x^2 + 4x - 21 = 0$ **(b)** $x^2 + 12 = -8x$ **(c)** $6x^2 = 5 - 13x$

8. A toy rocket sitting on a tower is launched vertically upward. The table shows its height, h, in metres, at t seconds.

t (s)	0	1	2	3	4	5	6	7	8
h (m)	16	49	57	85	88	81	64	37	0

(a) Justify the choice of a quadratic expression to model this data.

(b) Without graphing technology, make a scatter plot for the data. Manually fit a quadratic curve of best fit to the data. Use the graph to write a quadratic expression to model the data.

(c) Use your expression from (b) to answer these questions.

i. How high is the tower?

ii. When does the rocket reach its maximum height?

iii. What is the maximum height that the rocket reaches?

iv. How long did the flight last?

(d) Use graphing technology to find an equation for the curve of best fit for the data. Compare the answers for (b) to the answers found using technology.

9. The population of Steelsville is modelled by the equation $P = 6t^2 + 110t + 3000$, where P is the population and t is the time in years. When $t = 0$, the year is 2000.

(a) What was the population in 2000?

(b) What will be the population in 2020?

(c) What was the population in 1994?

(d) When will the population be 32 000?

(e) Explain why the population can never be 0 under this model.

10. A ticket to the school dance is \$6 and usually 250 students attend. The dance committee knows that for every \$1 increase in the price of a ticket, 25 fewer students attend the dance. What ticket price maximizes revenue?