3 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: What are the key properties of a quadratic relation?

A: The key properties are:

- In a table of values, the second differences are constant and not zero.
- The degree of the equation that represents the relation is 2.
- The graph has a U shape, which is called a parabola.
- Every parabola has a vertex that is the highest or lowest point on the curve.
- Every parabola has an axis of symmetry that passes through its vertex.

Study *Aid*

- See Lesson 3.1 and Lesson 3.2, Examples 1 to 4.
- Try Mid-Chapter Review Questions 1 and 2.

Q: What information can you easily determine from the factored and standard forms of a quadratic relation?

A: From the standard form $y = ax^2 + bx + c$, you can determine the y-intercept, which is c.

From the factored form $y = a(x - r)(x - s)$, you can determine

- the zeros, or x-intercepts, which are r and s
- the equation of the axis of symmetry, which is $x = \frac{r + s}{2}$
- the coordinates of the vertex, by substituting the value of the axis of symmetry for x in the relation
- the y-intercept, which is $a \times r \times s$

From both forms, you can determine the direction in which the parabola opens: upward when $a > 0$ and downward when $a < 0$.

Study *Aid*

- See Lesson 3.2, Example 2, and Lesson 3.3, Examples 1 to 3.
- Try Mid-Chapter Review Questions 3 to 7.

Q: If you are given information about a quadratic relation, how can you determine the equation?

A: If the graph has zeros, these can be used to write the equation of the quadratic relation in factored form. Then you can use a different point on the parabola to determine the coefficient a.

Study *Aid*

- See Lesson 3.3, Example 4.
- Try Mid-Chapter Review Questions 8 to 10.

EXAMPLE

The points $(-2, 0)$ and $(3, 0)$ are the zeros of a parabola that passes through $(4, 12)$. Determine an equation for the quadratic relation.

Solution

Use the zeros to write the equation $y = a(x + 2)(x - 3)$. Substitute the coordinates of the point $(4, 12)$ into the equation to determine the coefficient a.

$12 = a(4 + 2)(4 - 3)$

$12 = a(6)(1)$

$2 = a$

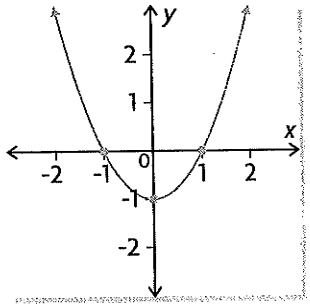
An equation for the quadratic relation is $y = 2(x + 2)(x - 3)$.

PRACTICE Questions

Lesson 3.1

1. State whether each relation is quadratic. Justify your answer.

a)



b) $y = 5x^2 + 3x - 1$

c)

x	0	1	2	3	4	5
y	3	2	1	0	1	2

2. Each table of values represents a quadratic relation. Decide, without graphing, whether the parabola opens upward or downward.

a)

x	−2	−1	0	1	2
y	0	−5	0	15	40

b)

x	−2	−1	0	1	2
y	−3	3	5	3	−3

Lesson 3.2

3. Graph $y = -x^2 + 6x$ to determine

a) the equation of the axis of symmetry

b) the coordinates of the vertex

c) the y-intercept

d) the zeros

4. The points $(-3, 8)$ and $(9, 8)$ lie on opposite sides of a parabola. Determine the equation of the axis of symmetry.

5. Use a graphing calculator to graph each relation. Determine the y-intercept, zeros, equation of the axis of symmetry, and vertex.

a) $y = x^2 + 8x + 15$

b) $y = -2x^2 + 16x - 32$

6. A soccer ball is kicked into the air. Its height, h, in metres, is approximated by the equation $h = -5t^2 + 15t + 0.5$, where t is the time in seconds since the ball was kicked.

a) From what height is the ball kicked?

b) When does the ball hit the ground?

c) When does the ball reach its maximum height?

d) What is the maximum height of the ball?

e) What is the height of the ball at $t = 3$? Is the ball travelling upward or downward at this time? Explain.

f) When is the ball at a height of 10 m?

Lesson 3.3

7. Determine the y-intercept, zeros, equation of the axis of symmetry, and vertex of each quadratic relation. Then sketch its graph.

a) $y = (x - 5)(x + 5)$

b) $y = -(x - 6)(x - 2)$

c) $y = 2(x - 1)(x + 3)$

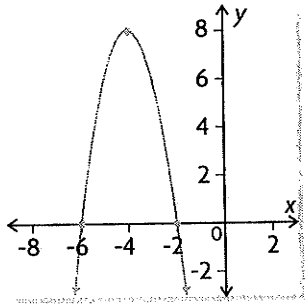
d) $y = -0.5(x + 4)^2$

8. The zeros of a parabola are -10 and 30. The parabola crosses the y-axis at 50.

a) Determine an equation for the parabola.

b) Determine the coordinates of the vertex.

9. Determine an equation for this quadratic relation.



10. Give an example of an equation of a quadratic relation whose vertex and x-intercept occur at the same point.

PRACTICE Questions

Lesson 3.1

1. State whether each relation is quadratic. Justify your decision.

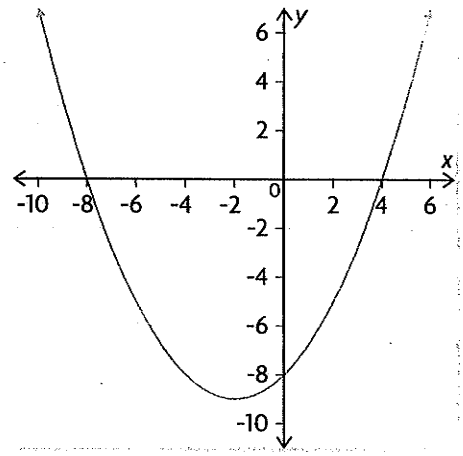
a) $y = 4x - 5$

b)

x	−3	−2	−1	0	1	2	3
y	56	35	18	5	−4	−9	−10

c) $y = 2x(x - 5)$

d)



2. Discuss how the graph of the quadratic relation $y = ax^2 + bx + c$ changes as a, b, and c are changed.

Lesson 3.2

3. Graph each quadratic relation and determine

i) the equation of the axis of symmetry

ii) the coordinates of the vertex

iii) the y-intercept

iv) the zeros

a) $y = x^2 - 8x$ **b)** $y = x^2 + 2x - 15$

4. Verify your results for question 3 using graphing technology.

5. The x-intercepts of a quadratic relation are -2 and 5, and the second differences are negative.

a) Is the y-value of the vertex a maximum value or a minimum value? Explain.

b) Is the y-value of the vertex positive or negative? Explain.

c) Calculate the x-coordinate of the vertex.

6. Create tables of values for three parabolas that go through the point (2, 7). How do you know that each table of values represents a parabola?

7. Use graphing technology to graph the parabola for each relation below. Then determine

i) the x-intercepts

ii) the equation of the axis of symmetry

iii) the coordinates of the vertex

a) $y = -x^2 + 18x$

b) $y = 6x^2 + 15x$

8. What does a in the equation $y = ax^2 + bx + c$ tell you about the parabola?

9. The Rudy Snow Company makes custom snowboards. The company's profit can be modelled with the relation $y = -6x^2 + 42x - 60$, where x is the number of snowboards sold (in thousands) and y is the profit (in hundreds of thousands of dollars).

a) How many snowboards does the company need to sell to break even?

b) How many snowboards does the company need to sell to maximize their profit?

Lesson 3.3

10. The x-intercepts of a parabola are -2 and 7, and the y-intercept is -28.

a) Determine an equation for the parabola.

b) Determine the coordinates of the vertex.

11. Determine an equation for each parabola.

a) The x-intercepts are 5 and 9, and the y-coordinate of the vertex is -2.

b) The x-intercepts are -3 and 7, and the y-coordinate of the vertex is 4.

c) The x-intercepts are -6 and 2, and the y-intercept is -9.

d) The vertex is (4, 0), and the y-intercept is 8.

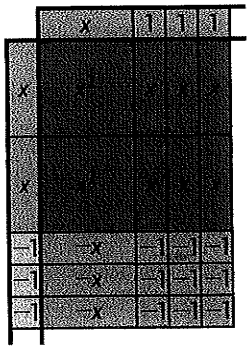
e) The x-intercepts are -3 and 3, and the parabola passes through the point (2, 20).

12. A bus company usually transports 12 000 people per day at a ticket price of $1. The company wants to raise the ticket price. For every $0.10 increase in the ticket price, the number of riders per day is expected to decrease by 400. Calculate the ticket price that will maximize revenue.

Lesson 3.4

13. Identify the binomial factors and their products.

a)



b)

	5x	−6
3x	$15x^2$	−18x
−4	−20x	24

14. Expand and simplify.

a) $(x + 5)(x + 4)$ **d)** $(4x + 5)(3x - 2)$

b) $(x - 2)(x - 5)$ **e)** $(4x - 2y)(5x + 3y)$

c) $(2x - 3)(2x + 3)$ **f)** $(6x - 2)(5x + 7)$

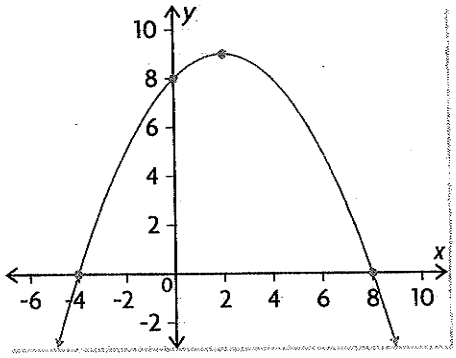
15. Expand and simplify.

a) $(2x + 6)^2$

b) $-2(-2x + 5)(3x + 4)$

c) $2x(4x - y)(4x + y)$

16. Determine the equation of the parabola. Express your answer in standard form.



Lesson 3.5

17. A model rocket is shot straight up into the air. The table shows its height, y, in metres after x seconds.

Time (s)	0	1	2	3	4	5	6
Height (m)	0.0	25.1	40.4	45.9	41.6	27.5	3.6

a) Sketch a curve of good fit.

b) Is the curve of good fit a parabola? Explain.

c) Determine the equation of your curve of good fit. Express your answer in standard form.

d) Estimate the height of the rocket after 4.5 s.

e) When is the rocket at a height of 20 m?

18. A sandbag is dropped into the ocean from a hot air balloon to make the balloon rise. The table shows the height of the sandbag at different times as it falls.

Time (s)	0	2	4	6	8	10
Height (m)	1200	1180	1120	1020	880	700

a) Draw a scatter plot of the data.

b) Sketch a curve of good fit.

c) Is the curve of good fit a parabola? Explain.

d) Determine the equation of your curve of good fit. Express your answer in standard form.

e) Estimate the time when the sandbag hits the water.

Lesson 3.6

19. Evaluate. Express your answers in rational form.

a) 2^{-3} **d)** $(-9)^0$

b) -5^{-1} **e)** 4^{-3}

c) $\left(\frac{2}{5}\right)^{-2}$ **f)** $-\left(\frac{1}{6}\right)^{-2}$

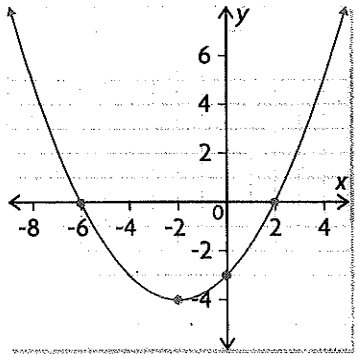
20. Which do you think is greater: $\left(\frac{1}{4}\right)^2$ or 3^{-2}?

Justify your decision.

21. For what postive values of x is x^2 greater than 2^x? How do you know?

3 Chapter Self-Test

1. State the zeros, vertex, and equation of the axis of symmetry of the parabola at the right.



2. The points $(-9, 0)$ and $(19, 0)$ lie on a parabola.

a) Determine an equation for its axis of symmetry.

b) The y-coordinate of the vertex is -28. Determine an equation for the parabola in factored form.

c) Write your equation for part b) in standard form.

3. Decide, without graphing, whether each data set can be modelled by a quadratic relation. Explain how you made your decision.

a)

x	-1	0	1	2	3
y	1	2	-3	-14	-31

b)

x	0	1	2	3	4
y	-4	-3	0	5	12

4. Sketch each graph. Label the intercepts and the vertex using their coordinates.

a) $y = (x - 6)(x + 2)$ **b)** $y = -(x - 6)(x + 4)$

5. The population, P, of a city is modelled by the equation $P = 14t^2 + 820t + 42\,000$, where t is the time in years. When $t = 0$, the year is 2008.

a) Determine the population in 2018.

b) When was the population about 30 000?

6. Expand and simplify.

a) $(2x - 3)(5x + 2)$ **b)** $(3x - 4y)(5x + 2y)$ **c)** $-5(x - 4)^2$

7. A toy rocket is placed on a tower and launched straight up. The table shows its height, y, in metres above the ground after x seconds.

Time, x (s)	0	1	2	3	4	5	6	7	8
Height, y (m)	16	49	72	85	88	81	64	37	0

a) What is the height of the tower?

b) How long is the rocket in flight?

c) Do the data in the table represent a quadratic relation? Explain.

d) Create a scatter plot. Then draw a curve of good fit.

e) Determine the equation of your curve of good fit.

f) What is the maximum height of the rocket?

8. In what ways is modelling a problem using a quadratic relation similar to using a linear relation? In what ways is it different?

9. Evaluate.

a) 7^{-2} **b)** -3^0 **c)** $-\left(\frac{2}{3}\right)^{-4}$ **d)** -5^{-3}

Process Checklist

- ✔ Question 2: Did you relate the characteristics of the graphical **representation** of the relation with its equation?
- ✔ Questions 5 and 7: Did you select appropriate **problem solving** strategies for each situation?
- ✔ Question 8: Did you make **connections** to **communicate** a variety of ways to relate modelling with linear and quadratic relations?