2.7 Using Coordinates to Solve Problems

GOAL

Use properties of lines and line segments to solve problems.

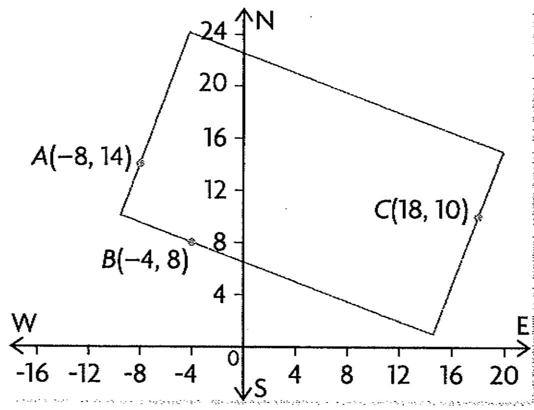
YOU WILL NEED

- grid paper
- ruler

LEARN ABOUT the Math

Rebecca is designing a parking lot. A tall mast light will illuminate the three entrances, which will be located at points *A*, *B*, and *C*. Rebecca needs to position the lamp so that it illuminates each entrance equally.

Parking Lot Entrances



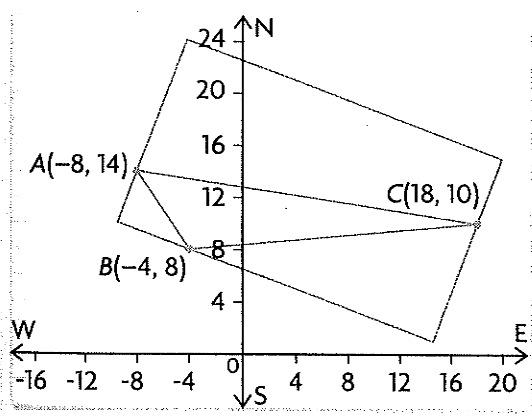
? How can Rebecca determine the location of the lamp?

EXAMPLE 1 **Solving a problem using a triangle property**

Determine the location of the lamp in the parking lot that Rebecca is designing.

Jack's Solution

The lamp should be placed the same distance from all three vertices of $\triangle ABC$.



If the lamp is the same distance from all three vertices, I reasoned that it would be at the centre of a circle that passes through all three vertices. I remembered that this point occurs where the perpendicular bisectors of the sides of the triangle intersect.

I decided to determine the perpendicular bisectors of *AB* and *BC*. I started with *AB*.

The midpoint of *AB* is

$$\left(\frac{-8 + (-4)}{2}, \frac{14 + 8}{2}\right) = (-6, 11).$$

To write an equation, I needed the slope and one point on the perpendicular bisector. I knew that the midpoint of *AB* would be on the perpendicular bisector, so I calculated this first.

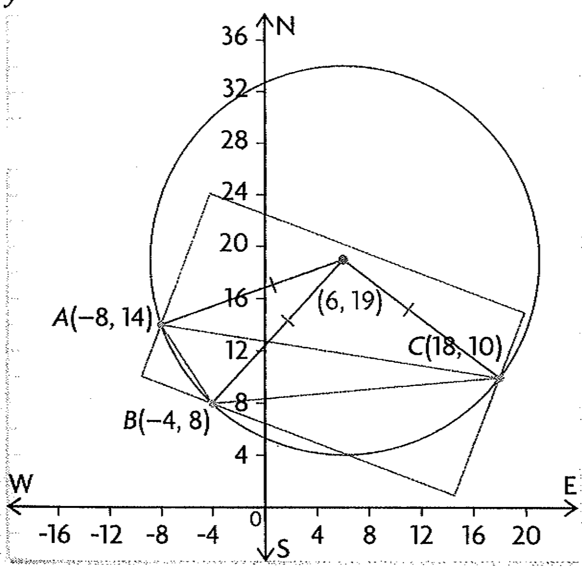
$$y = \frac{2}{3}x + 15$$
$$y = -11x + 86$$

To determine where the two perpendicular bisectors intersect, I set up their equations as a system of equations. I used the method of substitution to solve this system of equations.

At the point of intersection,

$$\frac{2}{3}x + 15 = -11x + 86$$
$$\frac{35}{3}x = 71$$
$$3\left(\frac{35}{3}\right)x = 3(71)$$
$$35x = 213$$
$$x = \frac{213}{35}$$
$$x \doteq 6.09$$
$$y = \frac{2}{3}\left(\frac{213}{35}\right) + 15$$
$$y = \frac{142}{35} + 15$$
$$y \doteq 19.06$$

First, I determined *x*. Then I substituted the value of *x* into the equation of the perpendicular bisector of *AB* to determine the value of *y*. I used the fractional value for *x* to minimize any rounding error.



If the lamp is placed at (6, 19), it will be about the same distance from each entrance. It will illuminate each entrance equally.

I rounded the values of *x* and *y* to the nearest integer.

Reflecting

A. Why is the intersection of two of the perpendicular bisectors the centre of the circle that Rebecca wants?

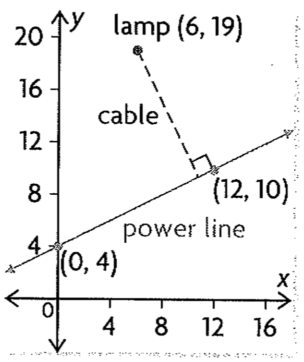
B. Why did Jack only need to determine the intersection of two of the perpendicular bisectors for the triangle?

APPLY the Math

EXAMPLE 2 Solving a problem using coordinates

The closest power line to the parking lot in Example 1 runs along a straight line that contains points (0, 4) and (12, 10). At what point on the power line should the cable from the lamp be connected? If each unit represents 1 m, how much cable will be needed to reach the power line? Round your answers to the nearest tenth.

Eden's Solution



I drew a diagram. The shortest distance from the lamp to the power line is the perpendicular distance. I drew this on my diagram.

To calculate the perpendicular distance, I had to determine the point where the perpendicular line intersects the power line. To do this, I had to determine the equations for the cable and the power line.

$$m = \frac{10 - 4}{12 - 0}$$
$$= \frac{6}{12}$$
$$= \frac{1}{2}$$

I determined the equation for the power line first. I already knew that the *y*-intercept is 4, so I just had to calculate the slope.

The equation of the power line is $y = \frac{1}{2}x + 4$.

The cable is perpendicular to the power line, so the slope of the equation for the power line is -2.

The cable from the lamp is perpendicular to the power line. The slope of the equation for the cable is the negative reciprocal of $\frac{1}{2}$, which is -2.

Therefore, an equation for the perpendicular line is $y = -2x + b$.

The point (6, 19) is on this line, so

$19 = -2(6) + b$
$19 = -12 + b$
$31 = b$

I used this slope to write an equation for the cable. Then I substituted the coordinates for the lamp into the equation to determine the value of *b*.

The equation of the perpendicular line from (6, 19) to the power line is $y = -2x + 31$.

$$y = \frac{1}{2}x + 4$$
$$y = -2x + 31$$
$$\frac{1}{2}x + 4 = -2x + 31$$
$$\frac{5}{2}x = 27$$
$$x = \frac{54}{5}$$
$$x = 10.8$$

I used substitution to solve the system of equations and determine the point where the two lines intersect.

The corresponding value of y is $y = \frac{1}{2}(10.8) + 4$

$= 9.4$

The cable from the lamp should be connected to the power line at point (10.8, 9.4).

$$\begin{aligned}\text{Length of cable} &= \sqrt{(10.8 - 6)^2 + (9.4 - 19)^2} \\ &= \sqrt{23.04 + 92.16} \\ &= \sqrt{115.2} \\ &\doteq 10.73\end{aligned}$$

I used the distance formula to calculate the length of cable that will be needed. I rounded my answer up to the nearest tenth of a metre to make sure I had extra cable.

About 10.8 m of cable will be needed to connect the lamp to the power line.

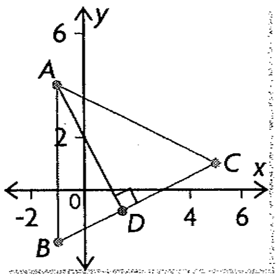
In Summary

Key Idea

- You can use the properties of lines and line segments to solve multi-step problems when you can use coordinates for some or all of the given information in the problem.

Need to Know

- When solving a multi-step problem, you may find it helpful to follow these steps:
  - Read the problem carefully, and make sure that you understand it.
  - Make a plan to solve the problem, and record your plan.
  - Carry out your plan, and try to keep your work organized.
  - Look over your solution, and check that your answers seem reasonable.
- Drawing a graph and labelling it with the given information may help you plan your solution and check your results.
- You may need to determine the coordinates of a point of intersection before using the formulas for the slope and length of a line segment.



CHECK Your Understanding

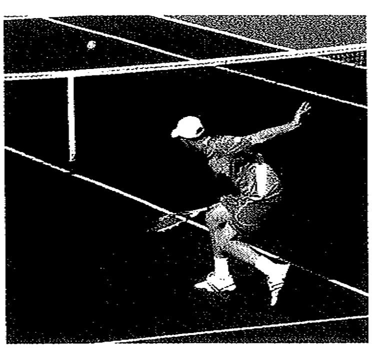
Questions 1 to 5 refer to the diagram at the left.

$\triangle ABC$ has vertices at $A(-1, 4)$, $B(-1, -2)$, and $C(5, 1)$. The altitude from vertex A meets BC at point D.

1. a) Determine the slope of BC.
   b) Determine the slope of AD.
   c) Determine the equation of the line that contains AD.
2. Determine the equation of the line that contains BC.
3. Determine the coordinates of point D.
4. Determine the lengths of BC and AD.
5. Determine the area of $\triangle ABC$.

PRACTISING

6. A triangle has vertices at $A(-3, 2)$, $B(-5, -6)$, and $C(5, 0)$.
   a) Determine the equation of the median from vertex A.
   b) Determine the equation of the altitude from vertex A.
   c) Determine the equation of the perpendicular bisector of BC.
   d) What type of triangle is $\triangle ABC$? Explain how you know.
7. Points $P(-9, 2)$ and $Q(9, -2)$ are endpoints of a diameter of a circle.
   K a) Write the equation of the circle.
   b) Show that point $R(7, 6)$ is also on the circle.
   c) Show that $\angle PRQ$ is a right angle.
8. $\triangle LMN$ has vertices at $L(3, 4)$, $M(4, -3)$, and $N(-4, -1)$. Use analytic geometry to determine the area of the triangle.
9. $\triangle DEF$ has vertices at $D(2, 8)$, $E(6, 2)$, and $F(-3, 2)$. Use analytic geometry to determine the coordinates of the orthocentre (the point where the altitudes intersect).
10. $\triangle PQR$ has vertices at $P(-12, 6)$, $Q(4, 0)$, and $R(-8, -6)$. Use analytic geometry to determine the coordinates of the centroid (the point where the medians intersect).
11. $\triangle JKL$ has vertices at $J(-2, 0)$, $K(2, 8)$, and $L(7, 3)$. Use analytic geometry to determine the coordinates of the circumcentre (the point where the perpendicular bisectors intersect).



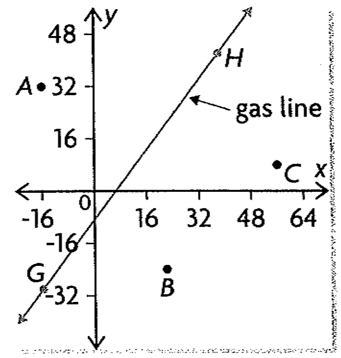
12. A university has three student residences, which are located at points $A(2, 2)$, $B(10, 6)$, and $C(4, 8)$ on a grid. The university wants to build a tennis court an equal distance from all three residences. Determine the coordinates of the tennis court.

13. Explain two different strategies you could use to show that points
C *D*, *E*, and *F* lie on the same circle, with centre *C*.

14. A design plan for a thin triangular computer component shows
A the vertices at points (8, 12), (12, 4), and (2, 8). Determine the coordinates of the centre of mass.

15. A stained glass window is in the shape of a triangle, with vertices at $A(-1, -2)$, $B(-2, 1)$, and $C(5, 0)$. $\triangle XYZ$ is formed inside $\triangle ABC$ by joining the midpoints of the three sides. The glass that is used for $\triangle XYZ$ is blue, but the remainder of $\triangle ABC$ is green. Determine the ratio of green to blue glass used.

16. Three homes in a rural area, labelled *A*, *B*, and *C* in the diagram at the right, are converting to natural gas heating. They will be connected to the gas line labelled *GH* in the diagram. On a plan marked out in metres, the coordinates of the points are $A(-16, 32)$, $B(22, -24)$, $C(56, 8)$, $G(-16, -30)$, and $H(38, 42)$.
    a) Determine the length of pipe that the gas company will need to connect the three houses to the gas line. Which homeowner will have the highest connection charge?
    b) Determine the best location for a lamp to illuminate the three homes equally.



17. Determine the type of triangle that is formed by the lines $x + y = 11$, $x - y = 1$, and $x - 3y = 3$. Justify your decision.

18. Archaeologists on a dig have found an outside fragment of an ancient
T circular platter. They want to construct a replica of the platter for a display. How could they use coordinates to calculate the diameter of the platter? Include a diagram in your explanation.

Career **Connection**

An archaeologist searches for clues about the lives of people in past civilizations. Most archaeologists are employed by a university or a museum.

19. Suppose that you know the coordinates of the vertices of a triangle. Describe the strategy you would use to determine the equation of each median and altitude that can be drawn from each vertex of the triangle to the opposite side.

Extending

20. A triangle has vertices at $P(-1, 2)$, $Q(4, -4)$, and $R(1, 2)$. Show that the centroid divides each median in the ratio 2:1.

21. A circle is defined by the equation $x^2 + y^2 = 10a^2$.
    a) Show that *RQ*, with endpoints $R(3a, a)$ and $Q(a, -3a)$, is a chord in the circle.
    b) Show that the line segment joining the centre of the circle to the midpoint of *RQ* is perpendicular to *RQ*.