1.3 Graphically Solving Linear Systems

GOAL

Use graphs to solve a pair of linear equations simultaneously.

YOU WILL NEED

- grid paper
- ruler
- graphing calculator

INVESTIGATE the Math

Matt's health-food store sells roasted almonds for \$15/kg and dried cranberries for \$10/kg.

❓ How can he mix the almonds and the cranberries to create 100 kg of a mixture that he can sell for \$12/kg?

A. Let x represent the mass of the almonds. Let y represent the mass of the cranberries.
  i) Write an equation for the total mass of the mixture.
  ii) Write an equation for the total value of the mixture.

B. Graph your equation of the total mass for part A. What do the points on the line represent?

C. Graph your equation of the total value for part A on the same axes. What do the points on this line represent?

D. Identify the coordinates of the point where the two lines intersect. State what each value represents. How accurately can you estimate these values from your graph?

E. The equations for part A form a **system of linear equations**. Explain why the coordinates for part D give the **solution to a system of linear equations**.

F. Substitute the coordinates into each equation to verify your solution.

system of linear equations

a set of two or more linear equations with two or more variables
For example, $x + y = 10$
$4x - 2y = 22$

solution to a system of linear equations

the values of the variables in the system that satisfy all the equations
For example, (7, 3) is the solution to
$x + y = 10$
$4x - 2y = 22$

Reflecting

G. Explain why you needed two linear relations to describe the problem.

H. Explain how graphing both relations on the same axes helped you solve the problem.

I. Explain why the coordinates of the point of intersection provide an ordered pair that satisfies both relations.

APPLY the Math

EXAMPLE 1 **Selecting a graphing strategy** to solve a linear system

Solve the system $y = 2x + 1$ and $x + 2y = -8$ using a graph.

Leslie's Solution

$$y = 2x + 1$$

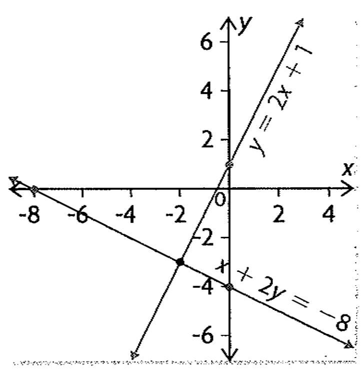
The slope of the line is 2.	At the y-intercept, $x = 0$.
$\frac{2}{1} = \frac{\text{rise}}{\text{run}}$	$y = 2(0) + 1$
	$y = 1$

I determined the slope and the y-intercept of the first equation.

$$x + 2y = -8$$

At the x-intercept, $y = 0$.	At the y-intercept, $x = 0$.
$x + 2(0) = -8$	$0 + 2y = -8$
$x = -8$	$\frac{2y}{2} = -\frac{8}{2}$
	$y = -4$

I determined the x- and y-intercepts of the second equation.



I graphed the first line (blue) by plotting the y-intercept and using the rise and run to plot another point on the line.

I graphed the second line (red) by plotting points $(-8, 0)$ and $(0, -4)$ and joining them with a straight line.

At the point of intersection, $x = -2$ and $y = -3$.
The solution is $(-2, -3)$.

I located the point of intersection and read its coordinates using the axes of my graph.

$y = 2x + 1$		$x + 2y = -8$	
Left Side	Right Side	Left Side	Right Side
y	$2x + 1$	$x + 2y$	-8
$= -3$	$= 2(-2) + 1$	$= -2 + 2(-3)$	
	$= -3$	$= -8$	

I checked my solution by substituting the x- and y-values into each equation.

EXAMPLE 2 Solving a problem using a graphing strategy

Ellen drives 450 km from her university in Kitchener-Waterloo to her home in Smiths Falls. She travels along one highway to Kingston at 100 km/h and then along another highway to Smiths Falls at 80 km/h. The journey takes her 4 h 45 min. What is the distance from Kingston to Smiths Falls?

Bob's Solution

Let x represent the distance that Ellen travels at 100 km/h. Let y represent the distance that she travels at 80 km/h.

I used letters to identify the variables in this situation.

The total trip is 450 km, so $x + y = 450$.

I wrote an equation for the total distance.

$$\frac{x}{100} + \frac{y}{80} = 4\frac{3}{4}$$

Since speed $= \frac{\text{distance}}{\text{time}}$, then time $= \frac{\text{distance}}{\text{speed}}$.
I wrote an equation to describe the total time (in hours) for her trip, where $\frac{x}{100}$ is the time spent driving at 100 km/h and $\frac{y}{80}$ is the time spent driving at 80 km/h.

$$x + y = 450$$

At the x-intercept, $y = 0$.

$$x + 0 = 450$$
$$x = 450$$

At the y-intercept, $x = 0$.

$$0 + y = 450$$
$$y = 450$$

I determined the x- and y-intercepts of the first equation.

$$\frac{x}{100} + \frac{y}{80} = 4\frac{3}{4}$$

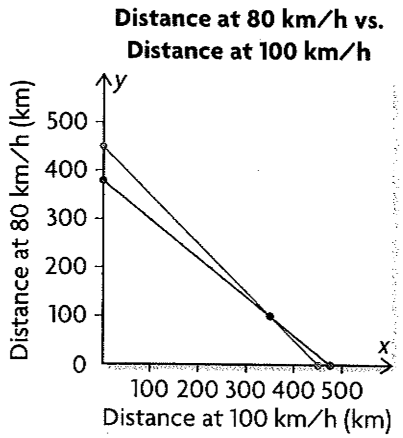
At the x-intercept, $y = 0$.

$$\frac{x}{100} + 0 = 4\frac{3}{4}$$
$$x = 100\left(4\frac{3}{4}\right)$$
$$x = 100\left(\frac{19}{4}\right)$$
$$x = 475$$

At the y-intercept, $x = 0$.

$$0 + \frac{y}{80} = 4\frac{3}{4}$$
$$y = 80\left(4\frac{3}{4}\right)$$
$$y = 80\left(\frac{19}{4}\right)$$
$$y = 380$$

I determined the x- and y-intercepts of the second equation.



I graphed each equation by plotting the x- and y-intercepts and joining them with a straight line.

The point of intersection is (350, 100), so the distance from Kingston to Smiths Falls is 100 km.

I determined the coordinates of the point of intersection. The y-coordinate of the point is the distance.

EXAMPLE 3 Selecting graphing technology to solve a system of linear equations

Hayley wants to rent a car for a weekend trip. Kelly's Kars charges \$95 for the weekend plus \$0.15/km. Rick's Rentals charges \$50 for the weekend plus \$0.26/km. Which company charges less?

Elly's Solution

Let x represent the distance driven in kilometres. Let y represent the total cost of the car rental in dollars.

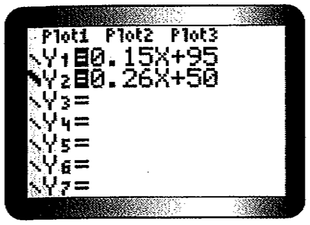
I chose x and y for the variables.

The cost to rent a car from Kelly's Kars is $y = 0.15x + 95$.

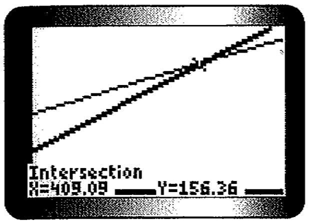
I wrote an equation for the cost to rent a car from Kelly's Kars. \$0.15x represents the distance charge and \$95 represents the weekend fee.

The cost to rent a car from Rick's Rentals is $y = 0.26x + 50$.

I wrote an equation for the cost to rent a car from Rick's Rentals. \$0.26x represents the distance charge and \$50 represents the weekend fee.



I used a graphing calculator. I entered the equation for Kelly's Kars in Y1 and the equation for Rick's Rentals in Y2. I used a thick line for Rick's Rentals to distinguish between the two lines.



The point of intersection is (409, 156), to the nearest whole numbers.

I graphed the lines and adjusted the window settings so that I could see both lines and the point of intersection.

The point of intersection occurred when $0 \leq X \leq 600$ and $0 \leq Y \leq 200$. I used the Intersect operation to determine the point of intersection.

If Hayley drives 409 km, both companies charge about $156.

If she plans to drive less than 409 km, Rick's Rentals charges less.

If she plans to drive more than 409 km, Kelly's Kars charges less.

I looked at my graph to determine which line is lower before and after the point of intersection.

Before the point of intersection, the thick line is lower, so Rick's Rentals charges less. After the point of intersection, the thin line is lower, so Kelly's Kars charges less.

Tech | *Support*

To graph with a thick line using a TI-83/84 graphing calculator, scroll to the left of Y2 and press ENTER.

Tech | *Support*

For help using a TI-83/84 graphing calculator to determine the point of intersection, see Appendix B-11. If you are using a TI-*n*spire, see Appendix B-47.

In Summary

Key Idea

- You can solve a system of linear equations by graphing both equations on the same axes. The ordered pair (x, y) at the point of intersection gives the solution to the system.

Need to Know

- You may not be able to determine an accurate solution to a system of equations using a hand-drawn graph.
- To determine an accurate solution to a system of equations, you can use graphing technology. When you use a graphing calculator, express the equations in slope y-intercept form.

CHECK Your Understanding

1. Decide whether each ordered pair is a solution to the given system of equations.

a) $(2, -1)$; $3x + 2y = 4$ and $-x + 3y = -5$

b) $(1, 4)$; $x + y = 5$ and $2x + 2y = 8$

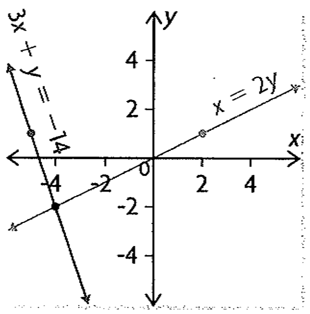
c) $(1, -2)$; $y = 3x - 5$ and $y = 2x - 4$

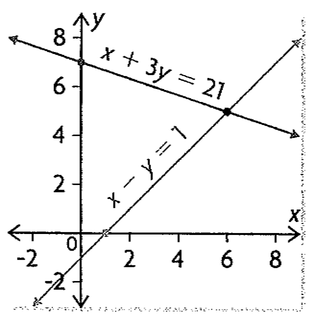
d) $(10, 5)$; $x - y = 5$ and $y = 5x - 40$

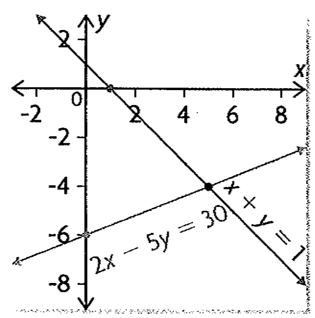
2. For each graph:

i) Identify the point of intersection.

ii) Verify your answer by substituting into the equations.

a)


b)


c)


PRACTISING

3. a) Graph the system $x + y = 5$ and $3x + 4y = 12$ by hand.

K **b)** Solve the system using your graph.

c) Verify your solution using graphing technology.

4. Alex needs to rent a minivan for a week to take his band on tour. Easyvans charges \$230 plus \$0.10/km. Cars for All Seasons charges \$150 plus \$0.22/km.

a) Write an equation for each rental company.

b) Graph your equations.

c) Which rental company would you recommend to Alex? Explain.

5. Solve each linear system by graphing.

a) $x + y = 3$
$x - y = 7$

b) $x + y = 8$
$4x - 2y = 8$

c) $y = 2x - 4$
$3x + y = 6$

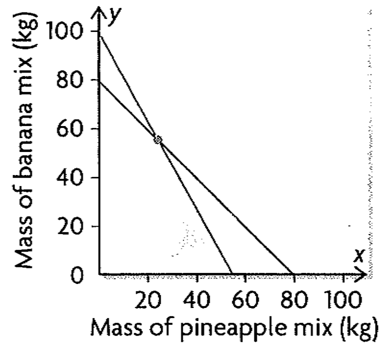
d) $2x + y = 10$
$y = x - 2$

e) $y = 3x - 5$
$y = -2x + 5$

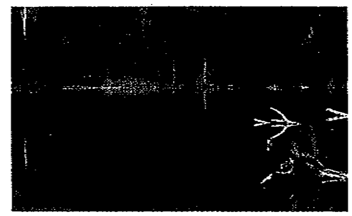
f) $6x - 5y - 12 = 0$
$-2x + 5y + 2 = 0$

6. Austin is creating a new "trail mix" by combining two of his best-selling blends: a pineapple–coconut–macadamia mix that sells for \$18/kg and a banana–papaya–peanut mix that sells for \$10/kg. He is making 80 kg of the new mix and will sell it for \$12.50/kg. Austin uses the graph shown at the right to determine how much of each blend he needs to use.
   a) Write the equations of the linear relations in the graph.
   b) From the graph, how much of each blend will Austin use?

Trail Mix



7. At Jessica's Java, a new blend of coffee is featured each week. This week, Jessica is creating a low-caffeine espresso blend from Brazilian and Ethiopian beans. She wants to make 200 kg of this blend and sell it for \$15/kg. The Brazilian beans sell for \$12/kg, and the Ethiopian beans sell for \$17/kg. How many kilograms of each kind of bean must Jessica use to make 200 kg of her new blend of the week?



8. When Arthur goes fishing, he drives 393 km from his home in Ottawa to a lodge near Temagami. He travels at an average speed of 70 km/h along the highway to North Bay and then at 50 km/h on the narrow road from North Bay to Temagami. The journey takes him 6 h.
   a) Write two equations to describe this situation.
   b) Graph your equations.
   c) Use your graph to determine the distance from North Bay to Temagami.

9. Joanna is considering two job offers. Phoenix Fashions offers \$1500/month plus 2.5% commission. Styles by Rebecca offers \$1250/month plus 5.5% commission.
   a) Create a linear system by writing an equation for each salary.
   b) What value of sales would result in the same total salary for both jobs?
   c) Which job should Joanna take? Explain your answer.

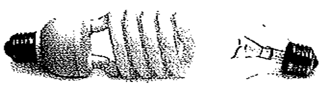
10. T Create a situation you can represent by a system of linear equations that has the ordered pair (10, 15) as its solution.

11. Six cups of coffee and a dozen muffins originally cost \$15.35. The price of a cup of coffee increases by 10%. The price of a dozen muffins increases by 12%. The new cost of six cups of coffee and a dozen muffins is \$17.06. Determine the new price of one cup of coffee and a dozen muffins.

12. Willow bought 3 m of denim fabric and 5 m of cotton fabric. The total bill, excluding tax, was \$22. Jared bought 6 m of denim fabric and 2 m of cotton fabric at the same store for \$28.
   a) Write a linear system you can solve to determine the price of denim fabric and the price of cotton fabric.
   b) Solve your system using a graph.
   c) How much will 8 m of denim fabric and 5 m of cotton fabric cost?

13. The drama department of a school sold 679 tickets to the school play, for a total of \$3370. Students paid \$4 for a ticket, and non-students paid \$7.
   a) Write a linear system for this situation.
   b) How many non-students attended the play? Solve the problem by graphing your system.

14. The equations $y = 2$, $y = 4x - 2$, and $y = -2x + 10$ form the sides
A of a triangle.
   a) Graph the triangle, and determine the coordinates of the vertices.
   b) Calculate the area of the triangle.



Environment ***Connection***

Fluorescent light bulbs decrease energy use and reduce pollution levels.

15. A regular light bulb costs \$0.65 to buy, plus \$0.004/h for the
C electricity to make it work. A fluorescent light bulb costs \$3.99 to buy, plus \$0.001/h for the electricity.
   a) Write a cost equation for each type of light bulb.
   b) Graph the system of equations using a graphing calculator. Use the window settings $0 \leq X \leq 2000$ and $0 \leq Y \leq 10$.
   c) After how long is the fluorescent light bulb cheaper than the regular light bulb?
   d) Determine the difference in cost after one year of constant use.

16. Rearrange the following sentences to describe the correct sequence of steps for solving a problem by graphing a linear system. Discard any sentences that do not belong in the description. Add any sentences that are needed to make the description clearer.
   - Label the graph.
   - Verify the solution by substituting into both equations.
   - Write two equations that describe the situation in the problem.
   - Determine the slope of each line by calculating the rise over the run.
   - Read the problem, and determine what you need to find.
   - Graph both equations on the same set of axes.
   - Choose the best strategy to graph each equation.
   - Determine the coordinates of the point of intersection.

Extending

17. a) Solve the linear system $3x - y - 11 = 0$ and $x + 2y + 1 = 0$.
   b) Show that the line with the equation $9x + 4y - 19 = 0$ passes through the point where the lines in part a) intersect.
   c) Determine the values of c and d if $9x + 4y - 19 = 0$ is written in the form $c(3x - y - 11) + d(x + 2y + 1) = 0$.

18. Solve the linear system $y = 2x - 1$, $4x - 3y = 7$, and $6x + y + 17 = 0$.

19. Solve each system of equations.
   a) $y = 2x^2$
      $y = -3x + 5$
   b) $y = \sqrt{x}$
      $y = x - 1$