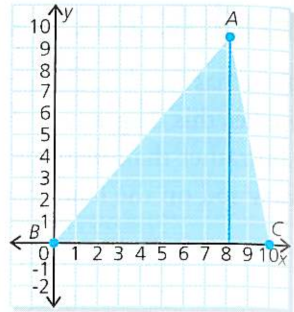
6. Generalize your results.

Can you always use the method in steps 1 to 5 to find the area of a non-right triangle? Are there triangles for which the method will not work?



(a) See if you can find an example for which this method does not work. Try various acute and obtuse triangles.

(b) Describe how to use the method above to find the area of ΔABC. Assume that the lengths of AB and BC, and the size of $\angle ABC$ are known, but the length of the altitude AD is not known.

(c) Write an equation or a formula that corresponds to your result in (b).

Focus 5.8

Key Ideas

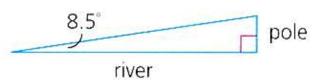
- If a problem involving the computation of a length or an angle can be represented with a right triangle, then it can be solved using trigonometric ratios.

Example 1

To evacuate some refugees, a bridge needs to be built across a river. The first step is to find out how wide the river is. A surveyor is on one side of the river, with a transit mounted on a tripod 1.2 m above the ground. An assistant stands on the other side of the river, holding a 3 m pole vertically. The angle of elevation from the transit to the top of the pole is 8.5°. How wide is the river?

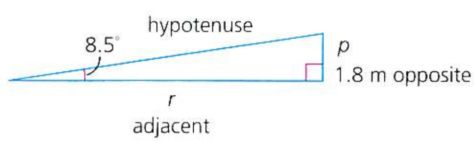
Solution

Sketch the situation.



The pole is 3 m tall, but the transit is 1.2 m above the ground. So, the actual height of the pole in the diagram is 3.0 m − 1.2 m or 1.8 m.

The triangle formed by the pole, the width of the river, and the line of sight from the transit to the top of the pole is right angled. So, the tangent ratio can be used.



Let r represent the width of the river. Let p represent the height of the pole.

$$\tan 8.5° = \frac{\text{opposite}}{\text{adjacent}} \qquad \text{Set up the ratio.}$$

$$\tan 8.5° = \frac{p}{r}$$

$$\tan 8.5° = \frac{1.8}{r}$$

$$r \tan 8.5° = \left(\frac{1.8}{\cancel{r}_1}\right)\cancel{r}^1$$

$$r = \frac{1.8}{\tan 8.5°}$$

$$r = \frac{1.8}{0.149\,451}$$

$$r = 12.044$$

$$r \doteq 12$$

The river is about 12 m wide.

Example 2

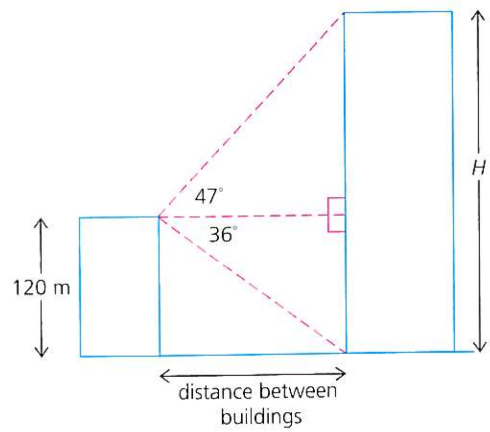
A video camera is mounted on the top of a 120 m tall building. When the camera tilts down 36° with the horizontal, it views the bottom of another building. If it tilts up 47° with the horizontal, it can view the top of the same building.

(a) How far apart are the two buildings?

(b) How tall is the building viewed by the camera?

Solution

Sketch the situation.



(a) Let x represent the distance between the buildings.
Then, in the right triangle shown,

$$\tan 36° = \frac{120}{x}$$

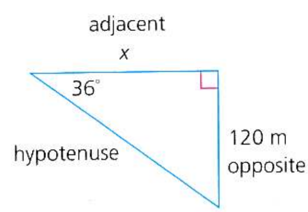
$$x \tan 36° = \left(\frac{120}{\cancel{x}_1}\right)\cancel{x}^1$$

$$x = \frac{120}{\tan 36°}$$

$$x = \frac{120}{0.726\,542\,25}$$

$$x = 165.165\,83\ldots$$

$$x \doteq 165$$



The buildings are about 165 m apart.

(b) Apply the value of x to the second triangle. Let h represent the difference in height between the two buildings. Let H represent the height of the taller building.

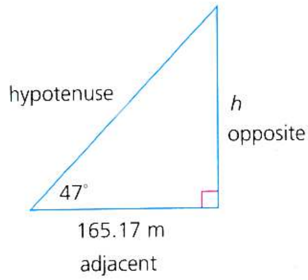
$$\tan 47° = \frac{h}{165}$$ Set up the ratio and substitute the known values.

$$h = 165 \tan 47°$$ Solve for h.

$$h \doteq 177$$

$$H = 120 + 177$$ Add the two heights.

$$H = 297$$



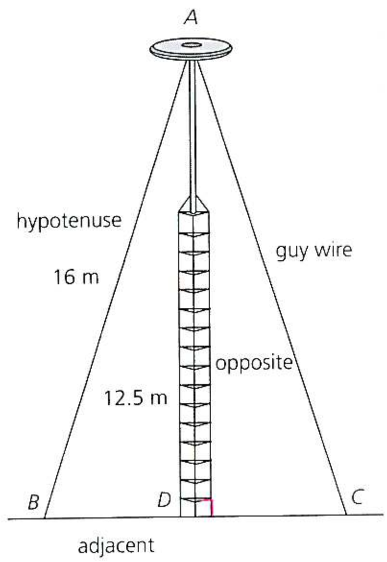
The second building is about 297 m high.

Example 3

A communications antenna is attached to the roof of a school and held in place with two 16 m guy wires. The antenna is 12.5 m tall.

(a) What angle do the wires make with the roof?

(b) At what distance from the base of the tower should the wires be secured to the roof?



Solution

(a) Draw a diagram. Name the sides of the triangle relative to $\angle B$ or $\angle C$. In this case, $\angle B$ is used in ΔABD.

Find the measure of $\angle B$.

$$\frac{12.5}{16} = \frac{\text{opposite}}{\text{hypotenuse}}$$ This is the sine of $\angle B$.

$$\sin B = \frac{12.5}{16}$$

$$\sin B = 0.781\ 25$$

$$\angle B = \sin^{-1}(0.781\ 25)$$

$$\angle B \doteq 51°$$

The wires make an angle of 51° with the roof.

(b) The distance to be determined corresponds to BD in the diagram.

$$\frac{BD}{16} = \frac{\text{adjacent}}{\text{hypotenuse}}$$ This is the cosine of $\angle B$.

$$\cos B = \frac{BD}{16}$$ $\angle B = 51°$

$$\cos 51° = \frac{BD}{16}$$

$$16(\cos 51°) = \overset{1}{\cancel{16}}\left(\frac{BD}{\cancel{16}_1}\right)$$

$$16(0.6293) = BD$$

$$10 \text{ m} \doteq BD$$

The wires should be attached around 10 m from the base of the antenna.

Practise, Apply, Solve 5.8

A

1. A tree that is 8.5 m tall casts a shadow 6 m long. At what angle are the sun's rays hitting the ground?

2. **Communication:** Jasmine is planning to do some rock climbing. Before she scales the cliff, she paces off 100 m from the base of the cliff and sights the top with a clinometer. The angle of elevation to the top is 80°. How high is the cliff? Show all your steps.

3. A ramp rises 2.5 m for every 5.5 m of run. What is the slope angle of the ramp?

4. A local building code states that the maximum slope for a set of stairs in a home is a 72 cm rise for every 100 cm of run. To one decimal place, what is the maximum angle at which a set of stairs can rise?

5. In Mexico, one of the Maya pyramids at Chichen Itza has stairs that rise about 64 cm for every 71 cm of run. Find the angle of rise of these stairs.

6. A communications tower 62 m tall has to be supported with cables running from the top of the tower to anchors in the ground on both sides of the tower. The cables must form an angle of 50°. How far from the base of the tower should the anchors be placed?

7. An airplane takes off from a runway near some mountains. The peak of the mountain is on the flight path 2.5 km from the end of the runway. The mountain is 2000 m high. What angle of ascent is needed to clear the mountain top?

B

8. To avoid slipping, a ladder should not be placed against a wall at an angle less than 45° with the ground. What is the minimum height up a wall that the top of a 10 m ladder should reach?

9. Laurier places at 10 m ladder against a wall at an angle of 70° with the ground. How high is the top of the ladder?

10. **Knowledge and Understanding:** A captain knows that his ship is due south of a lighthouse. His destination is 20 km due west of the lighthouse, on a course setting of 40° west of the lighthouse. How far south of the lighthouse is the ship?

11. The owner of an auto shop is designing a ramp that lets her mechanics work underneath a car parked on a platform. None of the mechanics are more than 2 m tall. The ramp is to meet the ground at an angle of 20°. How long should the ramp be?

12. Lee is standing at the top of a hill that is 200 m high. Using a clinometer, she sights the base of the hill at an angle of depression of 40° from the horizontal. If the slope of the hill is constant, how far will her walk be from the top of the hill to the base?

13. **Application:** A geologist has determined that an oil deposit lies under a lake. The lake is 150 m deep and the oil deposit is 1500 m below the bottom of the lake. Owing to environmental concerns, oil wells are not allowed in the lake itself and must be built on shore. The well is to be 1000 m from the point directly above the edge of the oil deposit.

(a) To minimize the cost of drilling, the drill has to be angled so that it pierces the deposit at the closest point. What angle should be used?

(b) The drill bit is extended using 10 m sections that are added on as the drill cuts through the earth. How many sections will be needed to reach the deposit?

14. Building codes often include tables like this one. These tables identify the allowable slope for ramps. Compute the maximum angle of incline for each vertical rise category in the table to one decimal place.

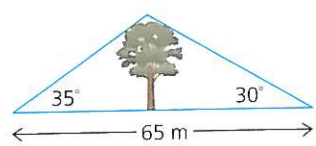
	Changes in Vertical Rise (mm)	Slope cannot be steeper than
(a)	0 to 15	1 : 2
(b)	15.1 to 50	1 : 5
(c)	50.1 to 200	1 : 10
(d)	More than 200	1 : 12

15. The pilot of an airplane is flying at 350 km/h. After one hour, she notices that owing to strong winds, she is 48 km west of her intended flight path. At what angle to her intended flight path has she been flying?

16. **Check Your Understanding:** Think of a real-life problem that can be modelled using a right triangle, in which the hypotenuse and an angle are known. Sketch the situation and explain how you could use trigonometry to find the unknown sides and angle.

C

17. Angle measurements were taken from two points on directly opposite sides of a tree as shown. How high is the tree?

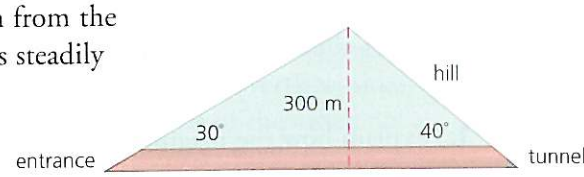


18. A regular hexagon has perimeter of 50 cm.

(a) Find the area of the hexagon.

(b) The hexagon is the base of a prism 100 cm tall. Find the volume and surface area of the prism.

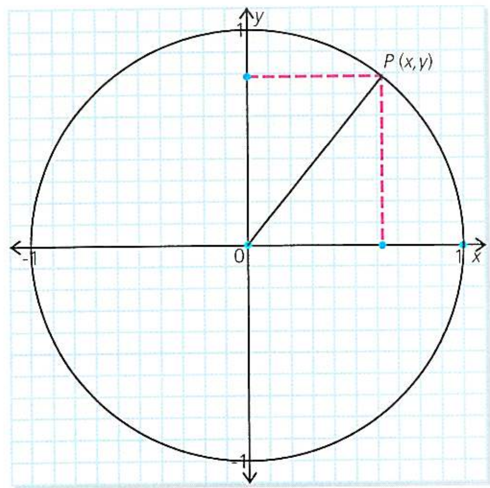
19. **Thinking, Inquiry, Problem Solving:** A tunnel is being dug through a hill. Ventilation shafts must be placed every 70 m from the entrance to the tunnel. On one side, the hill climbs steadily upward at an angle of 30°. The hill is steeper on the other side, which has a slope of 40°. The top of the hill is 300 m high.



(a) How many shafts must be drilled?

(b) Special corrugated metal pipes are used to line the shaft. These pipes come in 5 m sections. How many sections should the builder order?

20. Consider a circle centred at the origin, with a radius of 1 cm. Suppose $P(x, y)$ is a point in the first quadrant.



(a) Express the three trigonometric ratios for $\angle PAB$ in terms of the coordinates (x, y) of P.

(b) Suppose point P was moving counter clockwise around the circle at a speed of 1°/s (one degree per second). If P started at (1, 0), find the coordinates for P every 10 s until it returns to its starting position.

(c) Graph the x- and y-coordinates of P against time.

(d) Find the coordinates of P after 27 s.

(e) How would the graphs in (c) change if the radius of the circle had been 5 cm? 0.25 cm?

(f) How would the graphs in (c) change if the speed had been 2°/s? 0.5°/s?

The Chapter Problem—Mechanical Engineering

Consider the right triangle formed by the straight chain segment from the smaller wheel to the larger wheel, the radius of the larger wheel, and a line segment parallel to the segment joining the centres of the two wheels.

(a) Use this triangle to form all the possible trigonometric ratios that will give you information about the angle formed at the centre of the larger wheel by the radii to the chain's contact points on the wheel. Which ratio is most useful? Why? Use the trigonometric ratio to determine the angle.

(b) Use this angle to compute the length of chain that curves around the larger wheel.

(c) Repeat (b) for the smaller wheel.

(d) Compute the total chain length needed.