

Lesson 44: Angles in Standard Position

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1

Fast Five

- Evaluate $\sin(50^\circ)$ → illustrate with a diagram
- Evaluate $\sin(130^\circ)$ → illustrate with a diagram
- Evaluate $\sin(230^\circ)$ → illustrate with a diagram??
- Evaluate $\sin(320^\circ)$ → illustrate with a diagram??
- Evaluate $\sin(770^\circ)$ → illustrate with a diagram??
- Evaluate $\sin(-50^\circ)$ → illustrate with a diagram??
- Use your calculator and graph the function $f(x) = \sin(x)$ on the domain $-720^\circ < x < 720^\circ$

2

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2

BIG PICTURE

- The first of our keys ideas as we now start our Analytical Trig Unit:
- (1) **How do we use current ideas to develop new ones**

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3

BIG PICTURE

- The first of our keys ideas as we now start our Analytical Trig Unit:
- (1) **How do we use current ideas to develop new ones** → We will once again use RIGHT TRIANGLES to help develop new understandings

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4

BIG PICTURE

- The second of our keys ideas as we now start our Analytical Trig Unit:
- (2) **What does a TRIANGLE have to do with SINE WAVES**

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5

BIG PICTURE

- The second of our keys ideas as we now start our Analytical Trig Unit:
- (2) **What does a TRIANGLE have to do with SINE WAVES** → How can we REALLY understand how the sine and cosine ratios from right triangles could ever be used to create function equations that are used to model periodic phenomenon

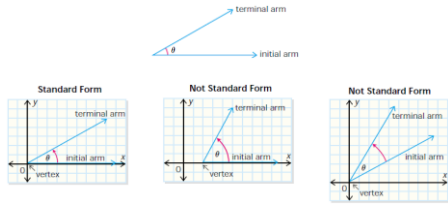
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6

(A) Angles in Standard Position

- Angles in standard position are defined as angles drawn in the Cartesian plane where the initial arm of the angle is on the x axis, the vertex is on the origin and the terminal arm is somewhere in one of the four quadrants on the Cartesian plane



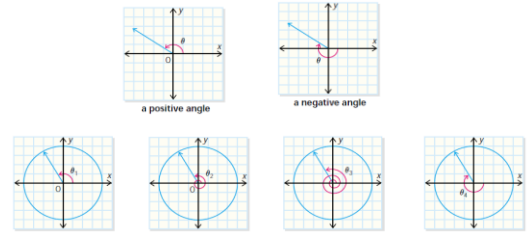
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7

(A) Angles in Standard Position

- To form angles of various measure, the terminal arm is simply rotated through a given angle



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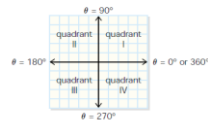
8

(A) Angles in Standard Position

- We will divide our Cartesian plane into 4 quadrants, each of which are a multiple of 90 degree angles

The x - y plane is divided into four quadrants by the x - and y -axes. If θ is a positive angle, then the terminal arm lies in

- quadrant I when $0^\circ < \theta < 90^\circ$
- quadrant II when $90^\circ < \theta < 180^\circ$
- quadrant III when $180^\circ < \theta < 270^\circ$
- quadrant IV when $270^\circ < \theta < 360^\circ$



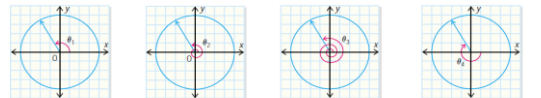
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9

(A) Coterminal Angles

- Coterminal angles** share the same terminal arm and the same initial arm.
- As an example, here are four different angles with the same terminal arm and the same initial arm.



If $\theta_1 = 120^\circ$, then

$$\theta_2 = 360^\circ + 120^\circ = 480^\circ$$

$$\theta_3 = 720^\circ + 120^\circ = 840^\circ$$

$$\theta_4 = -360^\circ + 120^\circ = -240^\circ$$

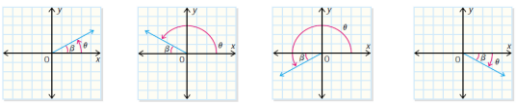
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10

(A) Principle Angles and Related Acute Angles

- The **principal angle** is the angle between 0° and 360° .
- The coterminal angles of 480° , 840° , and 240° all share the same principal angle of 120° .
- The **related acute angle** is the angle formed by the terminal arm of an angle in standard position and the x -axis.
- The related acute angle is always positive and lies between 0° and 90° .



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11

(B) Examples

Example 1

Determine the principal angle and the related acute angle for $\theta = -225^\circ$.

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12

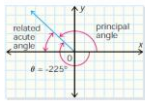
(B) Examples

Example 1

Determine the principal angle and the related acute angle for $\theta = -225^\circ$.

Solution

Sketch $\theta = -225^\circ$ terminating in quadrant II. Label the principal angle and the related acute angle.



The principal angle is the smallest positive angle that is coterminal to -225° . In this case, $360^\circ - 225^\circ = 135^\circ$. The related acute angle lies between the terminal arm and the x -axis. It is positive but less than 90° . In this case, $|-225^\circ - (-180^\circ)| = 45^\circ$. Or, using the principal angle, $180^\circ - 135^\circ = 45^\circ$.

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13

(B) Examples

Example 2

Determine the next two consecutive positive coterminal angles and the first negative coterminal angle for 43° .

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14

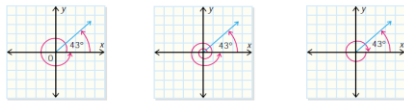
(B) Examples

Example 2

Determine the next two consecutive positive coterminal angles and the first negative coterminal angle for 43° .

Solution

Sketch each situation, showing the principal angle of 43° .



- The first positive coterminal angle for 43° is $360^\circ + 43^\circ = 403^\circ$.
- The second coterminal angle is $360^\circ + 360^\circ + 43^\circ = 763^\circ$.
- The first negative coterminal angle is $-360^\circ + 43^\circ = -317^\circ$.

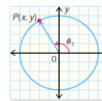
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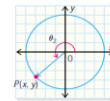
15

(C) Ordered Pairs & Right Triangle Trig

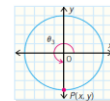
- To help discuss angles in a Cartesian plane, we will now introduce ordered pairs to place on the terminal arm of an angle



$90^\circ < \theta_1 < 180^\circ$
 θ_1 terminates in quadrant II.



$180^\circ < \theta_2 < 270^\circ$
 θ_2 terminates in quadrant III.



$P(x, y)$ lies in the negative y -axis.
 $\theta_3 = 270^\circ$

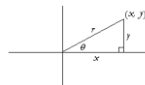
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16

(C) Ordered Pairs & Right Triangle Trig

- So to revisit our three basic trig ratios now in the context of the xy co-ordinate plane:



$$\sin \theta = \frac{y}{r} = \frac{y}{h}$$

$$\cos \theta = \frac{x}{r} = \frac{x}{h}$$

$$\tan \theta = \frac{y}{x} = \frac{y}{x}$$

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17

(C) Ordered Pairs & Right Triangle Trig

- Point $P(-3, 4)$ is on the terminal arm of an angle, θ , in standard position.
- (a) Sketch the principal angle, θ .
- (b) Determine the sine, cosine & tangent ratios of θ .
- (c) Determine the value of the related acute angle to the nearest degree.
- (d) What is the measure of θ to the nearest degree?

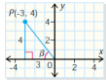
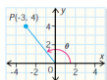
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18

(C) Ordered Pairs & Right Triangle Trig

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19

(C) Ordered Pairs & Right Triangle Trig - Examples

- Point $P(-9, 4)$ is on the terminal arm of an angle in standard position.
 - (a) Sketch the principal angle, θ .
 - (b) What is the measure of the related acute angle to the nearest degree?
 - (c) What is the measure of θ to the nearest degree?

Point $P(-5, -3)$ is on the terminal arm of an angle, θ , in standard position.

- (a) Sketch the principal angle, θ .
- (b) What is the measure of the related acute angle to the nearest degree?
- (c) What is the measure of θ to the nearest degree?
- (d) What is the measure of the first negative coterminal angle?

Point $P(-5, -8)$ is on the terminal arm of an angle, θ , in standard position. Determine all values of θ for $-540^\circ \leq \theta \leq 270^\circ$.

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20

(I) Homework

- Angles in Standard Position: Nelson 11, Chap 6.2, p442, Q1-10,12,15 (on-line link)

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21