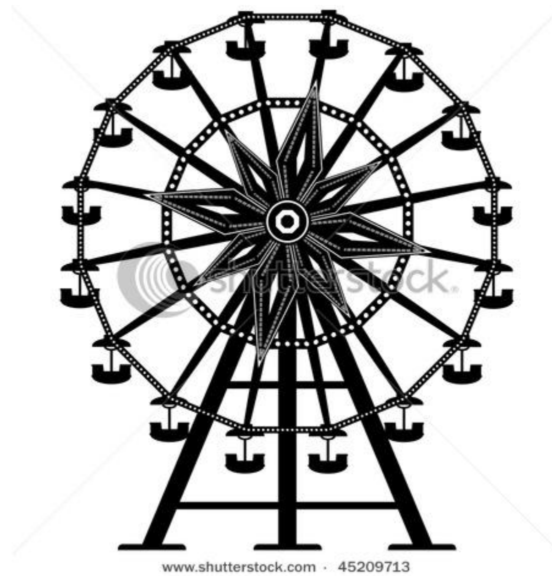
(A) Lesson Objectives

a. Introduce periodic phenomenon through several data driven investigations:
   i. the relationship between a riders height on a Ferris Wheel and the time of the ride
   ii. The relationship between the fraction of the moon that is visible and the day of the year
b. Introduce the key analysis features of periodic phenomenon

(B) Modelling Periodic Phenomenon – Riding on a Ferris Wheel

You are going for a ride on a Ferris wheel. The Ferris wheel rotates at a constant speed. It has a radius of 15 meters and the bottom of the wheel is 5 meters off the ground. It takes 60 seconds to go around the Ferris wheel one time. Again, you will be graphing the dependent variable, height (H), of your carriage in meters above the ground, at time (t) seconds.



1. You just get into your carriage at the ***MIDDLE*** of the wheel. What is your height when t=0? Plot this point on your graph.

2. What is the highest you will go? When will this happen? Plot this point on your graph.

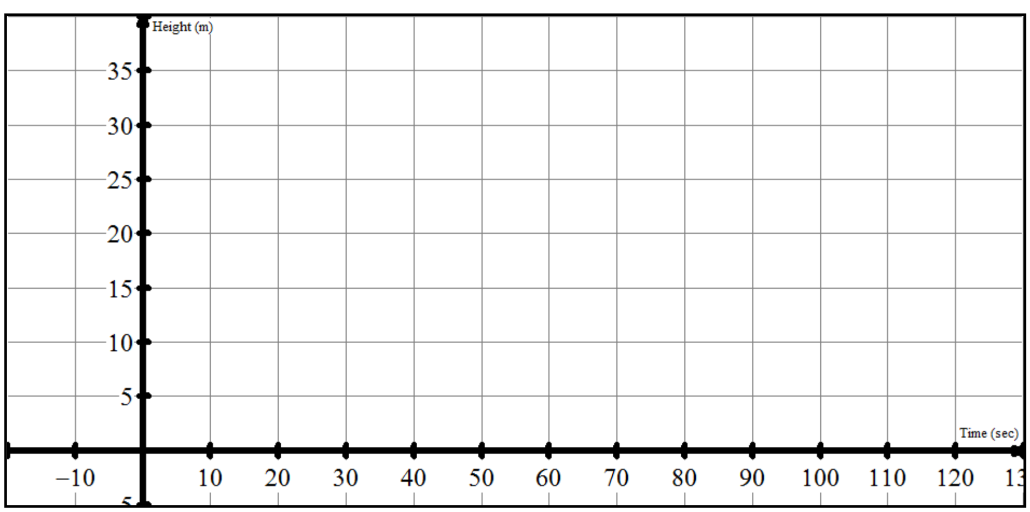
3. How high will you be after 30 seconds? Plot this point on your graph.

4. Is there another time (t) when you will be at the same height as above at 30 seconds? When will this be? Plot this point on your graph.

5. When will your height (h) be 5 meters? Plot this point on your graph.

6. Expand your graph to show your height on the Ferris wheel over 2 cycles of rotation.

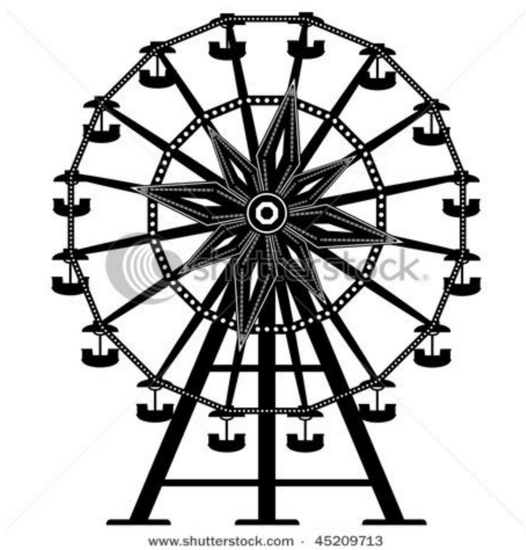
Time (sec)	Height (m)
0 sec	
15 sec	
30 sec	
45 sec	
60 sec	



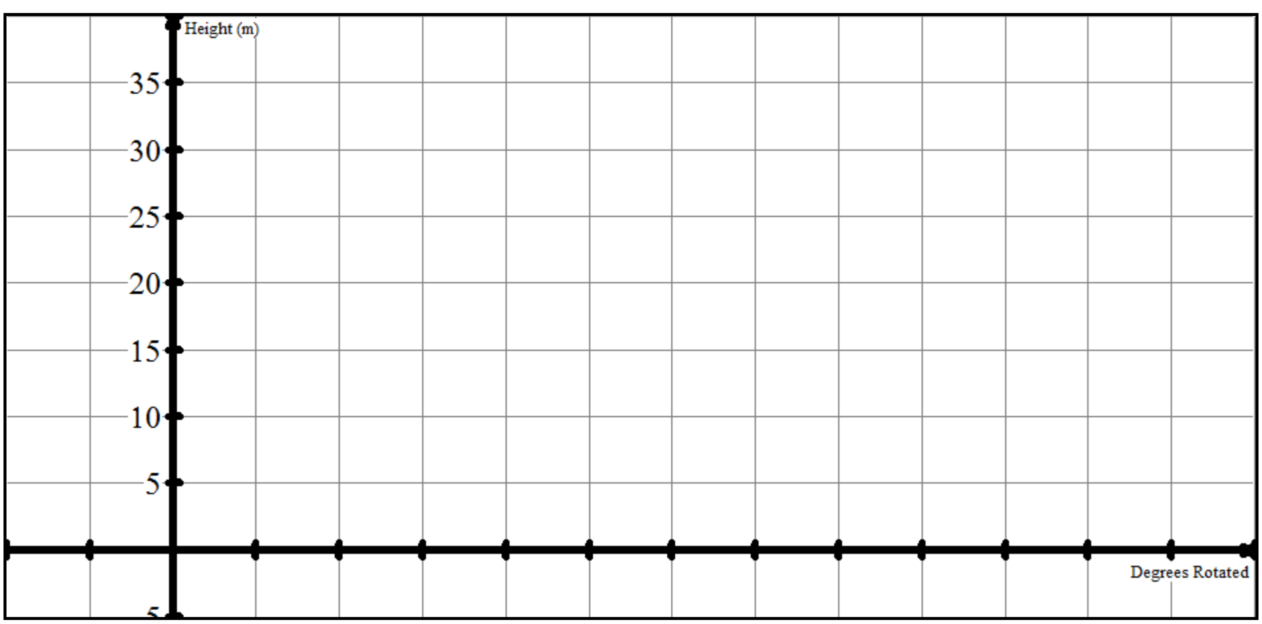
(C) Modelling Periodic Phenomenon – Riding on a Ferris Wheel

Now, let's make ONE change to our analysis. Again, you will be graphing the height (H), of your carriage in meters above the ground, but this time, let's change our INDEPENDENT variable to DEGREES of ROTATION

Again, you get into your carriage at the ***MIDDLE*** of the wheel and you go around twice.



Time (sec)	Degrees rotated	Height (m)
0 sec		
15 sec		
30 sec		
45 sec		
60 sec		



What is the amplitude?

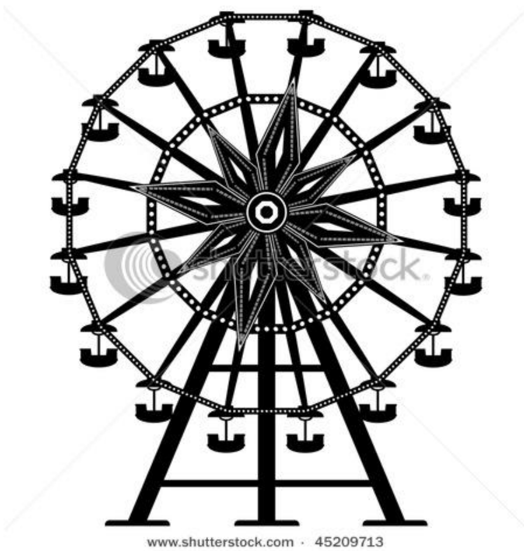
What is the period of rotation?

What is the equation of the axis of the curve?

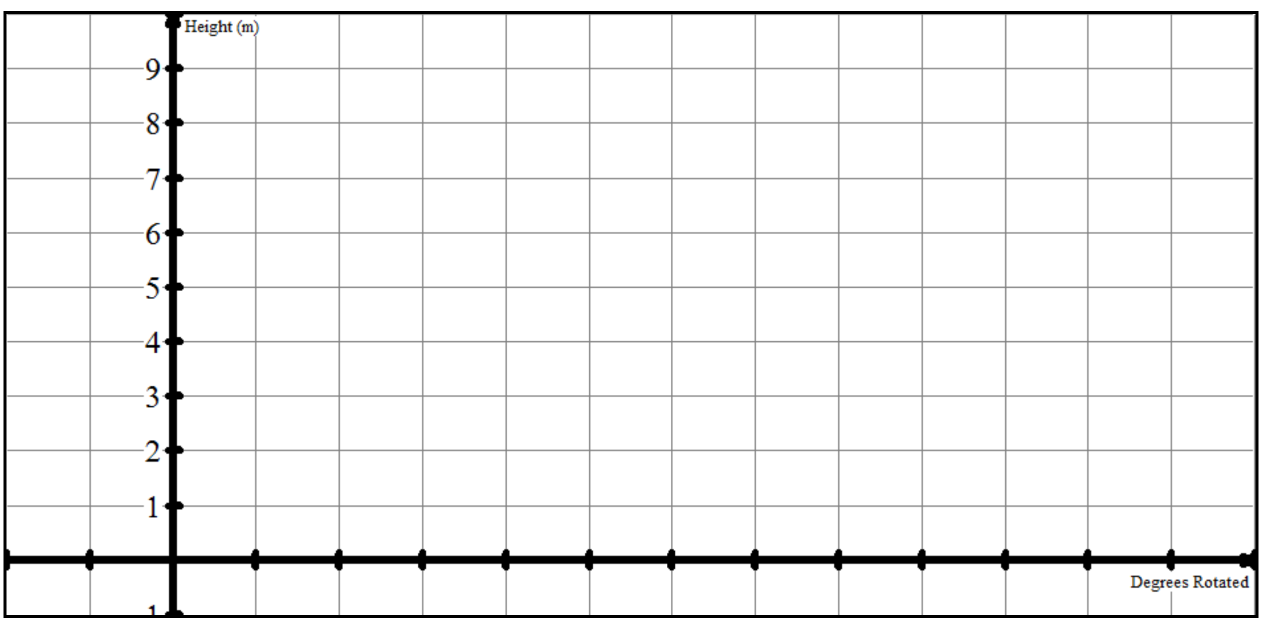
(D)Modelling Periodic Phenomenon – Riding on a Ferris Wheel

Now, let's make A SECOND change to our analysis. You will be graphing the height (H) of your carriage above the ground, let's keep our INDEPENDENT variable as DEGREES of ROTATION, but let's make our AMPLITUDE to be 1 meter

Again, you get into your carriage at the ***MIDDLE*** of the wheel and you go around twice.



Degrees rotated	Height (m)



What is the amplitude?

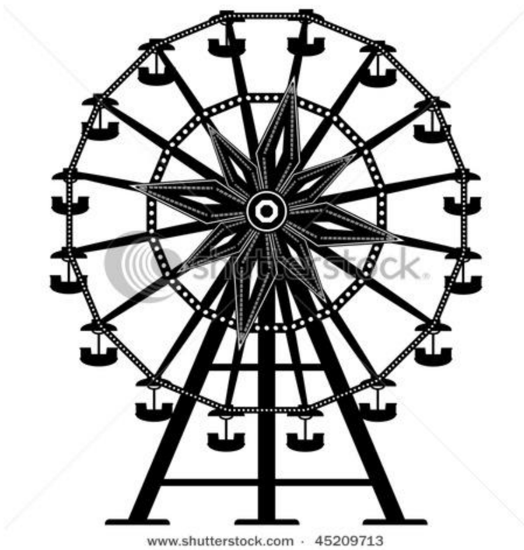
What is the period of rotation?

What is the equation of the axis of the curve?

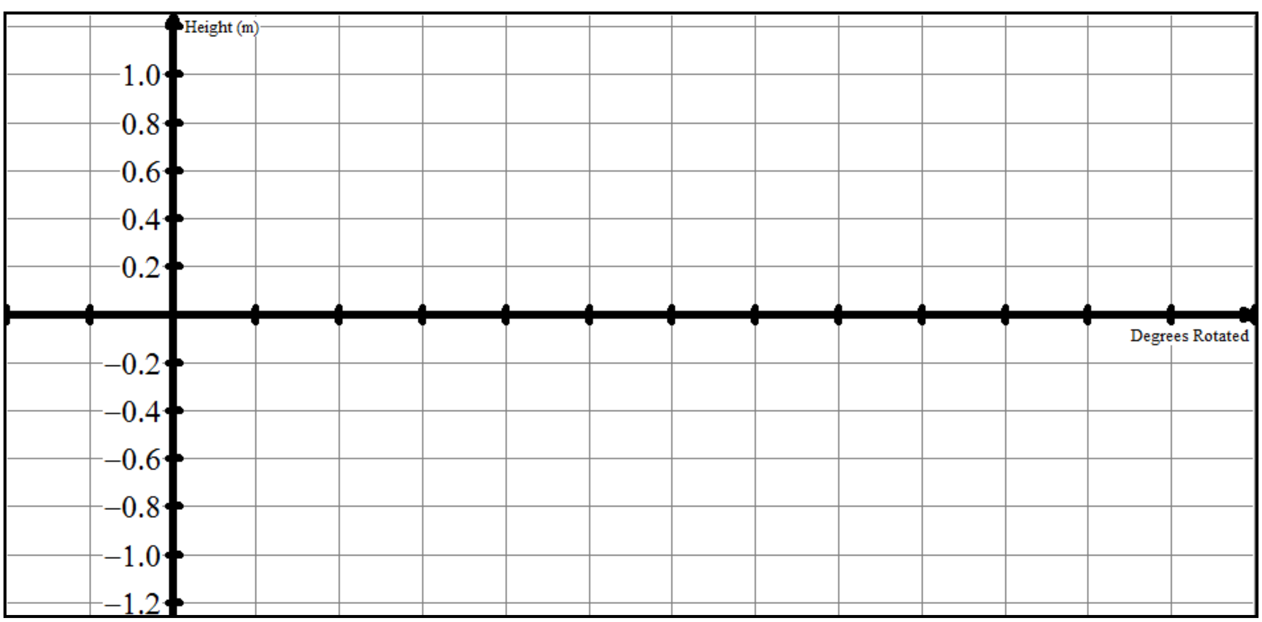
(E) Modelling Periodic Phenomenon – Riding on a Ferris Wheel

Now, let's make A FINAL change to our analysis. You will be graphing the height (H) of your carriage above the ground, let's keep our INDEPENDENT variable as DEGREES of ROTATION, our AMPLITUDE stays at 1 meter, but our AXIS OF THE CURVE will now be the X-AXIS (y = 0).

Again, you get into your carriage at the ***MIDDLE*** of the wheel and you go around twice.



Degrees rotated	Height (m)



What is the amplitude?

What is the period of rotation?

What is the equation of the axis of the curve?

(F) Parent Function: f(x) = sin(x)

a. The final graph you just produced is our PARENT FUNCTION.

b. The PERIOD of the graph/context refers to the CYCLE LENGTH (the length of one cycle). What is the period of this parent function?

c. All periodic phenomena have MAXIMUM and MINIMUM POINTS as well.
   i. Where are the maximum points of our parent function f(x) = sin(x)?
   ii. Where are the minimum points of our parent function f(x) = sin(x)?

d. All graphs of data that illustrates periodic phenomena have an EQUILIBRIUM AXES or AXES OF THE CURVE or MEDIAN LINE. Where would the axes of the curve be of our parent function f(x) = sin(x)?

e. The AMPLITUDE is the distance between median line and the maximum or minimum value of the graph. What is the AMPLITUDE of our parent function f(x) = sin(x)?

f. Graph 3 cycles of f(x) = sin(x) ON YOUR TI-84.

g. What is the DEPENDENT VARIABLE of our function and what does it represent?

h. What is our domain given the graph on your TI-84?

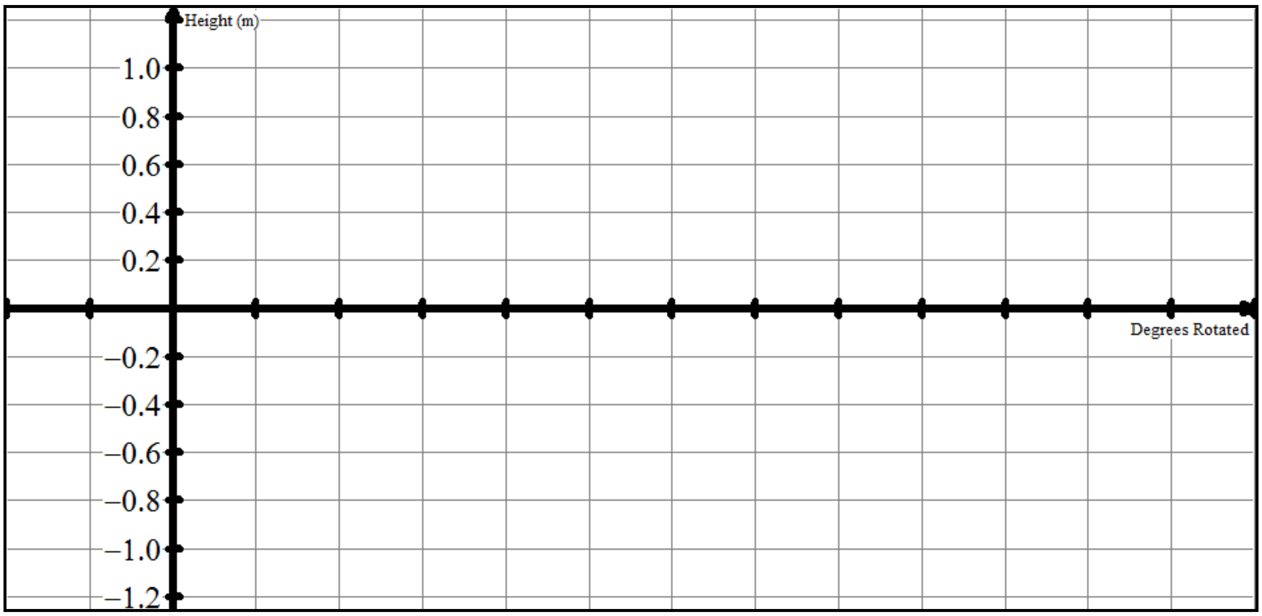
i. What is the INDEPENDENT VARIABLE of our function and what does it represent?

j. What is our range?

(G)<u>Second Parent Function: f(x) = cos(x)</u>

a. The next graph you will produced is our PARENT FUNCTION for y = cos(x). To understand the cosine function, we can revisit our Ferris Wheel context and START our rotation AT THE TOP of the Ferris Wheel. Again, we will go around twice (120 seconds or 720 degrees) with our radius as 1 unit.

b. Graph your “height” vs degrees on the grid below



c. The PERIOD of the graph/context refers to the CYCLE LENGTH (the length of one cycle). What is the period of this parent function?

d. All periodic phenomena have MAXIMUM and MINIMUM POINTS as well.
   i. Where are the maximum points of our parent function f(x) = sin(x)?
   ii. Where are the minimum points of our parent function f(x) = sin(x)?

e. All graphs of data that illustrates periodic phenomena have an EQUILIBRIUM AXES or AXES OF THE CURVE or MEDIAN LINE. Where would the axes of the curve be of our parent function f(x) = cos(x)?

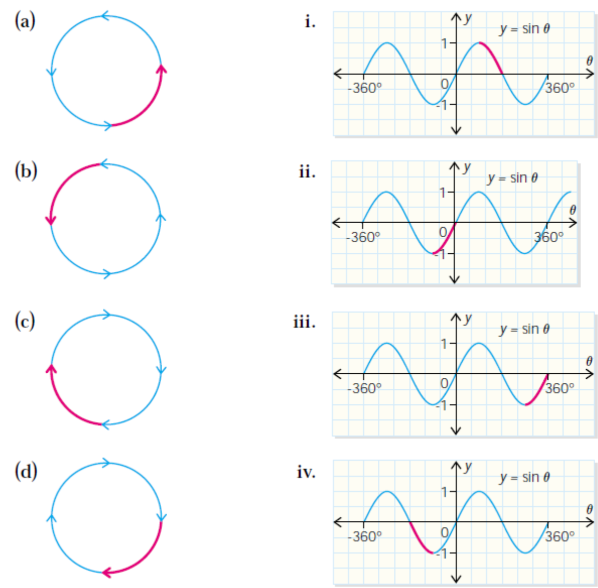
f. The AMPLITUDE is the distance between median line and the maximum or minimum value of the graph. What is the AMPLITUDE of our parent function f(x) = cos(x)?

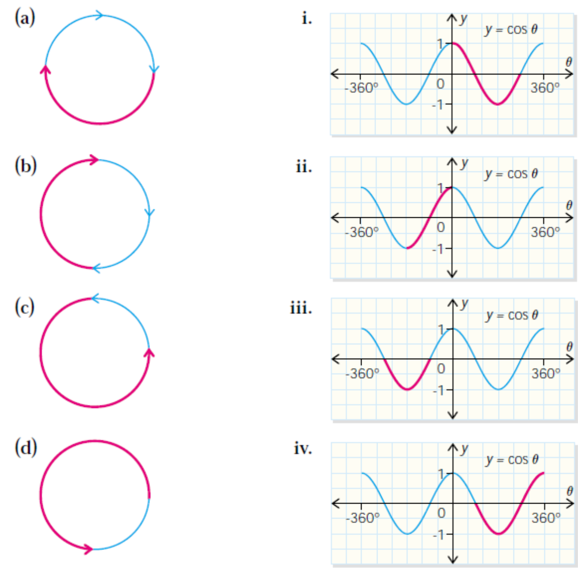
g. Graph 3 cycles of f(x) = cos(x) ON YOUR TI-84.

h. What is our domain given the graph on your TI-84?

i. What is our range?

Functions to Model Periodic Phenomena | Lesson 31





Consider the function $f(\theta) = \cos\theta$.

(a) Complete the table using the unit circle and sketch the graph.

θ	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
$f(\theta)$									

(b) State the coordinates of the maximum and minimum values of $f(\theta) = \cos\theta$ within the domain of the table.

(c) What are the coordinates of the zeros of the function within this domain?

(d) Show that $f(\theta) = f(-\theta)$ for all values of θ in the table.

Evaluate $y = \cos\theta$ for $0° \leq \theta \leq 540°$ when $y = -0.7$. Answer to the nearest degree.

Evaluate $y = \sin\theta$ for $-90° \leq \theta \leq 540°$ when $y = -0.3$. Answer to the nearest degree.

(a) Evaluate $h(t) = \cos(20t)°$ for $t = 3$.

(b) What is the value of t when $h(t) = 0.3$ for $0 \leq t \leq 18$?

(a) Evaluate $h(t) = 4\sin(30t)°$ for $t = 10$.

(b) What is the value of t when $h(t) = 3.2$ for $0 \leq t \leq 12$?

The height, h, of a basket on a water wheel at time t is given by $h(t) = \sin(6t)^\circ$, where t is in seconds and h is in metres.

(a) How high is the basket at 14 s?

(b) When will the basket first be 0.5 m under water?

The vertical distance in metres of a rider with respect to the horizontal diameter of a Ferris wheel is modelled by $h(t) = 5\cos(18t)^\circ$, where t is the number of seconds.

(a) To one decimal place, what is the rider's vertical distance with respect to the horizontal diameter of the wheel when $t = 8$ s? 16 s? 30 s?

(b) When is the rider first at 4.5 m? -3.2 m?

(c) When is the third time the rider is at -2.5 m?