

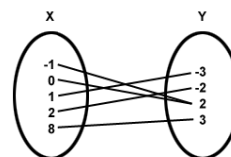
**(A) Lesson Objectives**

- a. Introduce composite functions and how it can be represented numerically, algebraically and graphically.
- b. Introduce graphical transformations
- c. Understand that graphical transformations are formed through composite functions

**(B) Introduction to Composite Functions.**

Review - We have explored the multiple representations of function:

- Literal
- Numerical Data (table-of-values and mapping diagrams)
- Graphical
- Algebraic



1. Let's consider a mapping diagram. Complete the mapping diagram for the function:

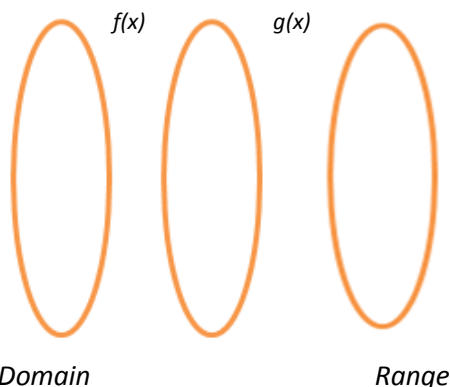
$f(x) = 2^x$ , where the Domain = {0, 1, 2, 3, 4}

What is the Range of  $f(x)$  \_\_\_\_\_



2. Sometimes functions undergo **more than one** mapping or transformation.

Let's consider these two functions:  $f(x) = 2^x$  and  $g(x) = x - 1$ , what would this mapping look like below. Let the Domain = {0, 1, 2, 3, 4}



Determine the Range \_\_\_\_\_

If our data is mapped via  $f(x)$  and then mapped by  $g(x)$ , the question then remains:

“What is the new equation of the twice mapped data that will allow us to get the same result in 1 step?”

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**3. Notation**

Given the functions  $f(x)$  and  $g(x)$  where  $x$  is mapped via  $f(x)$  FIRST and then mapped AGAIN via  $g(x)$ , then this IDEA or CONCEPT is represented by the notation:  $g(f(x))$  OR  $g \circ f(x)$

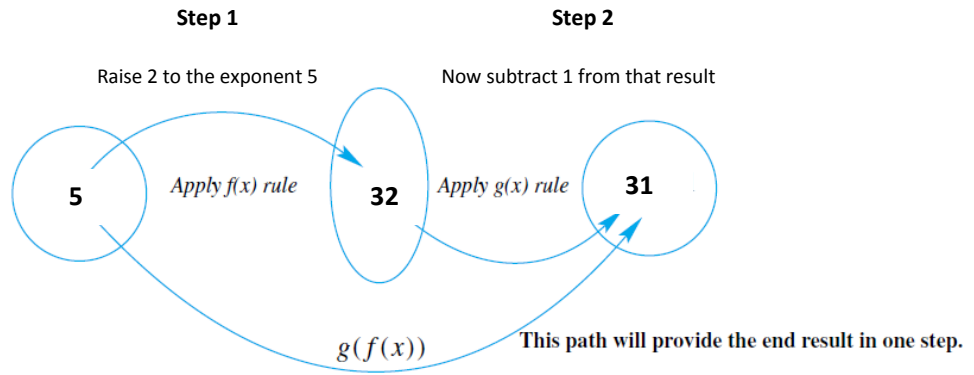
Consider the functions:

$$f(x) = 2^x$$

$$g(x) = x - 1$$

Find the value of:

$$g(f(x)) \text{ or } g \circ f(x)$$



#### 4. Working with Composite Functions

If  $f(x) = 2^x$  and  $g(x) = x + 1$  determine: *Model in a mapping diagram if needed*

|                |                |                |  |
|----------------|----------------|----------------|--|
| $g \circ f(0)$ | $g \circ f(3)$ | $g \circ f(x)$ |  |
| $f \circ g(0)$ | $f \circ g(3)$ | $f \circ g(x)$ |  |

If  $f(x) = 3^x$  and  $g(x) = x - 2$  determine: *Model in a mapping diagram if needed*

|                |                |                |  |
|----------------|----------------|----------------|--|
| $f \circ g(4)$ | $f \circ g(1)$ | $f \circ g(x)$ |  |
| $g \circ f(4)$ | $g \circ f(1)$ | $g \circ f(x)$ |  |

5. **Observations:** What are the key things I have noticed about composite functions?

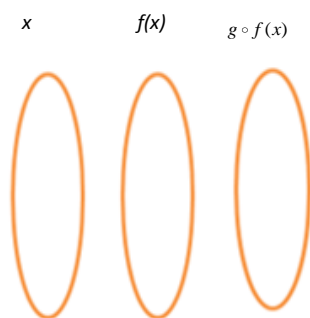
**(C) Graphical Representations of Composite Functions**

Consider the functions:

$f(x) = 2^x$  and  $g(x) = x - 1$

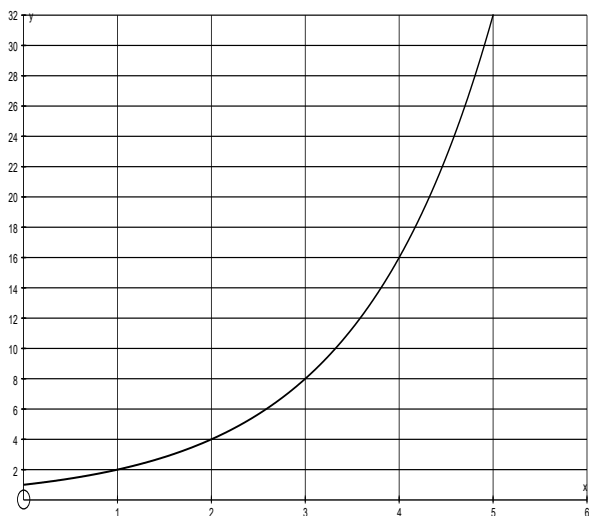
Determine  $g \circ f(x) =$  \_\_\_\_\_

Mapping Diagram



Here is the graph of  $f(x) = 2^x$

Graph the function  $y = g \circ f(x)$  using your GDC



Graphical transformations

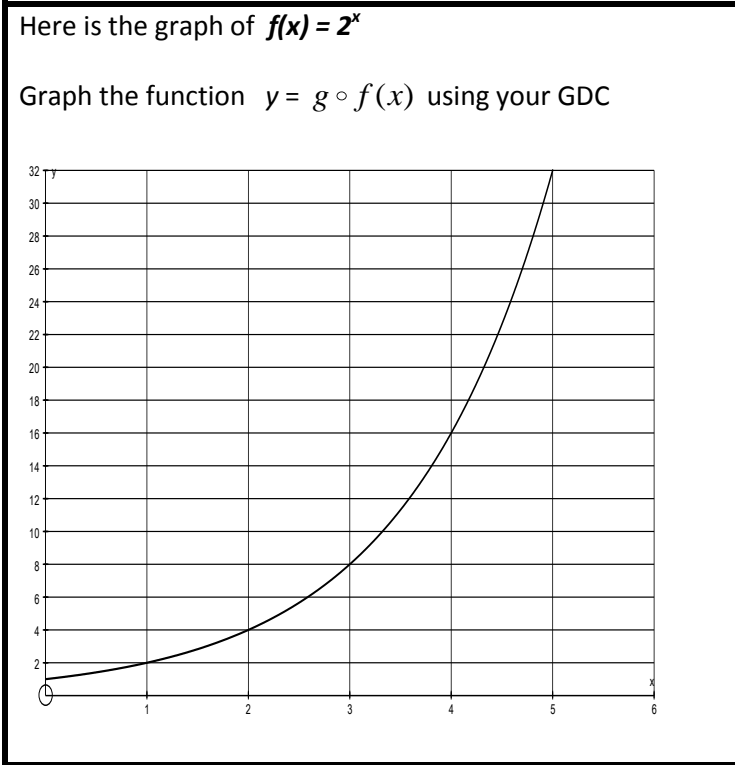
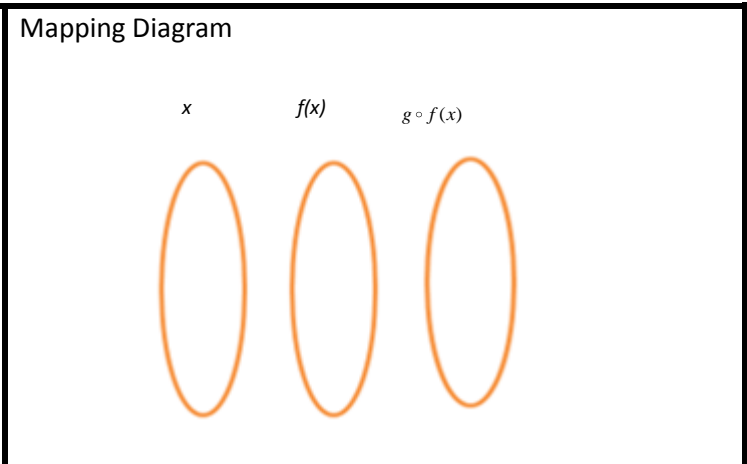
How has the graph of  $f(x) = 2^x$  transformed to the graph of  $y = g \circ f(x)$ ?

Explain graphical changes in detail.

|  |  |
|--|--|
|  |  |
|--|--|

Consider the functions:  
 $f(x) = 2^x$  and  $g(x) = x + 3$

Determine  $g \circ f(x) =$  \_\_\_\_\_



Graphical transformations

How has the graph of  $f(x) = 2^x$  transformed to the graph of  $y = g \circ f(x)$ ?

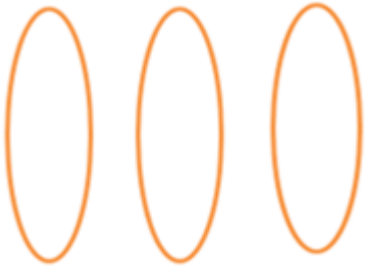
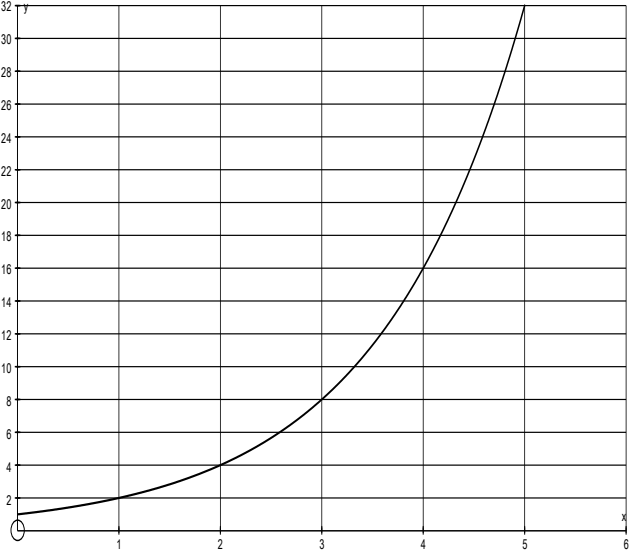
Explain graphical changes in detail.

Check for Understanding:

What do you think the graphical transformation would be for the function  $f(x) = 2^x - 5$  ? Explain your thinking.

# Lesson 19 - Exploring Graphical Transformations and Composite Functions Date \_\_\_\_\_

What do you think the 2 parent functions are that were composed to form  $f(x) = 2^x - 5$  ? Justify your conjecture.


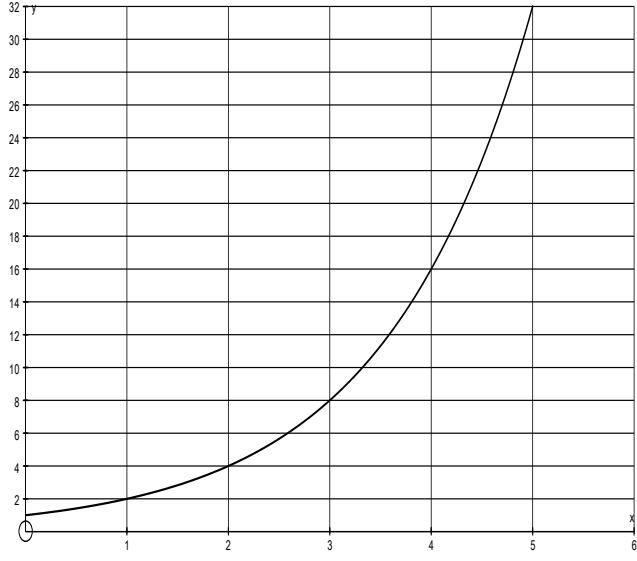
|                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                      |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Consider the functions:</p> <p><math>f(x) = 2^x</math> and <math>g(x) = x - 1</math></p> <p>Determine <math>f \circ g(x) =</math> _____</p>                                                                 | <p>Mapping Diagram</p> <div style="text-align: center; margin-bottom: 10px;"> <math>x</math>                  <math>g(x)</math>                  <math>f \circ g(x)</math> </div>  |
| <p>Here is the graph of <math>f(x) = 2^x</math></p> <p>Graph the function <math>y = f \circ g(x)</math> using your GDC</p>  | <p>Graphical transformations</p> <p>How has the graph of <math>f(x) = 2^x</math> transformed to the graph of <math>y = f \circ g(x)</math>?</p> <p>Explain graphical changes in detail.</p>                                                                          |

Try this!

What do you think the graphical transformation would be for the function  $y = 2^{x+3}$  ? Explain your thinking.

# Lesson 19 - Exploring Graphical Transformations and Composite Functions Date \_\_\_\_\_

What do you think the 2 parent functions are that were composed to form  $y = 2^{x+3}$  ? Justify your conjecture.

|                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                      |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Consider the functions:</p> <p><math>f(x) = 2^x</math> and <math>g(x) = x + 3</math></p> <p>Determine <math>f \circ g(x) =</math> _____</p>                                                                | <p>Mapping Diagram</p> <div style="text-align: center; margin-bottom: 10px;"> <math>x</math>                  <math>g(x)</math>                  <math>f \circ g(x)</math> </div>  |
| <p>Here is the graph of <math>f(x) = 2^x</math></p> <p>Graph the function <math>y = f \circ g(x)</math> using your GDC</p>  | <p>Graphical transformations</p> <p>How has the graph of <math>f(x) = 2^x</math> transformed to the graph of <math>y = f \circ g(x)</math> ?</p> <p>Explain graphical changes in detail.</p>                                                                         |

Understanding so Far:

Describe the transformation  $y = a^x$  has undergone to form  $y = a^x + b$ .

What are the parent functions that were composed to form  $y = a^x + b$

Describe the transformation  $y = a^x$  has undergone to form  $y = a^{x-c}$ .

What are the parent functions that were composed to form  $y = a^{x-c}$

**(D) Further Compositions**

Consider the functions:

$f(x) = 2^x$  and  $g(x) = 3x$

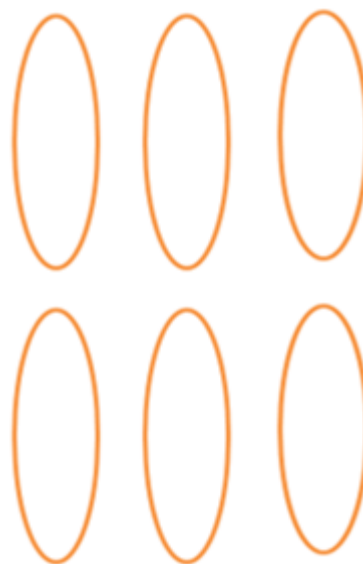
Determine  $f \circ g(x) =$  \_\_\_\_\_

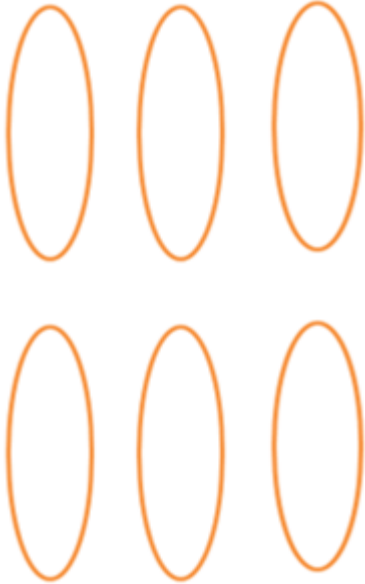
$f \circ g(2) =$  \_\_\_\_\_

Determine  $g \circ f(x) =$  \_\_\_\_\_

$g \circ f(2) =$  \_\_\_\_\_

Mapping Diagram



|                                                                                                                                                                                                                                                                                                                                                |                                                                                     |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| <p>Consider the functions:</p> <p><math>f(x) = 0.5^x</math> and <math>g(x) = -2x</math></p> <p>Determine <math>f \circ g(x) =</math> _____</p> <p style="padding-left: 40px;"><math>f \circ g(2) =</math> _____</p><br><p>Determine <math>g \circ f(x) =</math> _____</p> <p style="padding-left: 40px;"><math>g \circ f(2) =</math> _____</p> |  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|

**Extension:**

1. For the function:  $m(x) = 2a^x$  determine the parent functions that are composed to form  $m(x)$ .  
Show this composition in a mapping diagram

2. For the function:  $r(x) = a^{x+6} - 2$  determine the two parent functions that are composed to form  $r(x)$   
Show this composition in a mapping diagram

3. For the function:  $h(x) = a^{3x} + 1$  determine the two parent functions that are composed to form  $h(x)$



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Show this composition in a mapping diagram

- Describe what transformations the function  $f(x)$  undergoes to form  $p(x) = 2^{x-3} + 6$