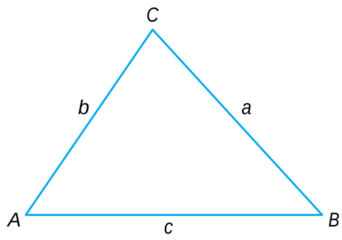
Extending Trigonometry Skills with Oblique Triangles

6.1

In earlier courses, you used the sine law and the cosine law to solve acute triangles. A triangle without a right angle is called an **oblique** triangle. Recall that in an acute triangle, ΔABC, with sides a, b, and c, these relations are true.

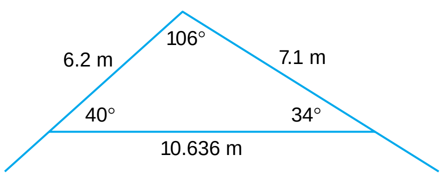


The Sine Law	The Cosine Law
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Can the sine law and cosine law be used to solve an obtuse triangle?

Part 1: Investigating the Cosine Law and Obtuse Triangles

Don is a carpenter who often frames houses. This blueprint shows a cross section of the roof structure for the house he is now working on.



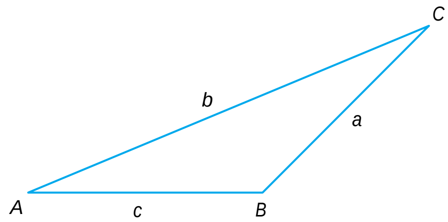
Think, Do, Discuss

1. Sketch the roof. Starting with the obtuse angle, label the vertices A, B, and C in a counterclockwise direction. Label the corresponding sides of the triangle.
2. Show that the cosine law is true for each acute angle in the diagram.
3. Determine cos 106°. How does this cosine differ from the cosines of the other two angles in the triangle?
4. How does the cosine of any obtuse angle differ from the cosine of any acute angle? Does this prevent the cosine law from being true for the obtuse angle?

5. Show that the cosine law is true for the obtuse angle.

6. Based on the results from step 5, can you say that the cosine law will work in all obtuse triangles? Explain.

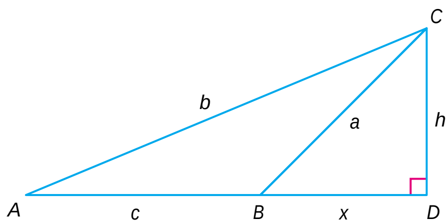
7. Examine this proof of the cosine law for any obtuse triangle, where $\angle B$, or $\angle ABC$, is obtuse.



Proof that $b^2 = a^2 + c^2 - 2ac \cos B$, when $\angle B > 90°$

i. Draw h perpendicular to side c extended. Let x be the length of BD and h be the length of CD.

ii. Since $CD \perp AD$, then ΔADC and ΔBDC are right triangles. Use the Pythagorean theorem to determine an expression for h^2 in both triangles.



In ΔADC,

$$(c + x)^2 + h^2 = b^2$$
$$h^2 = b^2 - (c + x)^2$$
$$h^2 = b^2 - (c^2 + 2cx + x^2)$$
$$h^2 = b^2 - c^2 - 2cx - x^2 \quad ①$$

In ΔBDC,

$$a^2 = x^2 + h^2$$
$$a^2 - x^2 = h^2 \quad ②$$

iii. Since h represents the height of both triangles, equations ① and ② are equal.

$$b^2 - c^2 - 2cx - x^2 = a^2 - x^2$$ Rearrange to isolate b^2.

$$b^2 = a^2 + c^2 + 2cx - x^2 + x^2$$ Simplify.

$$b^2 = a^2 + c^2 + 2cx$$

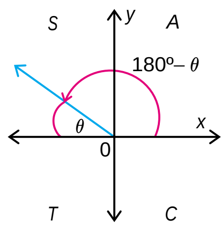
iv. ΔBDC is a right triangle. Use a primary trigonometric ratio to find an expression for x.

$$\cos \angle CBD = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos \angle CBD = \frac{x}{a}$$
$$a \cos \angle CBD = x$$

v. In ΔBDC,

$$\angle CBD = 180° - \angle ABC$$
$$= 180° - \angle B \qquad \text{where } \angle ABC = \angle B$$

For any obtuse angle $(180° - \theta)$, $\cos(180° - \theta) = -\cos\theta$.



Therefore,

$$\cos \angle CBD = \cos(180° - \angle ABC)$$
$$= \cos(180° - \angle B)$$
$$= -\cos B$$

Substitute into $a \cos \angle CBD = x$, from step iv.

$$a(-\cos B) = x$$
$$-a \cos B = x$$

Substitute this expression for x into $b^2 = a^2 + c^2 + 2cx$.

$$b^2 = a^2 + c^2 + 2c(-a\cos B)$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$

This is the result to be proven. Therefore, for any ΔABC, $b^2 = a^2 + c^2 - 2ac\cos B$. Similarly, by drawing perpendiculars to the other sides, you can prove that

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

8. In the proof above,

(a) What does h represent in ΔABC, ΔADC, and ΔBDC?

(b) Explain why the two expressions for h^2 can be set equal to each other in step iii.

(c) Explain why $\cos(180° - \theta) = -\cos\theta$ in step v.

9. What is the least information you need to know about a triangle to use the cosine law to calculate

(a) an unknown side?

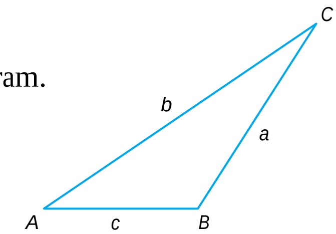
(b) an unknown angle?

Part 2: Investigating the Sine Law and Obtuse Triangles

Refer to your diagram of the roof cross section from Part 1.

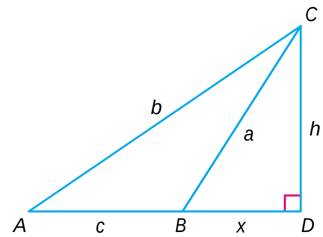
Think, Do, Discuss

1. Show that the sine law is true for each acute angle in the diagram.
2. Determine sin 106°. Other than value, does this sine differ from the sines of the other two angles in the triangle in any way?
3. Determine sin 74°. What do you notice?
4. When using the sine law to calculate the length of a side, is the calculation affected at all if the angle is obtuse, rather than acute or right?



5. Show that the sine law is true for the obtuse angle in the diagram.
6. Based on your results in step 5, can you say that the sine law will work in all obtuse triangles? Explain.
7. Examine this proof of the sine law for any obtuse triangle, where $\angle B$, or $\angle ABC$, is obtuse.

Proof that $a \sin B = b \sin A$ or $\frac{a}{\sin A} = \frac{b}{\sin B}$, where $\angle B > 90°$



i. Draw $CD \perp AB$ extended to point D. Label CD as h.

ii. In ΔCAD,

$$\sin A = \frac{h}{b}$$
$$\therefore \quad b \sin A = h$$

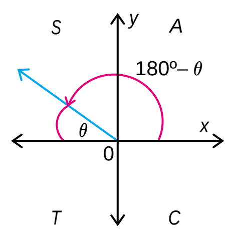
In ΔCBD,

$$\sin \angle CBD = \frac{h}{a}$$
$$\angle CBD = 180° - \angle ABC$$
$$= 180° - \angle B \qquad \text{where } \angle ABC = \angle B$$

Therefore,

$$\sin \angle CBD = \sin (180° - \angle ABC)$$
$$= \sin (180° - \angle B)$$

For any obtuse angle $(180° - \theta)$, $\sin (180° - \theta) = \sin \theta$.



Therefore,

$$\sin \angle CBD = \sin B$$

Substitute into $\sin (\angle CBD) = \frac{h}{a}$

$$\sin B = \frac{h}{a}$$
$$\therefore \quad a \sin B = h$$

iii. Because h represents the height of both triangles, it follows that

$$a \sin B = b \sin A \text{ or } \frac{a}{\sin A} = \frac{b}{\sin B}$$

Similarly, by drawing perpendiculars to the other sides, it may be proven that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

8. In the proof in step 7, explain why $\sin(180° - \theta) = \sin \theta$.

9. What is the least information you need to know about the triangle to use the sine law to calculate

(a) an unknown side?

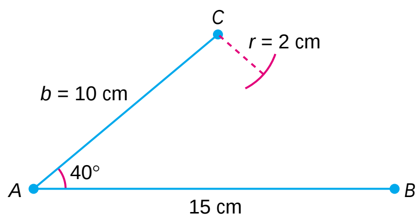
(b) an unknown angle?

Part 3: A Closer Look at the Sine Law — The Ambiguous Case

You can use the sine law when you know two side lengths of a triangle and the measure of an angle opposite one of the known sides. In this situation, does the given information always define one unique triangle? Dynamic geometry software can be used in this activity.

Think, Do, Discuss

1. Draw a line segment AB at least 15 cm long. Construct line segment AC so that $\angle CAB = 40°$ and $AC = b = 10$ cm.



2. Construct an arc that originates from point C with a radius of 2 cm. Does the arc intersect AB?

3. Repeat step 2 using the radii from the table. Record the number of triangles that can be drawn with each radius.

Radius, r (cm)	Number of Triangles
2	
4	
6	
8	
10	

4. Determine the radius that would allow you to draw exactly one triangle.

5. Calculate the value of $b \sin A$. What does this represent in your diagram?

6. For which values of r does the arc intersect AB twice?

7. For the radii that allowed you to create two triangles, determine the height of each triangle. What do you notice?

8. Repeat steps 1–3 using $\angle A = 120°$. Under what conditions do you get a triangle? do not get a triangle?

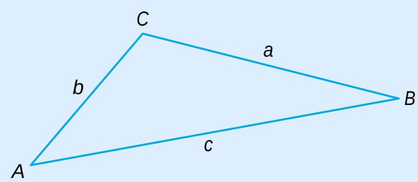
Focus 6.1

Key Ideas

- Any triangle without a right angle is called an **oblique triangle**. The cosine law and the sine law can be used to determine angles or sides in all triangles (acute, right, and obtuse).

- In ΔABC, with sides a, b, and c,

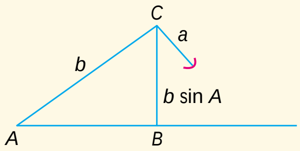
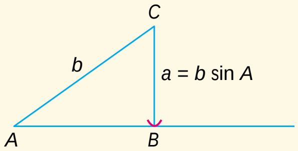
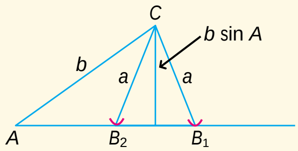
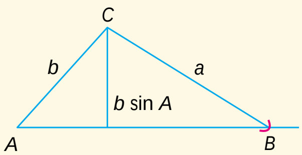
The Sine Law	The Cosine Law
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	$a^2 = b^2 + c^2 - 2bc \cos A$
or	$b^2 = a^2 + c^2 - 2ac \cos B$
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$c^2 = a^2 + b^2 - 2ab \cos C$

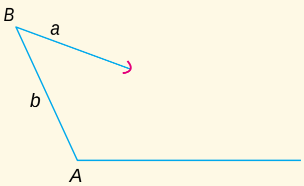
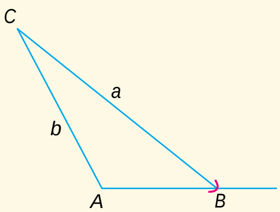


- To solve an oblique triangle, you need to know the measure of at least one side and any two other parts of the triangle. There are four cases in which this can happen.

Given Information	What Can Be Found	Law Required
1. Two angles and any side (AAS or ASA)	side	sine law
2. Two sides and the contained angle (SAS)	side	cosine law
3. Three sides (SSS)	angle	cosine law
4. Two sides and an angle opposite one of them (SSA)	angle	sine law

Case 4 is called the **ambiguous case** because sometimes it is possible to draw more than one triangle for the given information. In this case, there are four possible outcomes if $\angle A$ is acute, and two possible outcomes if $\angle A$ is right or obtuse. These possibilities are shown for the given $\angle A$, the given sides a and b, in ΔABC. The side opposite the given angle is always a, and $b \sin A$ represents the possible height of the triangle.

$\angle A < 90°$ (acute)	Conditions	Number and Type of Triangles Possible
	$a < b \sin A$	no triangle
	$a = b \sin A$	one right triangle
	$b \sin A < a < b$	two triangles—one acute, one obtuse
	$a \geq b$	one triangle

$\angle A > 90°$ (obtuse)	Conditions	Number and Type of Triangles Possible
	$a \leq b$	no triangle
	$a > b$	one obtuse triangle

Determining whether a given angle ($\angle A$) is acute or obtuse, and then comparing the size of a, b, and $b \sin A$ allows you to see which situation you are dealing with and, in turn, the number of possible solutions.

Example 1

Determine the measure of

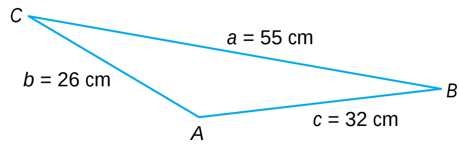
(a) $\angle A$ in ΔABC to the nearest degree, if $a = 55$ cm, $b = 26$ cm, and $c = 32$ cm

(b) $\angle B$ in ΔABC to the nearest degree, if $a = 3$ m, $b = 15$ m, and $\angle A = 16°$

(c) side r in ΔRST to one decimal if $\angle S = 130°$, $s = 50$ mm, and $t = 20$ mm

Solution

(a) All three sides (SSS) are known in ΔABC. Use the cosine law to determine $\angle A$. Sketch the triangle.



$a^2 = b^2 + c^2 - 2bc \cos A$	Substitute known values.
$55^2 = 26^2 + 32^2 - 2(26)(32) \cos A$	Solve for $\cos A$.
$\cos A = \dfrac{55^2 - 26^2 - 32^2}{-2(26)(32)}$	Simplify.
$\cos A = -0.796\ 274\ 038\ 5$	Determine the value of A using $\cos^{-1}$.
$\angle A = \cos^{-1}(-0.796\ 274\ 038\ 5)$	
$\angle A \doteq 143°$	

(b) Two sides and an angle opposite one side are given (SSA). This is the ambiguous case of the sine law, where the given angle is acute. Check the relationship between a, b, and $b \sin A$.

$$a = 3,\ b = 15,\ \angle A = 16°$$

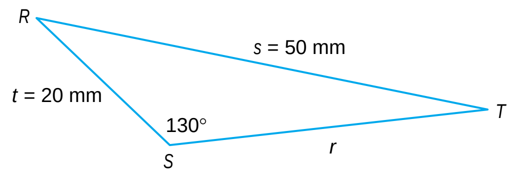
$$b \sin A = 15 \sin 16°$$
$$\doteq 4.1$$

Since $3 < 4.1$, then $a < b \sin A$. No triangle has these dimensions, so $\angle B$ cannot be determined.

Alternative solution using the sine law:

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$	Substitute known values.
$\dfrac{3}{\sin 16°} = \dfrac{15}{\sin B}$	Solve for $\sin B$.
$\sin B = \dfrac{15 \sin 16°}{3}$	Simplify.
$\sin B \doteq 1.378$	
$\angle B = \sin^{-1}(1.378)$	Solve for B using $\sin^{-1}$.
$\angle B =$ ERROR	Trying to determine the value of B using $\sin^{-1}$ on a calculator returns an error message, because the maximum of $\sin \theta$ is 1. $\angle B$ cannot be determined.

(c) Two sides and an angle opposite one side are known (SSA). This is the ambiguous case of the sine law, where the given angle is obtuse. In this case, $s > t$ (50 > 20), so only one obtuse triangle is possible. Sketch a diagram.



The ratio $\frac{r}{\sin R}$ has no known variables but must be eventually used to determine r. Also, $\angle R$ must be determined, but this cannot be done until $\angle T$ is known.

$$\frac{s}{\sin S} = \frac{t}{\sin T}$$ Substitute known values.

$$\frac{50}{\sin 130°} = \frac{20}{\sin T}$$ Solve for $\sin T$.

$$\sin T = \frac{20 \sin 130°}{50}$$ Simplify.

$$\sin T = 0.306\,417\,777\,2$$ Solve for T using $\sin^{-1}$.

$$\angle T = \sin^{-1}(0.306\,417\,777\,2)$$

$$\angle T \doteq 17.843\,481\,13°$$ Do not round because this value must be used to determine r.

Determine the value of R.

$$\angle R = 180° - (130° + 17.843\,481\,13°)$$
$$= 32.156\,518\,87°$$

To determine r, use the proportion $\frac{s}{\sin S} = \frac{r}{\sin R}$.

$$\frac{50}{\sin 130°} = \frac{r}{\sin 32.156\,518\,87°}$$

$$r = \frac{50 \sin 32.156\,518\,87°}{\sin 130°}$$

$$r \doteq 34.7$$

The value of r is about 34.7 mm.

Example 2

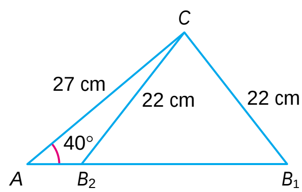
In ΔABC, $\angle A = 40°$, $a = 22$ cm, and $b = 27$ cm. Solve ΔABC. Round each angle to the nearest degree and each length to one tenth of a centimetre.

Solution

Two sides and an angle opposite one of the sides are given (SSA). This is the ambiguous case of the sine law where the given angle is acute. Check the relationship between a, b, and $b \sin A$.

We are given $a = 22$ cm and $b = 27$ cm. The value of $b \sin A$ must be determined.

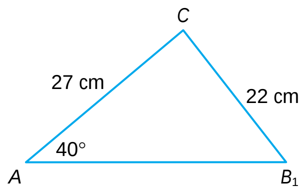
$$b \sin A = 27 \sin 40°$$
$$\doteq 17.4$$



Therefore, 17.4 < 22 < 27. As a result, $b \sin A < a < b$. There are two triangles to consider, one acute and one obtuse.

Case 1: ΔABC is acute.

Draw a well-labelled diagram.



Use the sine law to find $\angle B_1$.

$$\frac{a}{\sin A} = \frac{b}{\sin B_1}$$ Substitute known values.

$$\frac{22}{\sin 40°} = \frac{27}{\sin B_1}$$ Solve for $\sin B_1$.

$$\sin B_1 = \frac{27 \sin 40°}{22}$$ Simplify.

$$\angle B_1 = \sin^{-1}(0.788\,875\,702\,8)$$ Determine $\angle B_1$ using $\sin^{-1}$.

$$\angle B_1 \doteq 52°$$

Determine $\angle C$.

$$\begin{aligned}\angle C &= 180° - (40° + 52°)\\ &= 180° - 92°\\ &= 88°\end{aligned}$$

Determine side c using the sine law.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$ Substitute known values.

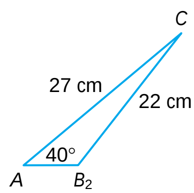
$$\frac{c}{\sin 88°} = \frac{22}{\sin 40°}$$ Solve for c.

$$c = \frac{22 \sin 88°}{\sin 40°}$$ Simplify.

$$c \doteq 34.2 \text{ cm}$$

Case 2: ΔABC is obtuse.

$\angle B_2$ is the supplement of $\angle B_1$ in Case 1. Therefore,



$$\begin{aligned}\angle B_2 &= 180° - 52°\\ &= 128°\end{aligned}$$

$$\begin{aligned}\angle C &= 180° - (40° + 128°)\\ &= 12°\end{aligned}$$

Determine the length of side c.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$ Substitute.

$$\frac{c}{\sin 12°} = \frac{22}{\sin 40°}$$ Solve for c.

$$c = \frac{22 \sin 12°}{\sin 40°}$$ Simplify.

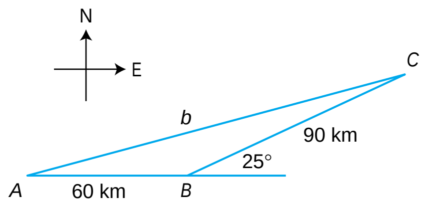
$$c \doteq 7.1 \text{ cm}$$

When ΔABC is acute, $\angle B = 52°$, $\angle C = 88°$, and $c = 34.2$ cm. When ΔABC is obtuse, $\angle B = 128°$, $\angle C = 12°$, and $c = 7.1$ cm.

Example 3

A boat travels 60 km due east. It then adjusts its course by 25° northward and travels another 90 km in this new direction. How far is the boat from its initial position to the nearest kilometre?

Solution



Draw a well-labelled sketch of the situation.

The distance required is side b in the diagram. In ΔABC, two sides and the angle between them are known (SAS). Use the cosine law to find side b.

$$\angle B = 180° - 25° = 155°$$

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \text{Substitute.}$$

$$b^2 = 90^2 + 60^2 - 2(90)(60) \cos 155° \quad \text{Simplify.}$$

$$b^2 = 8100 + 3600 - (10\,800)(-0.906\,307\,787)$$

$$b^2 = 21\,488.1241$$

$$b = \sqrt{21\,488.1241} \quad \text{Determine the square root and round.}$$

$$b \doteq 147$$

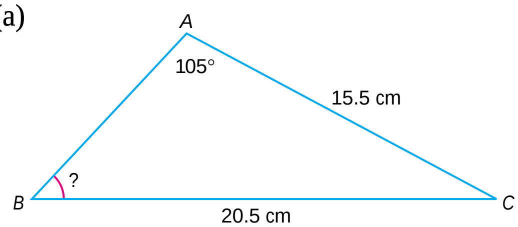
The boat is about 147 km from its initial position.

Practise, Apply, Solve 6.1

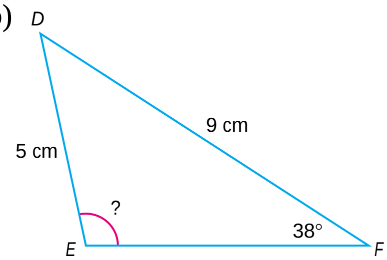
A

1. Determine the measure of the indicated angle to the nearest degree.

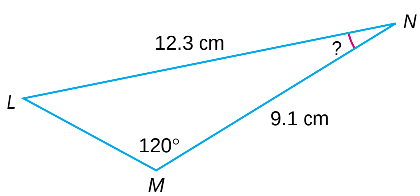
(a)



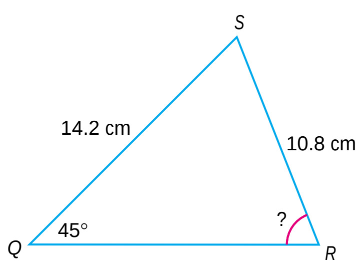
(b)



(c)

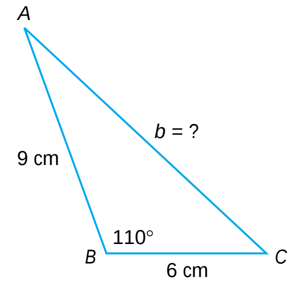


(d)



2. Determine the measure of the indicated side to one decimal place.

(a)



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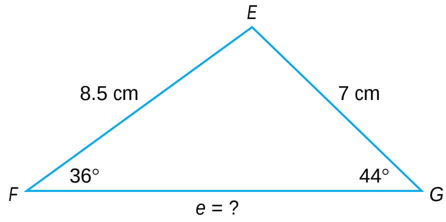
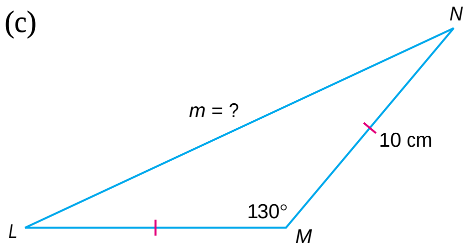
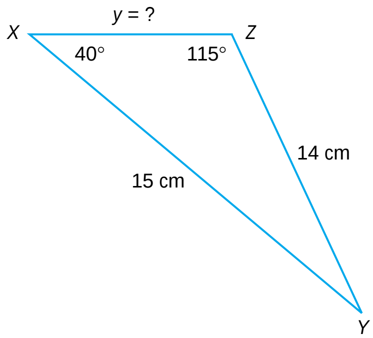


(c)

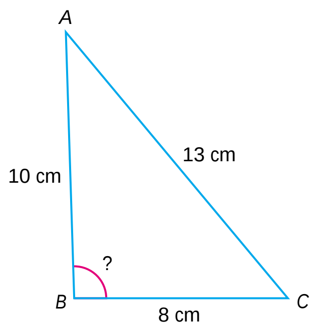


(d)

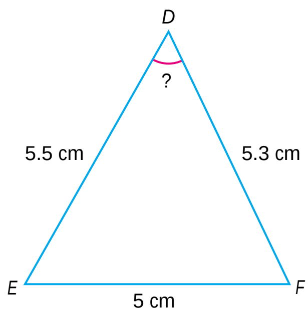


3. Determine the measure of the indicated angle to the nearest degree.

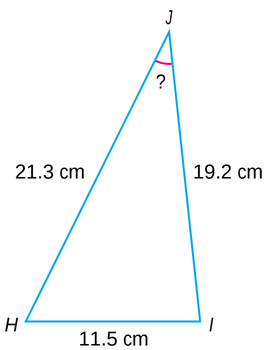
(a)



(b)



(c)



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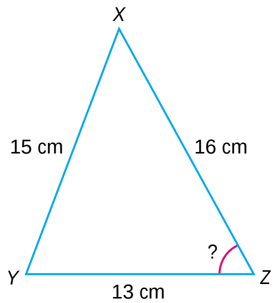
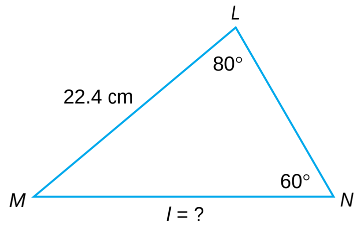


4. Determine the measure of the indicated side to one decimal place.

(a)



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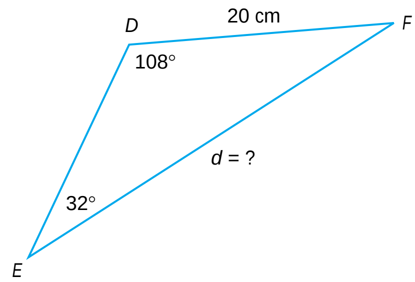

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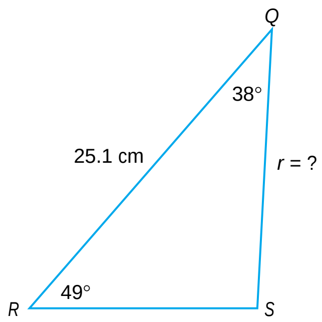

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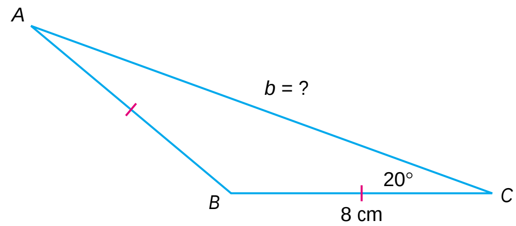
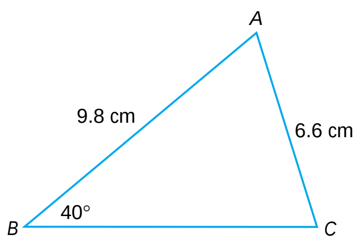


5. Each triangle is a rough sketch with the given information marked. Determine which triangles have no solution, one solution, and two solutions.

(a)



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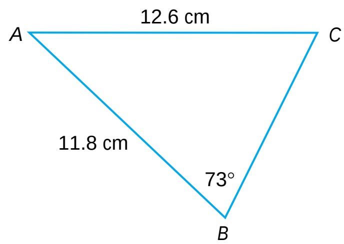

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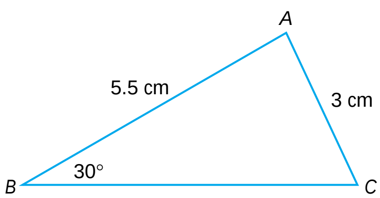

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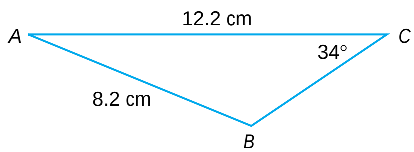


6. Given ΔABC.

i. sketch and label each triangle.
ii. calculate $b \sin A$ for each triangle.
iii. determine the number of possible solutions for the missing side, c.
iv. determine the length of side c to the nearest tenth of a unit.

(a) $a = 1.3$ cm, $b = 2.8$ cm, $\angle A = 33°$

(b) $a = 7.3$ m, $b = 14.6$ m, $\angle A = 30°$

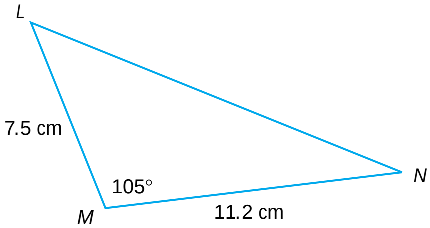
(c) $a = 7.2$ mm, $b = 9.3$ mm, $\angle A = 35°$

(d) $a = 24.3$ cm, $b = 17.2$ cm, $\angle A = 75°$

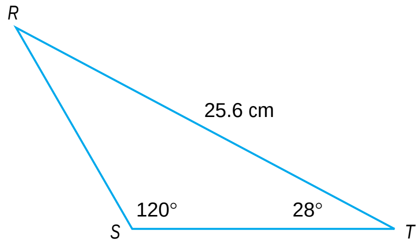
B

7. Solve each triangle. Express each angle to the nearest degree and each length to the nearest tenth of a unit.

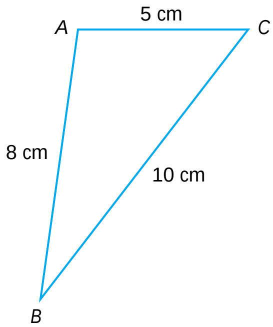
(a)



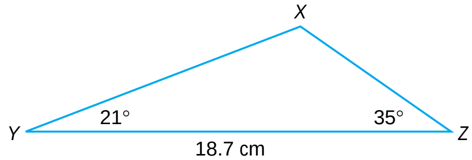
(b)



(c)



(d)



(e) ΔABC, $\angle A = 35°$, $\angle C = 40°$, $a = 12.8$ cm

(f) ΔLMN, $\angle L = 35°$, $m = 24$ cm, $n = 30$ cm

(g) ΔQRS, $\angle Q = 28°$, $\angle R = 60°$, $s = 15.2$ cm

(h) ΔDEF, $d = 12$ cm, $e = 14$ cm , $f = 16$ cm

8. Solve each triangle. Begin by sketching and labelling a diagram. Account for all possible solutions. Express each angle to the nearest degree and each length to the nearest tenth of a unit.

(a) ΔABC, $\angle A = 68°$, $a = 11.9$ cm, $b = 10.1$ cm

(b) ΔDEF, $\angle D = 52°$, $d = 7.2$ cm, $e = 9.6$ cm

(c) ΔHIF, $\angle H = 35°$, $h = 9.3$ cm, $i = 12.5$ cm

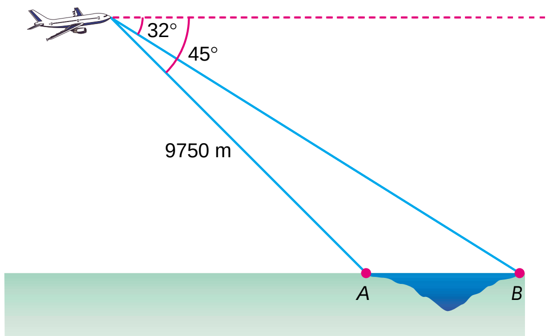
(d) ΔDEF, $\angle E = 45°$, $e = 81$ cm, $f = 12.2$ cm

(e) ΔXYZ, $\angle Y = 38°$, $y = 11.3$ cm, $x = 15.2$ cm

9. **Knowledge and Understanding:** In ΔDEF, $\angle D = 41°$, $d = 23$ cm, and $e = 27$ cm. Solve ΔDEF. Express each angle to the nearest degree and each length to one decimal place.

10. Show that the Pythagorean theorem is a special case of the cosine law. A diagram may be helpful.

11. From an airplane, a surveyor observes two points on the opposite shores of a lake, as shown. How far is it across the lake?



12. **Thinking, Inquiry, Problem Solving:** Mrs. Mardle says, "My lot is a triangular piece of property that is 430 m long on one side and the adjacent side is 110 m long. The angle opposite one of these sides is 35°."

(a) Is Mrs. Mardle right or wrong? Explain your reasoning.

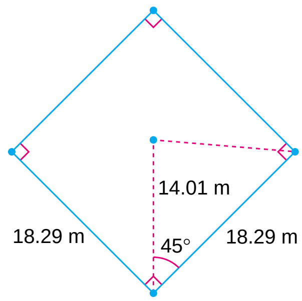
(b) Determine the other side and angles of this lot, if possible.

13. The pitcher's mound on a softball field is 14.01 m from home plate and the bases are 18.29 m apart.

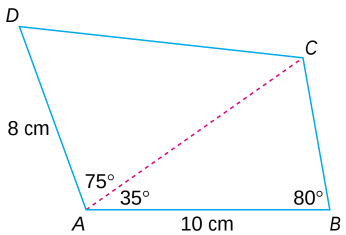
(a) How far is the pitcher's mound from first base?

(b) How far is first base from third base?

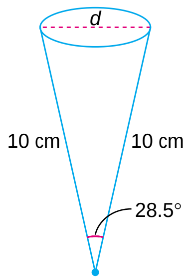
(c) How far is the pitcher's mound from second base?



14. Determine the length of *CD* to the nearest tenth of a metre.



15. A paper drinking cup is in the shape of a cone. The angle at the bottom of the cone is 28.5° and each side of the cup is 10 cm. Determine the diameter of the cup.



16. **Communication:** In ΔLMN, $\angle L$ is acute. Explain, with the help of a diagram, the relationship between $\angle L$, sides l and m, and the height of the triangle, for each of the following to occur.

(a) No triangle is possible.

(b) Only one triangle is possible.

(c) Two triangles are possible.

17. Two wires used to support a radio tower run along the same line but in opposite directions. They form an angle of 96° at the top of the tower. The wires are staked in the ground 25 m apart. One of the wires forms an angle of 44° with the ground.

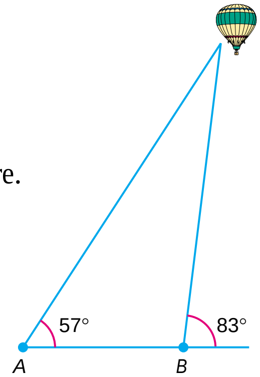
(a) How long is each wire?

(b) How high is the tower?

18. A sailboat on Lake Huron leaves Southampton and sails 20° west of north for 20 km. At the same time, a fishing boat leaves Southampton and sails 30° west of south for 15 km. At this point, how far apart are the boats to the nearest kilometre?

19. **Application:** A radio tower, at the top of a hill, casts a shadow 36 m long down the hill. The hill is inclined at an angle of 13° and the angle of elevation of the sun is 43°. How high is the tower?

20. From two different tracking stations, a weather balloon is spotted from two angles of elevation, 57° and 83°, respectively. The tracking stations are 15 km apart. Find the altitude of the balloon.



21. Find the perimeter of an isosceles triangle with a vertical angle of 100° and a base of 25 cm. Answer to the nearest tenth of a centimetre.

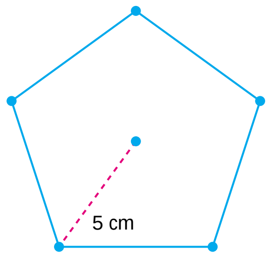
22. **Check Your Understanding:** An oblique triangle can be solved using the cosine law, the sine law, or a combination of both. Based on the given information, draw a triangle that represents each case. For each case, highlight all the given sides and angles on your diagram using a different colour. Under each diagram, summarize what could be found, the law that would be used, and the algebraic representation of the law.

C

23. The angles of a triangle are 120°, 40°, and 20°. The longest side is 10 cm longer than the shortest side. Find the perimeter of the triangle to the nearest hundredth of a centimetre.

24. In quadrilateral $QRST$, $QR = 3$ cm, $RS = 4$ cm, $ST = 5$ cm, and $TQ = 6$ cm. Also, diagonal RT is 7 cm. How long is the other diagonal to the nearest tenth of a centimetre?

25. A regular pentagon has all sides equal and all central angles equal. Calculate, to the nearest tenth, the area of the pentagon shown.



The Chapter Problem—What Time Is It?

CP1. Suppose a 50-cm long pendulum is pulled back 30 cm from the rest position. Determine the angle that the pendulum passes through on the first swing. That is, find the angle from its starting point to the point where it changes direction.

CP2. How far must the pendulum be pulled back to pass through

(a) a right angle? (b) an angle of 120°?

CP3. How long will the pendulum take to complete one swing? one period?