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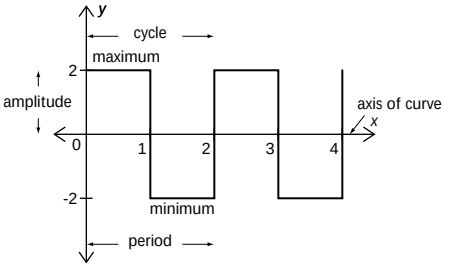
1. It has a repeating part.

2.



(a) cycle: exact portion of the curve that repeats
(b) period; length of one cycle
(c) amplitude: distance from axis of the curve to the maximum or minimum value
(d) axis of curve: horizontal line midway between maximum and minimum values of the curve
(e) maximum: uppermost point on the curve, minimum: lowest point on the curve

3. **(a)** Ice cream consumption follows a regular pattern as it increases from a low in the cooler months to a high in the warmer months and then decreases as the temperature cools. This cycle repeats each year.
(b) As temperatures cool, natural gases for heating purposes increase and as temperatures increase heating needs decrease. This cycle repeats each year.
(c) The pendulum swings from a point to the right of vertical, through the vertical position to a point on the left of vertical. It then swings back to its starting point.
(d) When the string is plucked it moves away from rest, then back through rest to the other side. This vibration, over a short period of time, could be considered as periodic although it will eventually stop.
(e) Temperatures rise and fall with the seasons in a predictable manner from one year to the next.
(f) The sound is repeated over and over again for the same length of time.

4. **(a)** The first cycle starts at $t = 0$ s and ends at $t = 10$ s. The minimum of the graph is V = 0 L and its maximum is V = 20 L. Repeat the given relation starting at $t = 10$ s.
(b) The first cycle starts at $t = 0$ S and ends at $t = 50$ s. The minimum of the graph is $h = 1$ m and its maximum is $h = 5$ m. Repeat the given relation starting at $t = 50$ s.

5. **(a)** maximum: 11 m, minimum: 1 m
(b) 60 s
(c) amplitude: 5 m, radius of the Ferris wheel
(d) $h = 6$ m

6. **(a)** vertex of angle is at origin and initial arm lies along positive x-axis
(b) angle is formed by a counterclockwise rotation of terminal arm
(c) angle is formed by a clockwise rotation of terminal arm
(d) angles that share the same terminal arm
(e) the angle θ between 0° and 360°
(f) the angle formed by terminal arm of an angle in standard position and x-axis

7. Quadrant I: 90° or $\frac{\pi}{2}$, Quadrant II: 180° or π, Quadrant III: 270° or $\frac{3\pi}{2}$, Quadrant IV: 360° or 2π

8. **(a)** rotate terminal arm clockwise to third quadrant, principal angle: 230°, related acute angle: 50°
(b) rotate terminal arm counterclockwise 360° + 140° to the second quadrant, principal angle: 140°, related acute angle: 40°
(c) rotate terminal arm clockwise 360° + 160° to the second quadrant, principal angle: 160°, related acute angle: 20°
(d) rotate terminal arm counterclockwise to the 4th quadrant, principal angle: 280°, related acute angle: 80°

9. **(a)** −658°, −298°, 62°, 422°, 782°, 1142°
(b) −497°, −137°, 223°, 583°

10. **(a)** −250°, 110°, 470°
(b) −530°, −170°, 190°, 550°

11. **(a)** Plot (−11, 14) on the coordinate grid. The principal angle is the angle between 0° and the terminal arm of the point P; Quadrant II
(b) 52° **(c)** 128°

12. **(a)** $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$, $x \neq 0$
(b) $(x, y) = (r\cos\theta, r\sin\theta)$

13. **(a)** Refer to Key Ideas 5.3 for graph of standard sine curve; extend domain from −360° to 360°.
(b) period: 360°, amplitude: 1, range = $\{y \mid -1 \leq y \leq 1, y \in \mathbf{R}\}$, axis: $y = 0$
(c) maximum: (−270°, 1), (90°, 1), minimum: (−90°, −1), (270°, −1), zeros:−360°, −180°, 0°, 180°, 360°

14. **(a)** Refer to Key Ideas 5.3 for graph of standard cosine curve; extend domain from −360° to 360°
(b) period: 360°, amplitude: 1, range = $\{y \mid -1 \leq y \leq 1, y \in \mathbf{R}\}$ axis: $y = 0$
(c) maximum: (−360°, 1), (0°,1), (360°, 1), minimum: (−180°, −1), (180°, −1), zeros: −270°, −90°, 90°, 270°

15. **(a)** Refer to Key Ideas 5.3 for graph of standard tangent curve; extend domain from −360° to 360°.
(b) $\tan\theta = \frac{y}{x}$, so function is undefined when $x = 0$
(c) There is a vertical asymptote where the function is undefined.

16. **(a)** $\sin\theta = -\frac{20}{\sqrt{689}}$, $\cos\theta = -\frac{17}{\sqrt{689}}$, $\tan\theta = \frac{20}{17}$
(b) $\theta = 230°$

17. 37.0 m

18. Both degrees and radians are used to determine the measure of an angle. Degree measure is based on a 360° rotation of a point around a circle and does not depend on the radius of the circle. Radian measure is also based on rotation about a circle but is dependent on the radius of the circle. For the unit circle, $\pi = 180°$.

20. **(a)** $\frac{\pi}{9}$, 0.3 **(b)** $\frac{-5\pi}{18}$, −0.9 **(c)** $\frac{8\pi}{9}$, 2.8
(d) $\frac{7\pi}{3}$, 7.3 **(e)** $\frac{-14\pi}{9}$, −4.9

21. **(a)** 45° **(b)** −225° **(c)** 480° **(d)** −120° **(e)** 330°

22. **(a)** 183°, rotate counterclockwise to between 8 and 9 o'clock
(b) −80°, rotate clockwise to between 5 and 6 o'clock
(c) 476°, rotate counterclockwise 360° to 11 o'clock
(d) 86°, rotate counterclockwise to between 12 and 1 o'clock
(e) 126°, rotate counterclockwise to 10 o'clock

23. Refer to Key Ideas 5.3 for graph of standard sine curve; extend domain from $\frac{-3\pi}{2}$ to $\frac{5\pi}{2}$.

24. **(a)** −5.4, −3.9, 0.8, 2.3 **(b)** −4.4, −1.9, 1.9, 4.4
(c) −4.9, −1.7, 1.4, 4.6

25. **(a)** 0.95 m
(b) 10 s; the ship rises and falls with the waves each 10 s
(c) 6.8 s, 8.2 s, 16.8 s, 18.2 s, 26.8 s, 28.2 s, 36.8 s, 38.2 s, 46.8 s, 48.2 s, 56.8 s, 58.2 s

26. The transformations are applied to the base graph of $y = \sin\theta$. a corresponds to the vertical stretch; for $a < 0$, the graph is reflected about the x-axis; the graph undergoes a horizontal stretch by

factor of $\frac{1}{k}$; b corresponds to the horizontal phase shift; $b > 0$ implies a phase shift of b units left, $b < 0$ implies a phase shift of b units right; d corresponds to a vertical translation; $d > 0$ indicates an upward shift of d units, $d < 0$ indicates a downward shift of d units.

27. **(a)** The period is 12, the number of months in a year.

(b) $y = \cos\theta$ has a maximum of 1 and a minimum of -1. Substitute -1 and 1 for $\cos\frac{\pi}{6}t$ to find the maximum and minimum temperatures.

28. **(a)** $\left(\frac{\pi}{2}, -3\right)$, $(\pi, 0)$ **(b)** $\left(\frac{\pi}{2}, 2\right)$, $(\pi, 1)$

(c) $\left(\frac{3\pi}{4}, 1\right)$, $\left(\frac{5\pi}{4}, 0\right)$ **(d)** $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{\pi}{2}, 0\right)$

29. **(a)** Apply the transformations as follows: horizontal stretch, phase shift, vertical stretch, vertical shift amplitude $= 2$, phase shift $= \frac{\pi}{6}$ right, period $= \frac{2\pi}{3}$, vertical shift $= -4$, min. $(0, -6)$, $\left(\frac{2\pi}{3}, -6\right)$, $\left(\frac{4\pi}{3}, -6\right)$, $(2\pi, -6)$, max. $\left(\frac{\pi}{3}, -2\right)$, $(\pi, -2)$, $\left(\frac{5\pi}{3}, -2\right)$, axis of symmetry $y = -4$

(b) Apply the transformations as follows: horizontal stretch, phase shift, vertical stretch, vertical shift; amplitude $= 3$, reflection in the y-axis, phase shift $= \frac{\pi}{4}$ left, period $= \pi$, vertical shift $= 2$, min. $\left(\frac{3\pi}{4}, -1\right)$, $\left(\frac{7\pi}{4}, -1\right)$, max $\left(\frac{\pi}{4}, 5\right)$, $\left(\frac{5\pi}{4}, 5\right)$, axis of symmetry $y = 2$

30. **(a)** amplitude: 2, period: 720°, phase shift: 120° right, vertical shift: $+1$

(b) amplitude: 3, period: 180°, phase shift: 45° left, vertical shift: -1

(c) amplitude: 2, period: $\frac{2\pi}{3}$, phase shift: $\frac{\pi}{6}$ right, vertical shift: $+1$

31. **(a)** Graphing the data gives the general shape of the trigonometric model and whether or not it is sinusoidal. It shows the maximum and minimum value and a starting point for the typical sinusoidal curve. These are necessary when completing the equations $y = a\sin k(\theta + b) + d$ or $y = a\cos k(\theta + b) + d$.

(b) The four key pieces of information are amplitude, phase shift, period, and vertical shift (axis of the curve).

amplitude $= \frac{\text{maximum} - \text{minimum}}{2}$, period $= \frac{2\pi}{k}$, vertical shift $= \frac{\text{maximum} + \text{minimum}}{2}$, phase shift: determine where the starting point of the graph has moved. If the starting point is right of 0, $b < 0$. If the starting point is left of 0, $b > 0$.

32. **(a)** $f(\theta) = 4\sin 3\theta$ **(b)** $f(t) = \cos\frac{\pi}{6}t + 5$

33. **(a)** Ice cream production rises to a maximum with warmer weather and then falls to a minimum with cooler weather. It is cyclical.

(b) $p(t) = 54\sin\frac{\pi}{6}(t - 3) + 222$ or $-54\cos\frac{\pi}{6}t + 222$

(c) The scatter plot and the graph follow the same pattern with the graph touching over 70% of the data points.

34. (1) Sketch the graph and interpolate. (2) Find points of intersection between two corresponding functions that make up the original equation. (3) Determine the zeros of the single corresponding function.

35. **(a)** yes **(b)** no

36. **(a)** 41.8°, 138.2° **(b)** 128.7°, 231.3° **(c)** 70.3°, 250.3°
(d) no solution **(e)** 132.8° **(f)** 133.6°

37. **(a)** 0.51, 5.78 **(b)** 3.42, 6.01
(c) 0.39, 1.96, 3.53, 5.10 **(d)** 4.71
(e) 0.39, 1.71, 2.48, 3.80, 4.58, 5.90
(f) 2.33

38. **(a)** 53.1°, 126.9° **(b)** 104.5°, 255.5°
(c) 71.6°, 251.6° **(d)** 30.0°, 150.0°, 210.0°, 330.0°

39. **(a)** 0.46, 3.61 **(b)** 1.05, 3.14, 5.24
(c) 0.52, 2.62, 4.71 **(d)** 1.57, 4.71

40. **(a)** maximum: July, minimum: Jan.
(b) Dec. and Jan **(c)** May and Aug.