5.1 Periodic Phenomena

Part 1: Investigating Periodic and Non-Periodic Models

Consider what kind of mathematical model would apply in each case.

1. A model rocket is launched. The table records its height above ground.

Time (s)	0	1	2	3	4	5	6
Height (m)	0	25	40	45	40	25	0

2. A technician counts the number of bacteria in a petri dish every hour for 6 h.

Time (h)	0	1	2	3	4	5	6
Bacteria Count	1500	2900	6100	12 400	25 100	49 800	100 500

3. A coach measures the velocity of air as a gymnast inhales and exhales after working out.

Time (s)	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Velocity of Air (L/s)	1.75	1.24	0	−1.24	−1.75	−1.24	0	1.24	1.75

Time (s)	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
Velocity of Air (L/s)	1.24	0	−1.24	−1.75	−1.24	0	1.24	1.75

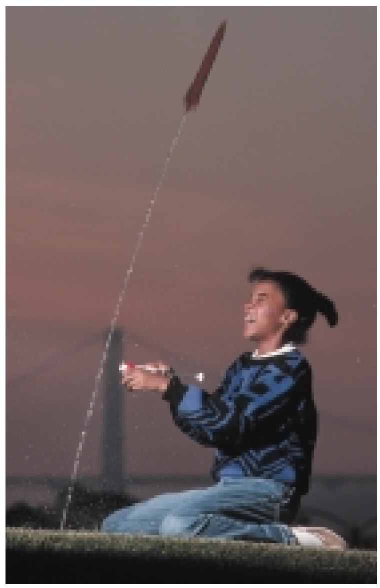
Think, Do, Discuss

1. (a) Draw and label a separate scatter plot for each situation.

(b) Which situation can be modelled with an exponential relation? Justify your answer. Draw the curve of best fit and comment on how well the model fits the data.

(c) Which situation can be modelled with a quadratic relation? Justify your answer. Draw the curve of best fit and comment on how well the model fits the data.

(d) Which situation is neither exponential nor quadratic but is still nonlinear? Draw a curve of best fit to match the data points. How well does the curve fit the model?



2. Use your graphs from step 1 to answer these questions.
   (a) How high is the rocket after 4.5 s?
   (b) About how many bacteria should there be after 8 h?
   (c) What is the velocity of the air after 6.75 s? Is the air being inhaled or exhaled at this time? Explain.

Steps 3 to 6 refer to the gymnast's record of breathing.

3. (a) How long does it take for one complete cycle of inhaling and exhaling?
   (b) What is the maximum velocity of the air? What is the minimum velocity?
4. (a) What is the velocity of air for the gymnast after 13 s? after 16 s?
   (b) Explain how the graph was extended to extrapolate the required information.
5. (a) Describe the shape of the graph of the gymnast's breathing.
   (b) Explain why the shape of the curve is reasonable for this case.
6. As time goes on, the gymnast will relax. What changes would you expect to see in the graph?

Part 2: Repeating Functions

Look up at the moon on a clear night. Sometimes the moon is full and the night sky is bright. At other times, there is a new moon with no visible light and the sky is dark. The moon is said to wax from dark to bright and wane back to dark.

Fraction of the Moon Visible at Midnight
Days 1 to 66 of the Year 2000

Day of the Year	1	2	3	4	5	6	10	15	20	21
Fraction of Moon Visible	0.25	0.18	0.11	0.06	0.02	0.00	0.11	0.57	0.99	1.00

Day of the Year	25	30	35	40	45	50	55	60	65	66
Fraction of Moon Visible	0.80	0.32	0.02	0.14	0.64	1.00	0.77	0.31	0.01	0.00

Source: US Naval Observatory, Washington.

Think, Do, Discuss

1. (a) Draw and label a scatter plot of the data.
   (b) Draw the curve of best fit.
2. (a) Starting with day 1, how many days does it take for the shortest complete pattern of the graph to repeat?
   (b) Starting with day 6, how many days does the graph take to repeat?
   (c) On what other day could the graph begin and still repeat?

3. (a) Extend the pattern of the graph to include the 95th day of the new millennium. Was the phase of the moon closer to a full moon or a new moon? Explain.
   (b) Extend the graph to predict the fraction of the moon that was visible on the summer solstice, June 21. Was the moon waxing or waning? Explain.

Part 3: Properties of Repeating Functions

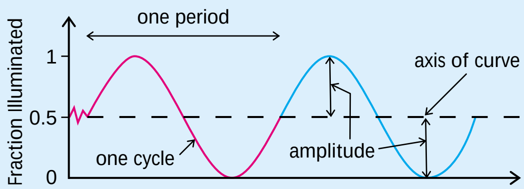
Graphs of periodic functions are self-replicating. That is, they repeat themselves over and over again. What are some other properties of periodic functions?

Think, Do, Discuss

1. Redraw the graph of the phases of the moon for the first three months of the year.
2. What is the maximum value of the curve? the minimum value? Explain the meaning of maximum and minimum in this case.
3. (a) Draw a straight, horizontal line through the graph that is halfway between the maximum and minimum values. This line is called the **axis of the curve** or **axis of the shape**. Write the equation of the axis of the curve using the maximum and minimum values.
   (b) The **amplitude** of a periodic function is the magnitude of the distance from this line to either the maximum or minimum value. How can the maximum and minimum values be used to calculate the amplitude?
   (c) Draw one complete cycle of the graph that begins on the line and ends on the line. Draw a different cycle that also begins and ends on the line. Explain how the two cycles are alike and not alike.

Focus 5.1

Key Ideas



- Repeating data forms a periodic function.
- A periodic function has a self-repeating graph.
- The cycle of a graph is the smallest complete repeating pattern of the graph.
- The length of one cycle is called the **period**.
- The horizontal line that is halfway between the maximum and minimum values of a periodic curve is called the **axis of the curve**.
- The equation of the axis of the curve is

$$y = \frac{\text{maximum value} + \text{minimum value}}{2}$$

- The magnitude of the vertical distance from the axis of the curve to either the maximum or minimum value is called the **amplitude** of the function. The amplitude, a, is calculated as

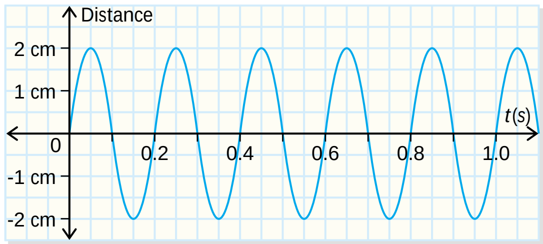
$$a = \frac{\text{maximum value} - \text{minimum value}}{2}$$

Example 1

Determine whether each graph is periodic or not.

(a) tip of vibrating meter stick

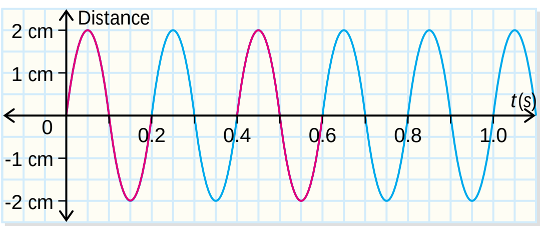
(b) movement of a piston in a combustion engine



Solution

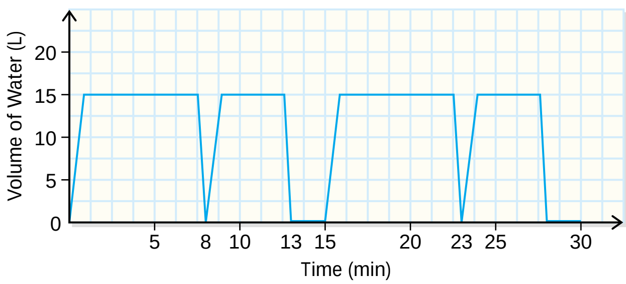
(a) The vibrating metre stick is not periodic because the graph does not repeat. The maximum and minimum values keep changing and it takes less and less time for the curve to complete one cycle.

(b) The movement of a piston is periodic because the graph repeats every 0.2 s and each cycle is exactly the same. The period is 0.2 s, the amplitude is 2 cm.



Example 2

The automatic dishwasher in a school cafeteria runs constantly through lunch. The graph shows the amount of water used as a function of time.



(a) Explain why the operation of the dishwasher is an example of a periodic function.

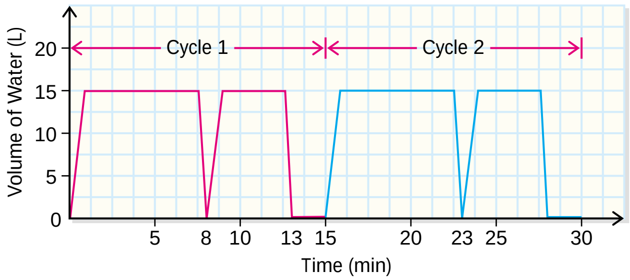
(b) What is the length of the period? What does one complete cycle mean in the context of the question?

(c) Extend the graph for one more complete cycle.

(d) How much water is used if the dishwasher runs through eight complete cycles?

Solution

(a) A periodic function has a repeating graph and each cycle is the same. The graph of the dishwasher repeats and each cycle is the same. This function is periodic.

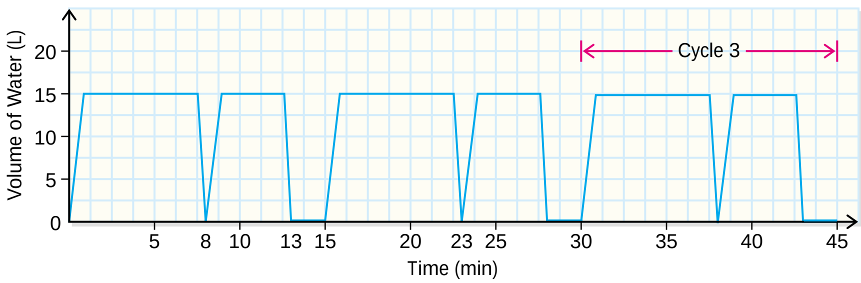


(b) The period is the length of one complete cycle. Calculate the period by subtracting the beginning time of a cycle from the ending time of a cycle.

$$15 - 0 = 15, \text{ or } 30 - 15 = 15$$

One complete cycle could mean an 8-min wash, followed by a 5-min rinse, followed by 2 min to unload and load dishes.

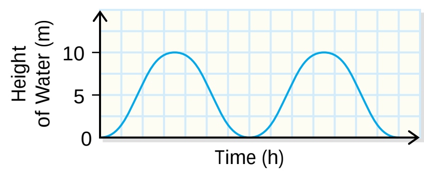
(c)



(d) The dishwasher uses 15 L to wash and another 15 L to rinse. Then eight cycles use $8(15 + 15) = 8(30)$. The washer uses 240 L.

Example 3

The Bay of Fundy, which is between New Brunswick and Nova Scotia, has the highest tides in the world. There can be no water on the beach at low tide, while at high tide the water covers the beach.



(a) Why can you use periodic functions to model the tides?

(b) What is the change in depth of water from low tide to high tide?

(c) Determine the equation of the axis of the curve.

(d) What is the amplitude of the curve?

Solution

(a) Tides resemble periodic functions because they repeat over a fixed interval of time.

(b) The water level at low tide is zero. The water level at high tide is 10 m. The change is 10 m. $10 - 0 = 10$

(c) The equation of the axis of the curve is $h = \frac{(10 + 0)}{2}$. Therefore, $h = 5$.

(d) The amplitude is

$$a = \frac{(10 - 0)}{2}$$
$$= 5$$

Practise, Apply, Solve 5.1

A

1. Match each situation with its graph. Describe what type of model best describes each situation.

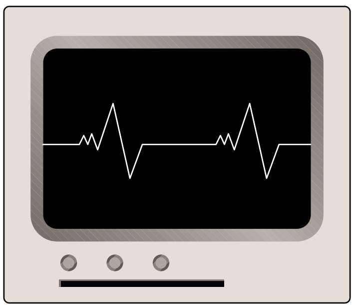
(a) the height of a shot put

(b) a record of a boy's heartbeat

(c) the distance a pendulum travels from its rest position

(d) the cost of a taxi ride

(e) the growth in the number of bacteria

i.


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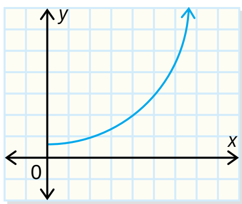

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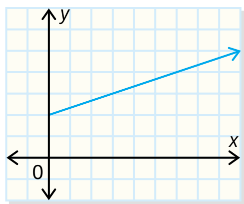

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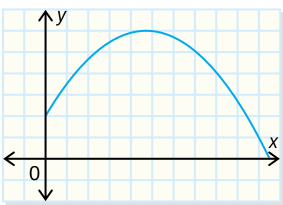

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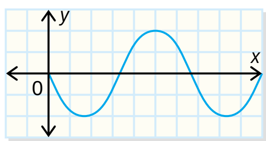


2. **Communication:** Explain why a periodic model can represent each situation.

(a) the average monthly high temperature in Sudbury

(b) the relationship between rabbit and coyote populations

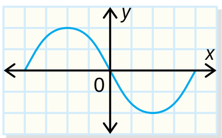
(c) the average monthly water level in Lake Superior

(d) the position of the sun at sunrise in relation to due east

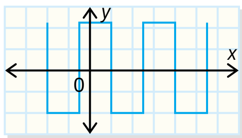
(e) the vibration of a tuning fork

3. Determine whether each graph is periodic or not. Justify your answer.

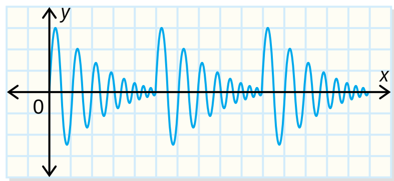
(a)



(b)

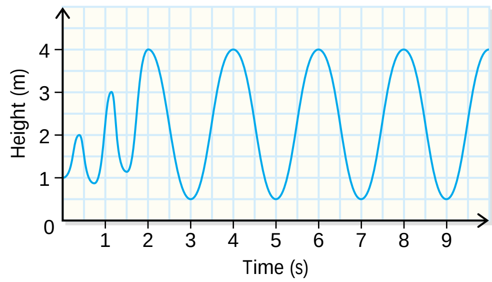


(c)



B

4. Nolan is jumping on a trampoline. The graph shows how high his feet are above the ground.



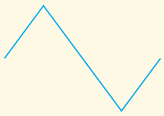
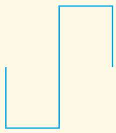
(a) How long does it take for Nolan's jumping to become periodic? What is happening during these first few seconds?

(b) How long is the period of the curve? Explain the meaning of period in the context of the problem.

(c) Write an equation for the axis of the periodic portion of the curve.

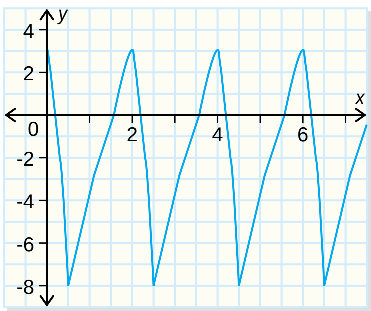
(d) What is the amplitude of the curve? Explain the meaning of amplitude in the context of the problem.

5. Sketch periodic graphs to satisfy the given properties.

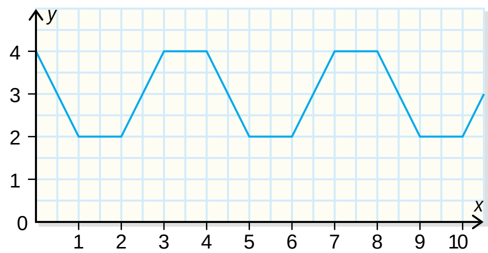
Shape	Period	Amplitude	Equation of Axis	Number of Cycles
	4	6	$y = 2$	2
	3	4	$y = 1$	3
	$\frac{1}{2}$	5	$y = -3$	2

6. State the period, amplitude, and the equation of the axis for each function.

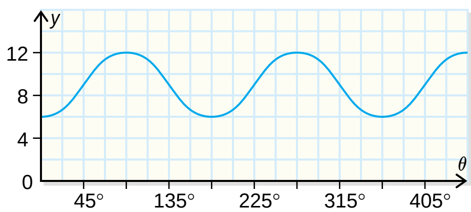
(a)



(b)



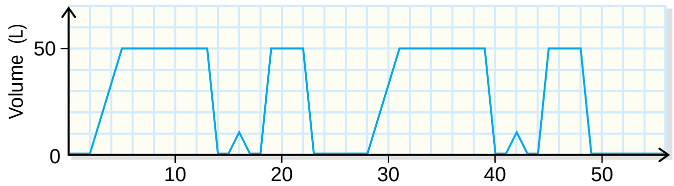
(c)



7. Draw an example of a periodic graph. Identify the properties of shape, period, amplitude, and axis of the shape. Show another student the graph and ask him or her to verify the four properties.

8. Create three differently shaped periodic graphs with amplitude 4 and period 5. Show two cycles of the graph.

9. **Application:** The graph shows the number of litres of water that a washing machine uses over several hours.



(a) There are several parts to each complete cycle of the graph. Explain what each part could mean in the context of "doing the laundry."

(b) What is the period of one complete cycle?

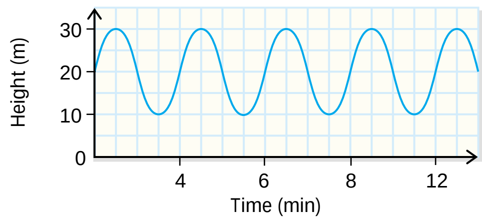
(c) What is the maximum volume of water used for each part of the cycle?

(d) What is the total volume of water used for one complete cycle?

(e) What volume of water represents the axis of the graph?

(f) State the amplitude of the graph.

10. **Knowledge and Understanding:** The graph shows John's height above the ground as a function of time as he rides a Ferris wheel.



(a) State the maximum and minimum height of the ride.

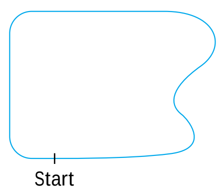
(b) How long does the Ferris wheel take to make one complete revolution?

(c) What is the amplitude of the curve? How does this relate to the Ferris wheel?

(d) Determine the equation of the axis of the curve.

11. Each year, around February 18, swallows travel 12 000 km from Goya, Argentina, to San Juan, Capistrano, California. They arrive in Capistrano about March 19 and stay until about October 23. Then they leave for their month-long journey back to Goya. Explain why the migration of the swallows of Capistrano is an example of periodic phenomena.

12. **Thinking, Inquiry, Problem Solving:** A race car driver is qualifying on a 3.2 km track as shown. He tries to drive each lap exactly the same way, slowing down at the corners and accelerating through the straightaways.

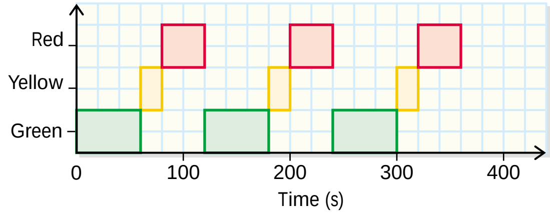


(a) Graph speed versus time for one lap around the track.

(b) Extend the graph to represent three laps around the track

(c) What does the period represent in this situation?

13. A traffic light changes colour over time as shown.



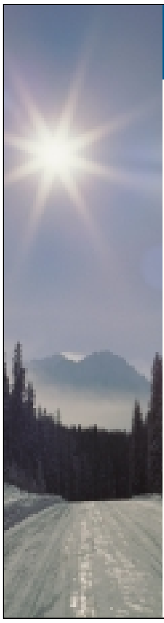
(a) Explain why the graph represents a periodic relation.

(b) Describe one complete cycle.

(c) What is the period of the graph?

(d) A 20-s advanced green arrow is added to the beginning of the cycle. What is the period now? Draw two full cycles of the graph.

14. **Check Your Understanding:** Write a definition for a periodic function. Use the concept of an independent variable and a dependent variable in your definition. Include an example of a periodic function and use your definition to explain why it is periodic.



The Chapter Problem—How Much Daylight?

In this section, you studied periodic phenomena. Apply what you learned to answer these questions about the Chapter Problem on page 404.

CP1. How can Naomi use the data in the table to schedule her guided fishing trips and maximize the possible catch within the fishing regulations on catch limits per day?

CP2. Explain why the data is periodic. What is the period of the data?

CP3. State the domain and range of the data.

CP4. (a) Graph average hours of daylight versus month. Refer to the start of the data as $t = 0$.

(b) What is the equation of the axis of the curve?

(c) What are the maximum and minimum values?

(d) What is the amplitude of the graph?