Practise, Apply, Solve 3.2, page 234

1. **(a)** D = {1, 3, 4, 5}, R = {3, 4, 5, 6, 7, 8}
(b) 1 to 3, 1 to 6, 3 to 5, 4 to 7, 5 to 4, 5 to 8
(c) not a function: 1 and 5 both map onto two different range values

2. **(a)** D = {1, 3, 4, 7}, R = {1, 2}; function: each domain value maps onto one range value
(b) D = {1, 4, 6}, R = {1, 2, 3, 5}; not a function: 1 maps onto both 2 and 3
(c) D = {0, 1, 2, 3}, R = {0, 1, 2, 3}; function: each domain value maps onto one range value
(d) D = {1, 2, 3}, R = {4, 5, 6}; not a function: 1 maps onto both 4 and 5
(e) D = {1, 2, 3}, R = {4}; function: each domain value maps onto one range value
(f) D = {2}, R = {4, 5, 6}; not a function: 2 maps onto different values

3. **(a)** $\sqrt{25} = 5$ **(b)** no
(c) When the square root button on the calculator is used, the output must be consistent. It must be a function.
(d) negative number **(e)** D = $\{x \mid x \geq 0, x \in \mathbf{R}\}$

4. **(a)** 25 maps onto ±5. Both 5 and −5 are valid square roots of 25.
(b) Every positive real number produces 2 different output values.
(c) not a function: at least one input value with more than one output value.
(d) No negative numbers can be used as input.

5. **i.** D = {−2, 0, 2, 3, 5, 6}, R = {1, 2, 3, 4, 5}
ii. 4, 2, 5, −2;
iii. They are not the same function because of order of operations.
iv. $f(2) = 5$ corresponds to (2, 5). 2 is x-coordinate of the point and $f(2)$ is y-coordinate of it.

6. **(a)** −3 **(b)** 3 **(c)** 7 **(d)** 2
(e) 4 **(f)** 13 **(g)** $3 - 2a$

7. **(a)** not a function: fails vertical line test
(b) function: passes vertical line test
(c) function: passes vertical line test
(d) not a function: fails vertical line test at end points
(e) not a function: fails vertical line test at right-hand part of curve
(f) not a function: fails vertical line test at most domain values
(g) function: passes vertical line test
(h) not a function: fails vertical line test at most domain values
(i) not a function: fails vertical line test (in a region, there are an infinite number of points above each other)

8. **i.** straight line through (0, 5) and $\left(-\frac{5}{3}, 0\right)$
ii. **(a)** 5 **(b)** 8 **(c)** 11 **(d)** 14 **(e)** 3
(f) 3 **(g)** 3 **(h)** 3 **(i)** 3 **(j)** 3
iii. first differences, part (j) is the slope of the straight line between (0, $g(0)$) and (4, $g(4)$). Since it is a linear relation, the slope is equal to the first differences.

9. **(a)** curve in first quadrant sloping down to right through (1, 1); curve in third quadrant through (−1, 1);
D = $\{x \mid x \neq 0, x \in \mathbf{R}\}$, R = $\{y \mid y \neq 0, y \in \mathbf{R}\}$; f is a function: graph passes vertical line test.
(b) straight line through (0, 6) and (3, 0); D = **R**, R = **R**; function: every x-value corresponds to one y-value.
(c) straight line through (0, 2) and (6, 0); D = **R**, R = **R**; function: passes vertical line test
(d) parabola opening upward with vertex (0, 3) through (−1, 4) and (1, 4): D = **R**, R = $\{y \mid y \geq 3, y \in \mathbf{R}\}$; function: passes vertical line test

10. There may be several brands of tires in the size requested, so different prices are possible for one tire size: therefore not a function. To be a function, the input needs to be more specific, so only one tire can be identified.

11. **i.** $y = 2x + 3$ **ii.** $g(x) = 2x + 3$ **iii.** 13, 7, 4, 13

12. **ii.** **(a)** 9 **(b)** 4 **(c)** 1 **(d)** 0 **(e)** 1
(f) −5 **(g)** −3 **(h)** −1 **(i)** 1 **(j)** 2
(k) 2 **(l)** 2 **(m)** 2
iii. first differences, second differences

13. **(a)** 49
(b) y-coordinate of the point on the graph with x-coordinate −2
(c) D = **R**, R = $\{y \mid y \geq -1, y \in \mathbf{R}\}$
(d) function: graph passes vertical line test
(e) The equation shows that every value of x yields only one possible value for $f(x)$.

14. **(a)** no
(b) This must be a function because of the answer to part (a). Its graph is a parabola that opens up, so it passes vertical line test.

15. **(a)** circle centred on origin, radius 5
(b) not a function: fails vertical line test for all values of x between −5 and 5.

16. **(a)** straight line through (0, −1) and (2, 5); D = **R**, R = **R**; function: passes vertical line test
(b) parabola opening downward with vertex (0, 10) through (−1.4, 0) and (1.4, 0); D = **R**, R = $\{y \mid y \leq 10, y \in \mathbf{R}\}$; function: passes vertical line test
(c) parabola opening upward with vertex ((2, −5) through (0, 7) and (4, 7); D = **R**, R = $\{y \mid y \geq -5, y \in \mathbf{R}\}$; function: passes vertical line test
(d) curve in first quadrant down to right through (1, 0), curve in fourth quadrant down to left through (−1, 1);
D = $\{x \mid x \neq 0, x \in \mathbf{R}\}$, R = $\{y \mid y > 0, y \in \mathbf{R}\}$; function: passes vertical line test
(e) parabola opening upward with vertex (1.5, −2.25) through (0, 0) and (3, 0); D = **R**, R = $\{y \mid y \geq -2.25, y \in \mathbf{R}\}$; function: passes vertical line test
(f) parabola opening upward with vertex (2, −4) through (0, 0) and (4, 0); D = **R**, R = $\{y \mid y \geq -4, y \in \mathbf{R}\}$; function: passes vertical line test
(g) line through (0, 5) and (3, 0); D = **R**, R = **R**; function: passes vertical line test

17. D = $\{x \mid x \geq 1\}$, R = $\{y \mid y \geq 2, y \in \mathbf{R}\}$

18. **(a)** If this relationship were not a function, customers would become upset if they found out that someone else received a better price for the same number of meals served.
(b) D = $\{x \mid x\ 0 \leq x \leq 200, x \in \mathbf{R}\}$,
R = $\{y \mid 1250 \leq y \leq 3700, y \in \mathbf{R}\}$
(c) The domain must have an upper limit because the room has a maximum seating capacity.
(d) Reasonable: the hall's owners would want to receive a minimum total payment. If fewer than 50 people attend, they will charge $1250 because they could rent the room to a larger group and make more money. They do not want to reserve a room, then discover only 10 people are coming − and only receive payment for those 10.

19. **(a)** parabola opening down, vertex (1.5, 20) through (0, 2) and (3, 0)
(b) D = $\{t \mid 0 \leq t \leq 3.1, t \in \mathbf{R}\}$, R = $\{h \mid 0 \leq h \leq 20, h \in \mathbf{R}\}$
(c) is a function: every value of t corresponds to a unique value of h.
(d) parabola opening to left with vertex (20, 1.5) through (0, 3) and (2, 0)

(e) not a function: each height between 1.6 m and 20 m corresponds to two different times; fails vertical line test

20. **(a)** Every quantity ordered must have a unique, reliable price.

(b) $D = \{x \mid x \geq 0, x \in \mathbf{R}\}$, $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$

(c) straight line from (0, 0) to (100, 400) (open dot); straight line at same angle from (100, 350) up

(d) 99 kg could cost more than 400 kg. Sample change: charge \$4/kg for the first 100 kg and then \$3.50/kg for the part of the order over 100 kg.

21. **(a)** $D = \{s \mid 0 \leq s \leq 16\,000, s \in \mathbf{R}\}$, $R = \{p \mid 300 \leq p \leq 1200, p \in \mathbf{R}\}$

(b) To ensure fairness: two employees with the same sales should receive the same pay.

(c) *Example*: weekly pay of \$300 for sales under \$2000; \$200 plus 5% commission for sales from \$2000 to \$8000; 7.5% sales commission for sales over \$8000.

22. **(a)** curved line up to right

(b) $D = \{i \mid 0 \leq i \leq 0.2, i \in \mathbf{R}\}$, $R = \{A \mid 10\,000 \leq A \leq 25\,000, A \in \mathbf{R}\}$

23. **(a)** Age is the independent variable, so if two members of the family are the same age and have different foot lengths, the relationship will not be a function. If they are all different ages, the relationship will be a function.

(b) Some of Sarah's friends are likely to be her age and are likely to have feet of different lengths, so the relationship will not be a function.

(c) The line of best fit will be a function.

(d) no; a person's age cannot be determined from foot length

24. *Example*: function: (x, y): (0, 3), (1, 4), (2, 5) Each value of x corresponds to a unique value of y. non-function: (x, y): (0, 3), (0, 4), (2, 5) The 0 corresponds to both 3 and 4.

25. **(a)** 13

(b) function: each binary number is converted to a unique decimal number

(c) 21, 3, 8 **(d)** $R = \{y \mid y \leq 15, y \in \mathbf{W}\}$ **(e)** 8 bits

26. **(a)** Answers will vary.

(b) rand does not appear to produce a unique output; for truly random numbers an unpredictable output is wanted.

(c) 0.943 597 402 5, 0.908 318 861, 0.146 687 829 2, 0.514 701 950 5, 0.405 809 641 8. These values are the same for everyone.

(d) This list is the same as before.

(e) For 0.4: 0.943 597 402 5, 0.908 318 861, 0.146 687 829 2
For 0.8: 0.943 597 402 5, 0.908 318 861, 0.146 687 829 2
For 1.2: 0.745 560 772 8, 0.855 900 597 1, 0.225 360 061 7
For 1.6: 0.745 560 772 8, 0.855 900 597 1, 0.225 360 061 7
For 2.0: 0.491 121 545 1, 0.711 801 117 9, 0.450 720 047 5
For 2.4: 0.491 121 545 1, 0.711 801 117 9, 0.450 720 047 5
For 2.8: 0.491 121 545 1, 0.711 801 117 9, 0.450 720 047 5

(f) The calculator truncates (eliminates any decimals) from the number stored to rand, then generates a list of numbers using that seed number. The random number obtained depends on the seed number and the number of times rand is chosen.

(g) Yes: rand is a function of the seed number and the number of times it is selected. For example, when 0 is stored in rand, and rand is selected 3 times, 0.146 687 829 2 is always produced. These are not random numbers. If an experiment is done using random numbers from the calculator, the result will not be random, but predictable.